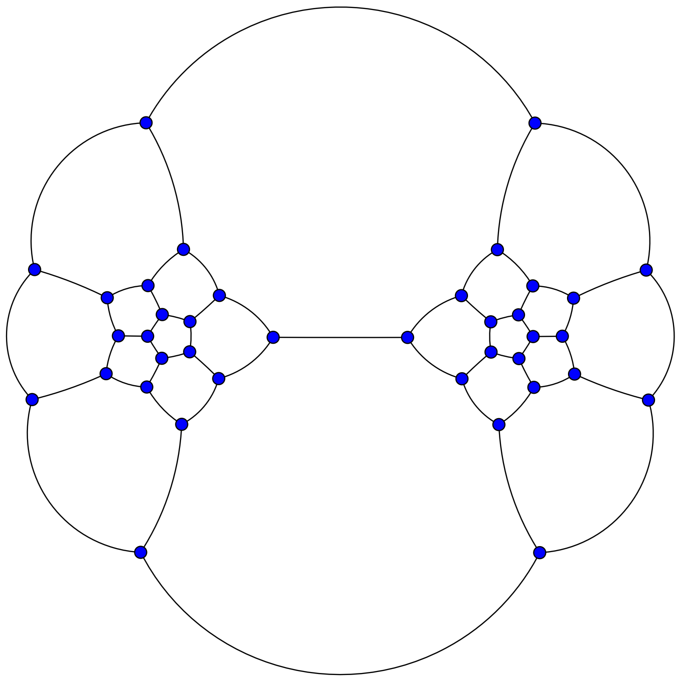
2nd Croatian Combinatorial Days, Zagreb, September 2018

CroCoDays 2018



Book of Abstracts

IMPRESSUM

Name of the conference: CroCoDays 2018 – 2nd Croatian Combinatorial Days
Organizer: Faculty of Civil Engineering, University of Zagreb
Place: Kačićeva 26, 10000 Zagreb, CROATIA
Dates: September 27–28, 2018

ORGANIZING AND PROGRAM COMMITTEE:
Tomislav Došlić, Zagreb, Croatia (Chairman)
Alan Filipin, Zagreb, Croatia
Snježana Majstorović, Osijek, Croatia
Ivica Martinjak, Zagreb, Croatia
Philippe Nadeau, Lyon, France
Riste Škrekovski, Ljubljana, Slovenia
Damir Vukičević, Split, Croatia
Ivana Zubac, Mostar, Bosnia Herzegovina
Petra Žigert Pleteršek, Maribor, Slovenia

LOCAL ORGANIZING COMMITTEE:
Nikola Adžaga
Tomislav Došlić
Alan Filipin
Ivica Martinjak
Ana Martinčić Špoljarić

EDITORS:
Tomislav Došlić, Ivica Martinjak

PUBLISHER:
Faculty of Civil Engineering, University of Zagreb

SPONSORS:
Faculty of Civil Engineering, University of Zagreb
Foundation of Croatian Academy of Sciences and Arts
Croatian Science Foundation – research project LightMol (Grant no. IP-2016-06-1142)

Abstracts were prepared by the authors.

Contents

Contributed talks

Detecting communities in directed acyclic networks

Suzana Antunović*
University of Split, Croatia
Damir Vukičević
University of Split, Croatia

Networks (or graphs) appear as dominant structures in different domains, including sociology, biology, neuroscience and computing. In most cases, these graphs are directed, which changes the semantics of the edges that are no longer symmetrical in the sense that the beginning vertex transfers some property or value to the end vertex, but not vice versa. Detecting community structure in complex networks is an interdisciplinary topic with many relevant areas of application. In order to detect communities in directed acyclic networks, apart from the direction of the edge, the requirement for topological ordering of the vertices should be taken into account. In other words, if the vertices are topologically ordered is such a way that $x_1 < x_2 < ... < x_n$, we are interested in dividing the network into communities $C_1, C_2, ..., C_k$ in such a way that:

$$\text{if } x_i < x_j,\ x_i \in C_i, x_j \in C_j \text{ then } C_i < C_j \text{ or } C_i = C_j$$

We present an algorithm in which the recursive placement of vertices in the corresponding communities gets the optimal division of the network into consecutive communities. The quality of the division is measured by modularity for directed networks Q_d.

A new algorithm for finding largest small polygons

Charles Audet
Ecole Polytechnique de Montreal, Canada

Pierre Hansen
HEC Montreal, Canada

Dragutin Svrtan*
University of Zagreb, Croatia

A small polygon is a convex polygon (in the euclidean plane) of unit diameter. The problem of determining the largest area of small n-gons was already studied by Reinhardt in 1922. He showed that for n odd the regular n-gon is optimal. For even n this is not the case. For $n = 6$ the largest area F_6, a plane hexagon of unit area can have, satisfies a 10th degree irreducible equation wit integer coefficients. This is the famous Graham's largest little hexagon (1975). R. L.Graham (with S. C.Johnson) needed factoring a 40-degree polynomial with up to 25-digit coefficients. Graham introduced the diameter graphs by joining the vertices at maximal distance. For $n = 6$ (resp. 8) there are 10 (resp. 31) possible diameter graphs. The case $n = 8$ was attacked by C. Audet, P. Hansen, F. Messine via global optimization (10 variables and 20 constraints) which produced (an approximate) famous Hansen's little octagon.

In this talk we report on a new algorithm which uses complex arithmetic and both elimination theory and iterated discriminants to compute a reduction for F_6 by the auxiliary polynomial of degree 14 (instead of 40) (a "missed opportunity" in Graham and Johnson's approach). Also, under axial symmetry conjecture, we obtaine an explicit equations for F_8 (resp F_{10}) of degree 42 (resp 152) via symbolic computations (sometimes involving almost 3000 digit numbers).

On some open problems in minimum coloring games

Jan Bok
Charles University in Prague, Czech Republic

I will talk about one particular type of combinatorial optimization game — minimum coloring games. This type of games was introduced by Deng, Ibaraki and Nagamochi in [1] along with many other types of games arising from problems of combinatorial optimization.

A *minimum coloring game*, assigned to graph $G = (V, E)$, is a pair (V, v_G) such that $v_G(S) = \chi(G[S])$, for all $S \subseteq V$. In classical cooperative game theory, the value $v_G(S)$ is interpreted as a cost that needs to be paid by the subset S. The main point is to study optimal allocation vectors for players in such game so that these allocations have some stability properties. The mostly studied allocation type is *core*.

I will give a necessary theoretical background. Then I will speak about the complexity of various cooperative game theory problems regarding minimum coloring games on the class of triangulated planar graphs and outerplanar graphs — for example core emptiness, stability, exactness, and largeness. I plan to sketch some of my preliminary results on these open problems.

References

[1] X. Deng, T. Ibaraki, H. Nagamochi, Algorithmic aspects of the core of combinatorial optimization games, *Mathematics of Operations Research* 24 (1999) 751–766.

Complexity of k-rainbow independent domination and some result on the lexicographic product of graphs

Simon Brezovnik
University of Maribor, Slovenia

A function $f : V(G) \to \{0, 1, \ldots, k\}$ is called a k-rainbow independent dominating function of G if $V_i = \{x \in V(G) : f(x) = i\}$ is independent for $1 \leq i \leq k$, and for every $x \in V_0$ it follows that $N(x) \cap V_i \neq \emptyset$, for every $i \in [k]$. The k-rainbow independent domination number, $\gamma_{\mathrm{ri}k}(G)$, of a graph G is the minimum of $w(f) = \sum_{i=1}^{k} |V_i|$ over all such functions.

In this talk I will show that the problem of determining whether a graph has a k-rainbow independent dominating function of a given weight is NP-complete for bipartite graphs and that there exists a linear-time algorithm to compute $\gamma_{\mathrm{ri}k}(G)$ of trees. We focus on the k-rainbow independent domination number of the lexicographic product of graphs and present a sharp lower and an upper bound for this number and give the exact formula in case $k = 2$.

Joint work with Tadeja Kraner Šumenjak, FKBV, University of Maribor, Maribor, Slovenia

Recent results on the magic of labyrinth patterns and fractals

Ligia L. Cristea
University of Graz, Austria

An $n \times n$ pattern is obtained by starting with the unit square, dividing it into $n \times n$ congruent smaller subsquares and coloring some of them in black (which means that they will be cut out), and the rest in white. Sierpiński carpets are (self-similar) fractals in the plane that originate from the well-known Sierpiński carpet, constructed as follows: the unit square is divided into $n \times n$ congruent smaller subsquares and m of them are cut out, according to a given $n \times n$ pattern (called the generator of the carpet). This construction step is repeated with all the remaining subsquares ad infinitum. The resulting object is a fractal of Hausdorff and box-counting dimension $\log(n^2 - m)/\log(n)$, called a Sierpiński carpet. By using special patterns, which we called **labyrinth patterns**, and which we described with the help of the graph associated to a pattern, we created and studied a special class of carpets, called labyrinth fractals. Labyrinth fractals are self-similar, and mixed labyrinth fractals are not self-similar. Very recently, we studied an even more general class of labyrinth fractals, called supermixed labyrinth fractals and solved a conjecture on mixed labyrinth fractals. It is very interesting to see how some of the facts and the suitable tools or arguments change when passing to a new family of labyrinth fractals, and some still hold. During my talk I shall present results from recently published or submitted research.

The results stem from joint work with Bertran Steinsky, Gunther Leobacher and Paul Surer.

Canonical form of positive definite matrix

Mathieu Dutour Sikirić
Rugjer Bošković Institute, Zagreb, Croatia

The fundamental problem of graph isomorphism is to check if two graphs are isomorphic. This problem has attracted recent interest and there are very efficient programs for solving that problem. For other combinatorial structures, there are ways for reducing the problem to a graph. The graph isomorphism programs provide another feature and it is the canonical form of a graph which is of great interest for enumeration problems. Positive definite quadratic forms are widely used in geometry of numbers and the relevant notion of equivalence is arithmetic equivalence. In this talk we build a canonical form for positive definite quadratic forms and we shortly consider extensions to other settings.

Maximum External Wiener Index of Graphs

Barbara Ikica
University of Ljubljana, Slovenia

The Wiener index of a graph, defined as the sum of distances between all pairs of vertices, is the oldest and most studied distance-based topological index that has found many applications in chemistry as it is correlated with various molecular quantities. The sum can be conveniently split into two parts, the external and the internal Wiener index, i.e., $W = W_{ex} + W_{in}$. The aim of this talk is to explore extremal properties of the former, W_{ex}, which refers to the sum of distances between two vertices, of which at least one must be a leaf. More precisely, the maximum value of the external Wiener index amongst all graphs is attained by a balanced double broom, which gives an affirmative answer to a strengthened version of the conjecture proposed by Gutman, Furtula, and Das by considering all graphs instead of considering only the class of trees. Finally, we discuss a potential extension of this work.

Theory and Simulation of Double Roman Domination Number on Cardinal Product of Graphs

Antoaneta Klobučar
University of Osijek, Croatia,

Ana Klobučar*
University of Zagreb, Croatia

On a graph $G = (V, E)$ is defined a function $f : V \to \{0, 1, 2, 3\}$ and V_i ($i \in \{0, 1, 2, 3\}$) are sets of vertices assigned the value i by function f. The function f is a *double Roman dominating function* (DRDF) on G if it satisfies the following conditions:

1. If $f(v) = 0$, then the vertex v has at least two neighbors in V_2 or at least one neighbor in V_3.
2. If $f(v) = 1$, then the vertex v has at least one neighbor in $V_2 \cup V_3$.

Double Roman domination number $\gamma_{dR}(G)$ is the minimum weight of double Roman dominating functions on $G = (V, E)$. Double Roman dominating function of G with the weight $\gamma_{dR}(G)$ is called a γ_{dR}-*function* of G.

We will determine some bounds for double Roman domination numbers on cardinal product of any two graphs and some exact values for the cardinal product of paths and cycles.

Also, we will give example of visual simulation of a battle between Romans and barbarians on cardinal product of graphs to determine efficiency of double protection. The simulation is organized in alternating turns played by the Romans and the barbarians. Each turn has complexity of $\mathcal{O}(n)$.

A conjecture about perfect matchings motivated by quantum mechanics

Mario Krenn
University of Vienna, Austria

Recently, we found a surprising bridge between quantum mechanical experiments and Graph Theory. The resulting state of the experimental configuration is the coherent sum of perfect matchings in the graph. The link allows making statements about the possibility of creating specific quantum states with state-of-the-art photonic technology.

I will talk about one open and particularly important question. The question concerns the existence of a bi-colored graph **G**, for which the *inherited vertex coloring* of **G**'s perfect matching fulfill a specific property [1]. I conjecture that such a graph cannot exist, which is significant for the understanding of photonic quantum experiments.

I will define the question formally, give examples, explain a special case solution, and hope for feed-back and ideas from the experts in the field of combinatorics.

References

[1] https://mathoverflow.net/questions/301657/graphs-with-only-disjoint-perfect-matchings-with-coloring

Homology groups of generalized polyomino type tilings

Edin Liđan
University of Bihać, Bosnia and Herzegovina

A polyomino is a plane geometric figure formed by joining one or more equal squares edge to edge and it may be regarded as a finite subset of the regular square tiling with a connected interior. Polyomino tiling problem asks is it possible to properly cover a finite region M consisting of cells with polyomino shapes from a given set $\mathcal{T}$. There are numerous generalizations of this questions towards symmetrical and asymmetrical tilings, higher dimension analogs, polyomino types in other regular lattice grids (triangular, hexagonal), etc. However, the problem in all cases in general is NP-hard and we can give definite answer only in limited number of cases.

This enthralling problem from recreational mathematics attracts attention of both mathematicians and non-experts. Conway and Lagarias in [1] assigned to each set of tiles $\mathcal{T}$ the homology and the homotopy group of tilings and formulated a necessary condition for existence of a proper tilings of a finite region M, and their ideas are further developed by Reid in [2]. This powerful idea allows natural generalization to a much wider class of combinatorial tilings. In this talk we study problem of tiling a surface S subdivided in finite 'combinatorial' grid which may fail to be regular with finite set of polyomino like shapes $\mathcal{T}$ and define the homology group $H_S(\mathcal{T})$. We present some new results together with illustrative examples explaining the application of the homology group of generalized polyomino type tilings in combinatorial and topological context. This is joint work with Đorđe Baralić.

References

[1] J. H. Conway, J. C. Lagarias, Tilings with polyominoes and combinatorial group theory, *Journal of Combinatorial Theory*, Series A 53 (1990) 183 – 208.

[2] M. Reid, Tile homotopy groups, *L'Enseignement Mathématique* 49 (2003) 123 – 155.

Graphs preserving total distance upon vertex removal

Snježana Majstorović
University of Osijek, Croatia

The *total distance* or *Wiener index* $W(G)$ of a connected graph G is defined as the sum of distances between all (unordered) pairs of vertices in G. In 1991, Šoltés posed the following problem:

Find all such graphs G that the equality $W(G) = W(G - v)$ holds for all their vertices v.

Until now, only one such graph is known: it is a cycle with 11 vertices.

Motivated by Šoltés's problem we construct an infinite family of unicyclic graphs which preserve total distance after removal of a particular vertex. We prove that there are infinitely many unicyclic graphs with this property even when we fix the length of the cycle.

We also show that for every graph G there are infinitely many graphs H such that G is an induced subgraph of H and $W(H) = W(H - v)$ for some vertex $v \in V(H) \setminus V(G)$.

For $k \geq 3$ we show that there are infinitely many graphs G with a vertex v of degree k for which $W(G) = W(G - v)$. We prove the existence of such graphs when the degree is $n - 1$ or $n - 2$.

Finally, we show that dense graphs cannot be solutions of Šoltes's problem.

Our contribution shows that the class of graphs, whose total distance does not change when a particular vertex is removed, is rich.

Periodic Parallelogram Polyominoes

Philippe Nadeau
University of Lyon, France

Parallelogram polyominoes form a simple class of polyominoes which frequently occur in combinatorics or statistical physics. Their enumeration with respect to half perimeter is in particular given by the celebrated Catalan numbers. Bousquet-Mlou and Viennot then refined this enumeration by taking into account the area of the polyomino.

We introduce new objects called periodic parallelogram polyominoes and enumerate them following the ideas of Bousquet-Mlou and Viennot. These objects appeared naturally while studying a certain class of infinite permutations, namely 321-avoiding affine permutations, in joint work with R. Biagioli and F. Jouhet.

Domino Tilings of Euclidean and Hyperbolic Boards

László Németh
University of Sopron, Hungary

Several studies deal with tilings with squares and dominoes of the well-known regular square mosaic in Euclidean plane, but we are not aware of any concerned with the hyperbolic regular square mosaics. In the presentation, we examine the tiling problem with colored squares and dominoes of one type of the possible hyperbolic generalization of $(2\times n)$-board. The recurrence sequence describing the number of the different tilings on this board is a fourth order linear homogeneous recurrence sequence and it is a generalization of the Fibonacci sequence, which has also connection to the tilings.

In the presentation, we also introduce the 3-dimensional Euclidean $(2\times 2\times n)$-board on which we examine the tiling problem with colored cubes and bricks.

References

[1] T. Komatsu, L. Németh, L. Szalay, Tilings of hyperbolic $(2 \times n)$-board with colored squares and dominoes, *Ars Math. Contemp.* 15 (2018) 337–346.

[2] L. Németh, Tilings of $(2\times 2\times n)$-board with colored cubes and bricks, submitted.

Bayesian Statistics by Means of Heuristic Methods

Antonio Nuić
University of Osijek, Croatia

Bayesian statistics differs from classical or frequentist statistics in the approach to probability. In Bayesian statistics, we make assumption about distribution and then update it regarding data. This process is based on a Bayes' formula

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)},$$

which is used to make conclusions about posterior distribution. For better understanding of the principle an example will be presented. When it comes to other aspects of posterior distribution (we may want to calculate, for example $P(\theta \in A|y_1, ..., y_n)$), we need to calculate complex integrals. In Bayesian literature, Monte Carlo methods are often used to approximate these integrals. We will explain importance of Monte Carlo methods in Bayesian statistics, and provide simple example of how Monte Carlo methods are used in Bayesian inference.

On a Generalization of the Carré Arithmétique de Fermat and on Weighted Sums of the Type $\sum_{k=0}^{n} k^l a_k^{(r)}$

Daniele Parisse
Airbus Defence and Space GmbH, Manching, Germany

Let $(a_n)_{n\in\mathbb{N}_0}$, be a sequence of natural numbers. We construct an $r \times n$-table $A(r,n), r, n \geq 0$, containing in the $(r+1)$st row the sequence of the partial sums of the preceding sequence in the rth row. In this way one obtains the sequences $(a_n^{(r)})_{n\in\mathbb{N}_0}$ known as the *hypersequences of* $(a_n)_{n\in\mathbb{N}_0}$ *of the* r*th generation.* For example, the constant sequence $a_n := 1, n \geq 0$, generates the *carré arithmétique de Fermat* consisting of the figured numbers.

We determine at first the general term of the table, then by shifting the nth column of the table by n steps downwards we obtain a triangular array. After determining the general term of this array, we consider the sequence defined by the sums of the rows.

In the second place, we shall prove that the weighted sums of the type $\sum_{k=0}^{n} k^l a_k^{(r)}, l, r \geq 0$, can be expressed in terms of the hypersequences $(a_n^{(r+m+1)})_{n\in\mathbb{N}_0}, m = 0, 1, 2, \ldots, l$.

Finally, in the last part we shall apply the general theory to five special cases, namely the constant sequence $a_n := 1, n \geq 0$, the sequence $a_0 = 1, a_n = j, n \geq 1, j \in \mathbb{N}_0$, generating the r-dimensional figurate numbers, the Dirichlet sequence defined as the characteristic function of 1 for all $n \geq 0$, the Fibonacci sequence and the Lucas sequence.

Mathematical model for determining octanol-water partition coefficient of alkanes

Jelena Sedlar
University of Split, Croatia

In order to predict chemical properties of molecules many topological indices were introduced in graph theory, among them the first Zagreb index $M_1(G)$ and the forgotten index $F(G)$ of a graph G. These two indices were tested on the benchmark dataset of 18 octane isomers recommended by the International Academy of Mathematical Chemistry and it was established they correlate well with the same two chemical properties, while for eleven other properties the correlation is not satisfactory. Yet, their linear combination $M_1(G) + \lambda F(G)$ for $\lambda = -0.140$ correlates well with the third property, namely the octanol-water partition coefficient. In order to eliminate free parameter λ we introduce a new index under the name Lanzhou index and denoted by $Lz(G)$, which depends only on the properties of the graph G. Comparing these two indices it turns out they both correlate well with the octanol-water partition coefficient of octanes, while for nonanes $Lz(G)$ way outperforms $M_1(G) + \lambda F(G)$ for $\lambda = -0.140$. Finally, we prove several mathematical properties of the newly introduced index.

Some results on unique-maximum coloring of plane graphs

Riste Škrekovski
FIS Novo Mesto, Slovenia

A *unique-maximum* coloring of a plane graph is a proper vertex coloring by natural numbers where on each face α the maximal color appears exactly once on the vertices of α. Fabrici and Göring proved that six colors are enough for any plane graph and conjectured that four colors suffice. Thus, this conjecture is a strengthening of the Four Color Theorem. Wendland later decreased the upper bound from six to five.

We first show that the conjecture holds for various subclasses of planar graphs but then we disprove it for planar graphs in general. So, we conclude that the facial unique-maximum chromatic number of the sphere is not four but five.

Additionally, we will consider some open problems.

(Joint work with Vesna Andova, Bernard Lidický, Borut Lužar, and Kacy Messerschmidt)

Some Distance-Based Topological Indices of Partial Cubes

Niko Tratnik
University of Maribor, Slovenia

Theoretical molecular structure-descriptors (also called topological indices) are usually graph invariants calculated from the molecular graph of a chemical compound. Partial cubes constitute a large class of graphs with a lot of applications and include, for example, many families of chemical graphs (benzenoid systems, trees, phenylenes, cyclic phenylenes, polyphenylenes). In this talk, some new methods for computing several distance-based topological indices of partial cubes will be presented. In particular, we will show how the cut method can be used to compute the Szeged index, the edge-hyper Wiener index, and the Steiner hyper-Wiener index of partial cubes. Note that a cut method is a powerful tool for the investigation of such topological indices. Moreover, some results regarding the (edge-) Wiener index, the (edge-) Szeged index, and the PI index of benzenoid systems will be mentioned.

Two inequalities: a geometric and a combinatorial

Darko Veljan
University of Zagreb, Croatia

We shall present and prove two inequalities, one geometric and one combinatorial. They both are interesting not only by themselves, but also have some interesting consequences and interpretations, the first one in hyperbolic geometry and the second in probability.

Coloring and Multicoloring of Graphs to Secure a Secret

Tanja Vojković
University of Split, Croatia

Vertex coloring and multicoloring of graphs are well known subjects in graph theory, as well as their applications. In vertex multicoloring, each vertex is assigned some subset of a given set of colors. We propose a new kind of vertex coloring and multicoloring, motivated by the situation of sharing a secret amongst graph vertices and securing it from the actions of some number of attackers. We name the multicoloring a **highly a-resistant vertex k-multicoloring**, where a is the number of the attackers, and k the number of colors. For different assumptions of the action of attackers we define different vertex colorings and multicolorings, and analyze their chromatic numbers.

MMR vaccination rates predicted by combining epidemic spread models, game theory, Fibonacci trading, and Google search statistics

Damir Vukičević*
University of Split, Croatia
Chris T. Bauch
University of Waterloo, Ontario, Canada

Analysis of stock price movements based on the golden ratio is widely utilized in technical market trading, where such movements are known as Fibonacci retracements. This phenomenological approach assumes that the size of market corrections is determined by the golden ratio. Some theoretical models of vaccine refusal are qualitatively consistent with this pattern of over-sold or over-bought markets. Here, we apply Fibonacci retracement to movements of measles-mumps-rubella (MMR) vaccine uptake during an extended episode of vaccine refusal in England from 1995 to 2017. We use a game theoretical model of vaccine refusal and empirical data on risks and epidemiology to retrospectively predict the retracements during this time period, finding good agreement with the data. The analysis of vaccine uptake is supplemented with an analysis of Google searches during the same time period that confirms the timing of shifts between Fibonacci retracement phases. We conclude that such phenomenological approaches to predicting time evolution of vaccine uptake during episodes of vaccine refusal could complement existing approaches that Cavour mechanistic modeling. Ensemble forecasting methods that include diverse methods such as Fibonacci retracement could potentially enable public health to improve scenario planning and restore vaccine uptake more rapidly, thus phenomenological approaches should be further researched.

A note on the Maximal Matchings in Rooted Product of Certain Graphs

IVANA ZUBAC
University of Mostar, Bosnia and Herzegovina

A matching M in a graph G is maximal if it cannot be extended to a larger matching in G. The enumerative properties of maximal matchings are much less known and researched than for maximum and perfect matchings. In this paper we present the recurrences and generating functions for the sequences enumerating maximal matchings in rooted product of some classes of graphs.

Resonance graphs of kinky benzenoid systems are daisy cubes

Petra Žigert Pleteršek
University of Maribor, Slovenia

Recently introduced daisy cubes are interesting isometric subgraphs of n-cubes Q_n, induced by intervals between the maximal elements of a poset $(V(Q_n), \leq)$ and the vertex $0^n \in V(Q_n)$. We will show that the resonance graph, which reflects the interaction between Kekulé structures of aromatic hydrocarbon molecules, is a daisy cube, if the molecules considered can be modeled with the so called kinky benzenoid systems, i.e. catacondensed benzenoid systems without linear hexagons.

Posters

The Golden Ratio within Regular Polyhedra and Geometrical Construction

Andrea Behin
University of Osijek, Croatia

We say that line segment is divided into the golden ratio if the ratio between the length of its longer part and its smaller part is equal to the ratio between segment's entire length and the longer part. This well known value, usually denoted by the Greek letter φ, is the irrational number $\frac{1+\sqrt{5}}{2}$. In addition to the golden rectangle, the golden ratio appears in regular pentagon and regular icosahedron. Here we present a construction named golden window where one can find a few golden proportions. Some further constructions are also discussed.

While the golden ratio is well understood in natural sciences and mathematics, it also appears in social areas including economy. Possibly it plays an important role in the stock market by means to determine critical points that cause an asset's price to reverse. Presented figures are calculated and drawn in GeoGebra by the author.

On the asymmetry measure of Minkowski

Katherina von Dichter
Technical University Munich, Germany

For an n-dimensional convex and compact set K, its asymmetry measure of Minkowski $s(K)$ [1] is the smallest $\rho > 0$ such that $-K \subset x + \rho K$, for some $x \in \mathbb{R}^n$. It is known that

- $s()$ is continuous and affine invariant,
- $1 \leq s(K) \leq n$,
- $s(K) = 1$ if and only if K is symmetric,
- $s(K) = n$ if and only if K is an n-dimensional simplex.

It will be explained how the asymmetry measure of Minkowski sharpens several classical results, and how we have been able to improve recent ones.

Joint work with René Brandenberg (Technische Universität München) and Bernardo González Merino (Universidad de Sevilla)

References

[1] H. Minkowski, Allgemeine Lehrsätze über konvexe Polyeder, *Gesamelte Abhandlungen*, vol. 2, (Teubner, Leipzig-Berlin, 1911), pp. 103–121.

Atiyah-Sutcliffe Conjectures for Almost Collinear Configurations and Some New Conjectures for Symmetric Functions

Dragutin Svrtan
University of Zagreb

Igor Urbiha
Zagreb University of Applied Sciences, Croatia

In 2001 Sir M. F. Atiyah formulated a conjecture $(C1)$ and later with P. Sutcliffe two stronger conjectures $(C2)$ and $(C3)$. These conjectures, inspired by physics (spin-statistics theorem of quantum mechanics), are geometrically defined for any configuration of points in the Euclidean three space. The conjecture $(C1)$ is proved for $n = 3, 4$ and for general n only for some special configurations (M. F. Atiyah, M. Eastwood and P. Norbury, D. Đoković).

Here we shall explain some new conjectures for symmetric functions which imply $(C2)$ and $(C3)$ for almost collinear configurations. Computations up to $n = 6$ are performed with a help of `Maple` and J. Stembridge's package `SF` for symmetric functions. For $n = 4$ we have also verified the conjectures $(C2)$ and $(C3)$ for some infinite families of tetrahedra.

References

[1] M. Atiyah. The geometry of classical particles. *Surveys in Differential Geometry* (International Press) **7** (2001).

[2] M. Atiyah. Configurations of points. *Phil. Trans. R. Soc. Lond. A* **359** (2001), 1375–1387.

[3] M. Atiyah and P. Sutcliffe. Polyhedra in Physics, Chemistry and Geometry, To appear in the Milan Journal of Mathematics

[4] M. Atiyah and P. Sutcliffe. The geometry of point particles. arXiv:hep-th/0105179 v2, 15 Oct 2001.

[5] M. Eastwood and P. Norbury. A proof of Atiyah's conjecture on configurations of four points in Euclidean three-space. *Geometry & Topology* **5** (2001), 885–893.

[6] D.Ž. Đoković, Proof of Atiyah's conjecture for two special types of configurations, arXiv:math.GT/0205221 v4, 11 June 2002. *Electron. J. Linear Algebra* **9** (2002), 132–137.

[7] D.Ž. Đoković, Verification of Atiyah's conjecture for some nonplanar configurations with dihedral symmetry, arXiv:math.GT/0208089 v2, 13 Aug 2002.

[8] I. G. Macdonald *Symmetric functions and Hall polynomials* 2^{nd} edition, Oxford University Press, 1995.

[9] A. F. Möbius Über eine Methode, um von Relationen, welche der Longimetrie angehören, zu entsprechenden Sätzen der Planimetrie zu gelangen, Ber. Verhandl. königl. Sächs. Ges. Wiss. Leipzig Math. Phys. Cl., 1852., 41–45.

List of participants

1. Suzana Antunović,
   University of Split, Croatia
   santunovic@gradst.hr

2. Andrea Behin,
   University of Osijek, Croatia
   abehin@mathos.hr

3. Jan Bok,
   Charles University in Prague, Czech Republic
   bok@iuuk.mff.cuni.cz

4. Simon Brezovnik,
   University of Maribor, Slovenia
   simon.brezovnik2@um.si

5. Loretta Cristea,
   University of Graz, Austria
   strublistea@gmail.com

6. Katherina von Dichter,
   Technical University Munich, Germany
   kate1611@ukr.net

7. Tomislav Došlić,
   University of Zagreb, Croatia
   doslic@grad.hr

8. Mathieu Dutour Sikirić,
   Ruđer Bošković Institute, Croatia
   mathieu.dutour@gmail.com

9. Alan Filipin,
   University of Zagreb, Croatia
   filipin@grad.hr

10. Ivana Grgić,
    University of Split, Croatia
    ipletiko@fesb.hr

11. Barbara Ikica,
    University of Ljubljana, Slovenia
    barbara.ikica@gmail.com

12. Ana Klobučar,
    University of Zagreb, Croatia
    ana.klobucar@fsb.hr

13. Antoaneta Klobučar,
    University of Osijek, Croatia
    antoaneta.klobucar@os.t-com.hr

14. Biserka Kolarec,
    University of Zagreb, Croatia
    bkudelic@agr.hr

15. Tadeja Kraner Šumenjak,
    University of Maribor, Slovenia
    tadeja.kraner@um.si

16. Mario Krenn,
    University of Vienna, Austria
    mario.krenn@univie.ac.at

17. Edin Liđan,
University of Bihać, Bosnia and Herzegovina
lidjan_edin@hotmail.com

18. Tea Lovrić,
University of Zagreb, Croatia
tea.lovric998@gmail.com

19. Snježana Majstorović,
University of Osijek, Croatia
smajstor@mathos.hr

20. Ivica Martinjak,
University of Zagreb, Croatia
imartinjak@phy.hr

21. Philippe Nadeau,
University of Lyon, France
nadeau@math.univ-lyon1.fr

22. László Németh,
University of Sopron, Hungary
nemeth.laszlo@uni-sopron.hu

23. Antonio Nuić,
University of Osijek, Croatia
anuic@mathos.hr

24. Daniele Parisse,
Airbus Defence and Space GmbH, Germany
daniele.parisse@t-online.de

25. Ines Petrić,
University of Osijek, Croatia
ipetric@mathos.hr

26. Jelena Sedlar,
University of Split, Croatia
jsedlar@gradst.hr

27. Dragutin Svrtan,
University of Zagreb, Croatia
dsvrtan@math.hr

28. Riste Škrekovski,
FIS Novo Mesto, Slovenia
riste.skrekovski@gmail.com

29. Aleksandra Tepeh,
University of Maribor, Slovenia
aleksandra.tepeh@gmail.com

30. Sanja Tipurić-Spužević,
University of Mostar, Bosnia and Herzegovina
sanja.spuzevic@fpmoz.sum.ba

31. Niko Tratnik,
University of Maribor, Slovenia
niko.tratnik@um.si

32. Darko Veljan,
University of Zagreb, Croatia
darko.veljan@gmail.com

33. Tanja Vojković,
University of Split, Croatia
tanja@pmfst.hr

34. Damir Vukičević,
University of Split, Croatia
vukicevi@pmfst.hr

35. Ivana Zubac,
University of Mostar, Bosnia and Herzegovina
ivana.fsr@gmail.com

36. Petra Žigert Pleteršek,
University of Maribor, Slovenia
petra.zigert@um.si

Notes