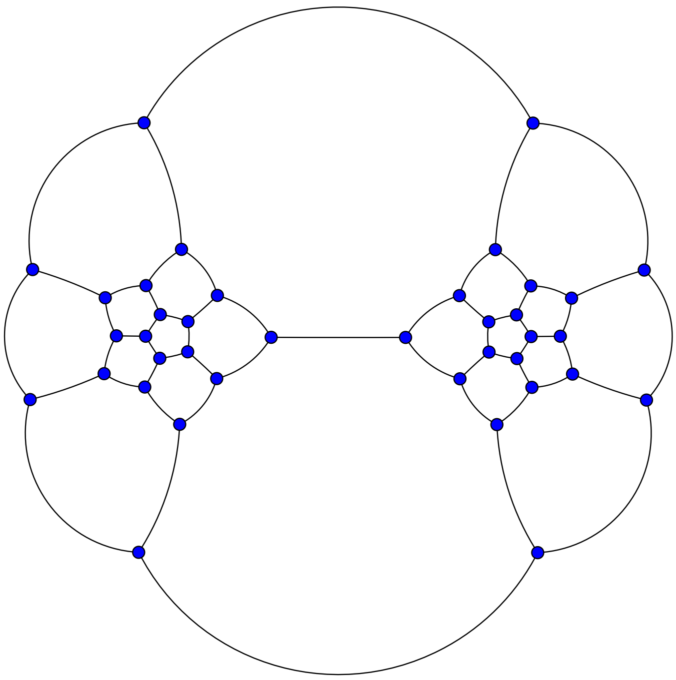
3rd Croatian Combinatorial Days, Zagreb, September 2020

CroCoDays 2020



Book of Abstracts

IMPRESSUM

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Contents

Acyclic, star and injective colouring of H-free graphs

Jan Bok
Charles University, Prague, Czech Republic

A k-colouring c of a graph G is a mapping $V(G) \to \{1, 2, \ldots\ k\}$ such that $c(u) \neq c(v)$ whenever u and v are adjacent. The corresponding decision problem is Colouring. A classical complexity result on Colouring is a well-known dichotomy for H-free graphs, which was established twenty years ago (a graph is H-free if and only if it does not contain H as an induced subgraph). We say that colouring is acyclic, star, or injective if any two colour classes induce a forest, star forest or disjoint union of vertices and edges, respectively. In my talk, I will focus on the corresponding decision problems for these special types of colourings.

We give almost complete classifications for the computational complexity of Acyclic Colouring, Star Colouring and Injective Colouring for H-free graphs. If the number of colours k is fixed, that is, not part of the input, we give full complexity classifications for each of the three problems for H-free graphs. From our study we conclude that for fixed k the three problems behave in the same way, but this is no longer true if k is part of the input.

This is a joint work with Nikola Jedličková, Barnaby Martin, Daniël Paulusma and Siani Smith.

New Methods for Calculating the Degree Distance (Schultz Index) and the Gutman Index

Simon Brezovnik
Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia

In the talk I will present some new methods for calculating the two well-known topological indices, the degree distance and the Gutman index. The Wiener index of a double vertex-weighted graph can be computed from the Wiener indices of weighted quotient graphs with respect to a partition of the edge set that is coarser than Θ^*-partition. This result immediately gives a method for computing the degree distance of any graph. The degree distance and the Gutman index of an arbitrary phenylene can be expressed by using its hexagonal squeeze and inner dual. In addition, we will observe how these two indices of a phenylene can be obtained from the four quotient trees. Furthermore, reduction theorems for the Wiener index of a double vertex-weighted graph will be presented. Finally, a formula for computing the Gutman index of a partial Hamming graph will be obtained.

Joint work with Niko Tratnik, Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia and Faculty of Education, University of Maribor, Slovenia

Triangular labyrinth patterns, sets and fractals

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An *n-triangular pattern* is obtained by dividing an equilateral triangle of side length 1 into $n \times n$ congruent equilateral triangles and colouring some of them in black (which means that they will be cut out), and the rest in white. In particular, the famous Sierpiński gasket can be obtained as the limit set of an interative construction with such a pattern (with a lot of symmetries). Obvoiusly, a generating pattern for this fractal is the 2-pattern obtained by dividing an equilateral in 4 equilateral triangles of side-length 1/2, and then colouring the central one in black and the rest in white. Then, at each iteration step, replace every white triangle by the scaled image of the initial pattern. Let us notice that here only upright white patterns occur, while in general in a triangular pattern both upright and upside down triangles. This leads naturally to the idea of using a system of two patterns for the interative construction.

An *n-triangular labyrinth patterns system* consists of two n-triangular patterns, the white one (with white and black triangles) and the yelllow one (with yellow and black triangles), which both have three characteristic properties, two of which relate them to each other.

Triangular labyrinth fractals are (self-similar) fractals in the plane. They are dendrites obtained as limits of the two nested sequences of labyrinth sets (the white and the yellow one) generated by a triangular labyrinth patterns system. We focus on lengths of paths in labyrinth sets, and subsequently on properties of arcs in the resulting dendrites. It is interesting to explore which features encountered in square labyrinth fractals can be found in their triangular relatives and whether/how the triangular shape impacts the results.

The results stem from joint work with my colleague Paul Surer from Vienna.

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Packing stars in fullerenes

Tomislav Došlić
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A perfect star packing in a graph G is a spanning subgraph of G whose every component is isomorphic to the star graph $K_{1,3}$. We investigate which fullerene graphs allow such packings. We also consider generalized fullerene graphs and packings of some other small graphs into classical and generalized fullerenes. Several open problems are listed.

Coloring the Voronoi tesselation of lattices

Mathieu Dutour Sikirić
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The chromatic number of a lattice is the least number of colors one needs to color the interiors of the cells of the Voronoi tessellation of a lattice so that no two cells sharing a facet are of the same color. We compute the chromatic number of the root lattices, their duals, and of the Leech lattice, we consider the chromatic number of lattices of Voronoi's first kind, and we investigate the asymptotic behaviour of the chromatic number of lattices when the dimension tends to infinity. In passing, we explain the parameter space of Voronoi tesselation of lattice with L-type and C-type.

List homomorphism problems for signed graphs

Nikola Jedličková
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We consider homomorphisms of signed graphs from a computational perspective. In particular, we study the list homomorphism problem seeking a homomorphism of an input signed graph (G, σ), equipped with lists $L(v) \subseteq V(H), v \in V(G)$, of allowed images, to a fixed target signed graph (H, π). The complexity of the similar homomorphism problem without lists (corresponding to all lists being $L(v) = V(H)$) has been previously classified by Brewster and Siggers, but the list version remains open and appears difficult. Both versions (with lists or without lists) can be formulated as constraint satisfaction problems, and hence enjoy the algebraic dichotomy classification recently verified by Bulatov and Zhuk. By contrast, we seek a combinatorial classification for the list version, akin to the combinatorial classification for the version without lists completed by Brewster and Siggers. We illustrate the possible complications by classifying the complexity of the list homomorphism problem when H is a (reflexive or irreflexive) signed tree. It turns out that the problems are polynomial-time solvable for certain caterpillar-like trees, and are NP-complete otherwise. The tools we develop will be useful for classifications of other classes of signed graphs, and we mention some follow-up research of this kind; those classifications are surprisingly complex.

Joint work with Jan Bok, Richard Brewster, Toms Feder, and Pavol Hell.

Properties of the total and double total domination number on hexagonal grid

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Let G be a graph with the vertex set $V(G)$ and edge set $E(G)$. A set $D \subset V(G)$ is a total dominating set of a graph G if every vertex y in $V(G)$ is adjacent to some vertex in D. The total domination number $\gamma_t(G)$ is the cardinality of the smallest total dominating set.

Here we determine the upper and lower bound for total domination number and exact values and upper bound for double total domination number on hexagonal grid $H_{m,n}$ with m hexagons in a row and n hexagons in a column. Further, we explore the ratio between total domination number and number of vertices of $H_{m,n}$ when m and n tend to infinity.

Keywords: total domination number, double total domination number, hexagonal grid, molecular graph

Constructing partial geometries with prescribed automorphism groups

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Partial geometries $pg(s,t,\alpha)$ are partial linear spaces of order (s,t) such that for every non-incident point-line pair (P,ℓ), there are exactly α points on ℓ collinear with P. Special cases include Steiner 2-desings and their duals, nets and their duals (transversal designs), and generalized quadrangles. We shall discuss algorithms for constructing partial geometries with prescribed automorphism groups and present some preliminary computational results.

Simplicial complexes arising from polyomino tilings

Edin Liđan
University of Bihać, Bosnia and Herzegovina

We consider tilings of a given region R in the square grid by a finite set of polyomino shapes T and define a simplicial complex $K_{(T,R)}$. It appears that topology and combinatorics of $K_{(T,R)}$ acquire some important information about tilings and number of distinct placements of the shapes from T in R.

This is joint work with Đorđe Baralić.

Metric vs. edge metric dimension of a graph

Snježana Majstorović
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Given a connected graph G, the metric (edge metric) dimension of G is the cardinality of the smallest ordered set of vertices that uniquely identifies every pair of distinct vertices (edges) of G by means of distance vectors to such a set. We give solutions to three open problems on (edge) metric dimension of graphs.

(i) For which integers r, t, $n \geq 1$, with r, $t \leq n-1$ there exists a graph G of order n with $\dim(G) = r$ and $\text{edim}(G) = t$?

(ii) Is it possible to bound $\dim(G)$ or $\text{edim}(G)$ from above by some constant factor of $\text{edim}(G)$ or $\dim(G)$, respectively?

(iii) The only known family of graphs for which the inequality $\dim(G) > \text{edim}(G)$ holds is the Cartesian product of two cycles $C_{4r} \square C_{4t}$. Are there any other families of such graphs?

New Class of Binomial Sums and Their Applications

Jovan Mikić
J.U. SŠC "Jovan Cvijić", Modriča, Republic of Srpska, Bosnia and Herzegovina

We introduce a notion that we call a "M-sum" and use it to examine the divisibility properties of some binomial sums.

There is a close relationship between M-sum and D-sum. By using D-sums, we give recurrence relations for M-sums.

We present three applications of our new sums. The first application is for a known alternating binomial sum studied by Calkin.

The second application is for the following binomial sum S: $\mathbb{Z}^{l+2}_{>=0} \to \mathbb{Z}$,

$$S(2n, m, a_1, a_2, \ldots, a_l) = \sum_{k=0}^{2n} (-1)^k \binom{2n}{k}^m \prod_{i=1}^{l} \binom{a_i + k}{k} \binom{a_i + 2n - k}{2n - k};$$

which arises as a generalization of a known identity connected with a famous Dixon's formula. We prove that, if m is a positive integer, then $S(2n, m, a_1, a_2, \ldots, a_l)$ is divisible by $\binom{2n}{n}$ and $\binom{a_i+n}{n}$ for all $i = 1, \ldots, l$.

As the third application, we present an interesting connection between our new sums and one Theorem by Guo, Jouhet and Zeng.

Sequences on the square Zig-zag shapes

László Németh[1]
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One of several approaches to generalize Pascal's arithmetic triangle, the hyperbolic Pascal triangle [1], based on a regular hyperbolic square mosaic can be figured as a digraph, where the vertices and the edges are the vertices and the edges of a well defined part of a regular hyperbolic square lattice, respectively. Furthermore each vertex possesses a value, giving the number of different shortest paths from the fixed base vertex. On the hyperbolic Pascal triangle linked to mosaic $\{4,5\}$ one can found the Fibonacci sequence if one follows a specific zig-zag path.

The authors [1, 2] proved that all the integer linear homogeneous recurrence sequences $\{f_i\}_{i\geq 0}$ defined by

$$f_i = \alpha f_{i-1} \pm f_{i-2}, \qquad (i \geq 2),$$

where $\alpha \in \mathbb{N}$, $\alpha \geq 2$, and $f_0 < f_1$ are positive integers with $\gcd(f_0, f_1) = 1$ appear in the hyperbolic Pascal triangle along different zig-zag paths. This interesting result inspired the authors to examine zig-zag paths on certain parts of the Euclidean square mosaic.

In the presentation, we define a so-called square k-zig-zag shape as an infinite part of the regular square grid in Euclidean plane. Considering the shape as a k-zig-zag digraph, we give values of its vertices according to the number of the shortest paths from a base vertex. The diagonal and zig-zag paths provide several integer sequences, whose higher-order homogeneous recurrences are determined by the help of a special matrix recurrence.

Key words: Zig-zag digraph, recurrence sequence, zig-zag sequence.

MSC 2010: 11B37, 11Y55, 05C38, 05A10.

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Vertex and edge metric dimensions of graphs with edge disjoint cycles

Jelena Sedlar
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In a graph G, cardinality of the smallest ordered set of vertices that distinguishes every element of $V(G)$ is the (vertex) metric dimension of G.Similarly, the cardinality of such a set is the edge metric dimension of G, if it distinguishes $E(G)$. In this paper these invariants are considered first for unicyclic graphs, and it is shown that the vertex and edge metric dimensions obtain values from two particular consecutive integers, which can be determined from the structure of the graph. In particular, as a consequence, we obtain that these two invariants can differ for at most one for the same unicyclic graph. Next we extend the results to graphs with edge disjoint cycles showing that the two invariants can differ at most by c, where c is the number of cycles in such a graph. We conclude the paper with a conjecture that generalizes the previously mentioned consequences to graphs with prescribed cyclomatic number c by claiming that the difference of the invariant is still bounded by c.

Permutations avoiding a simsun pattern

Matteo Silimbani
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A permutation π avoids the simsun pattern τ if π avoids the consecutive pattern τ and the same condition applies to the restriction of π to any interval $[k]$. Permutations avoiding the simsun pattern 321 are the usual simsun permutation introduced by Simion and Sundaram. Deutsch and Elizalde enumerated the set of simsun permutations that avoid in addition any set of patterns of length 3 in the classical sense. In this paper we enumerate the set of permutations avoiding any other simsun pattern of length 3 together with any set of classical patterns of length 3. The main tool in the proofs is a massive use of a bijection between permutations and increasing binary trees.

Mixed Atiyah Determinants for Graphs in Euclidean or Hyperbolic Space

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2020 Mathematics Subject Classification. 11Cxx, 51Mxx, 68W30

In 2001 Sir Michael Atiyah, inspired by physics (Berry Robbins problem of a geometric explanation of the spin statistics theorem of quantum mechanics) associated a remarkable determinant to any n distinct points in Euclidean 3-space (or hyperbolic 3-space), via an elementary construction. Although the problem of nonvanishing of the Atiyah determinants is very intricate (Atiyah's first conjecture), we associate a mixed Atiyah determinant to any graph with the given points as vertices. For the sum of all mixed determinants we can prove an identity ($n! - conjecture$) which implies that for any configuration of n distinct points in hyperbolic 3-space at least one of the associated mixed determinants is nonzero (this result, presented in 2014 at International Congress of Mathematicians (ICM) in Seoul,S.Korea and in the Seminaire Lotharingien de Combinatoire (SLC) in Strobl, Austria, gives a "probabilistic" verification of Atiyah-Sutcliffe conjectures for arbitrary n points in hyperbolic 3-space. This result is now called Svrtan's $n!$ Formula (see J. Malkoun, Determinants, Choices and Combinatorics, Discr. Math. $342(2019)250 - 255$, page 253). For two stronger Atiyah-Sutcliffe conjectures, for four Euclidean points, a direct geometric proof was obtained ten years ago by the speaker and presented first at MATH/CHEM/COMP 2010, Dubrovnik, Croatia, 7-12.06.2010, Program & Book of Abstracts/Graovac Ante, et al(ur.). Zagreb, HUM naklada d.o.o.. page 62. Later in 2014 another proof is obtained by M.J.Khuzam and M.J.Johnson (SIGMA 10(214), 70, 9 pages), via linear programming. Both proofs use the famous Eastwood-Norbury formula for the 4-pt Atiyah determinant (hyperbolic analogue of which is not yet completely known!). Our proof uses new six shear coordinates and the formulas are short, but the later proof uses twelve (dependent) parameters and the formulas are much longer. In this talk we shall explain a more direct proof of Eastwood Norbury type formulas for $n = 4$ and even for $n = 5$. The later formula has almost 100000 terms!

For more information :
https://bib.irb.hr/prikazi-rad& rad=553790
http://www.emis.de/journals/SLC/wpapers/s73vortrag/svrtan.pdf

Keywords: complex polynomials associated to points (in euclidean or hyperbolic 3-space), Atiyah determinant, symbolic computations, $n!-$ formula

Mixed metric dimension of cactus graphs

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In a graph G, the cardinality of the smallest ordered set of vertices that distinguishes every element of $V(G) \cup E(G)$ is called the mixed metric dimension of G. In this paper we first establish the exact value of the mixed metric dimension of a unicycic graph G which is derived from the structure of G. We further consider graphs G with edge disjoint cycles in which a unicyclic restriction G_i is introduced for each cycle C_i. Applying the result for unicyclic graph to each G_i then yields the exact value of the mixed metric dimension of such a graph G. The obtained formulas for the exact value of the mixed metric dimension yield a simple sharp upper bound on the mixed metric dimension, and we conclude the paper conjecturing that the analogous bound holds for general graphs with prescribed cyclomatic number.

Resonance graphs of catacondensed even ring systems

Niko Tratnik
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A catacondensed even ring system (shortly CERS) is a simple bipartite 2-connected outerplanar graph with all vertices of degree 2 or 3. The resonance graph of a CERS models interactions among the perfect matchings (in chemistry known as Kekule structures) of the given CERS. In this talk, we will firstly generalize the binary coding procedure of perfect matchings from catacondensed benzenoid graphs to all CERS. In addition, CERS with isomorphic resonance graphs will be considered. Furthermore, we will show that two even ring chains are evenly homeomorphic iff their resonance graphs are isomorphic. Finally, CERS whose resonance graphs are daisy cubes will be characterized, which generalizes greatly the result known for kinky benzenoid graphs. Some open problems will be also presented.

This is joint work with Simon Brezovnik and Petra Žigert Pleteršek

Euler's inequalities improved in plane and space

Darko Veljan
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As we know back from Euler, the circumcircle of a tringle is at least as long as the triangle's incircle, $R \geq 2r$. For a tetrahedron, the circumsphere has at least nine times larger surface area than the insphere, $R^2 \geq 9r^2$. We improved both Euler's inequalities in terms of symmetric functions of the side lengths of the triangle or the tetrahedron in question, thus obtaining an intrinsic inequality unlike the ambient, dimension-like Euler's inequality $R \geq nr$ for an n-simplex in dimension n. We shall also discuss the hyperbolic version and some related topics such as Grace-Danielsson's inequality.

Beauty of the Canvas Aspect Ratios 1.357 and 1.441

Damir Vukičević
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Recently, collection of more than 223 thousand paintings have been analyzed and it was established that average aspect ratio for portraits is 1.357:1 and for landscape-oriented paintings is (close to) 1.441:1. Using wisdom of the crowd theory, these two numbers should be related to some universal beauty that surpasses individual personal preferences. We show that indeed these values are related to important mathematical proportions (arithmetical mean, Kepler triangle, golden section) and that difference between aspect ratios of vertically and horizontally oriented paintings is related to peripheral vision field. These aspect ratios can be used by painters and frame manufacturers to amplify the beauty of artistic compositions taking into consideration psychology of perception – our ability to innocuously register proportion as beauty. Very few real numbers are so special, that they should be widely known in the artistic world (e.g. golden ratio). It might be that these two numbers could deserve such status.

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Notes