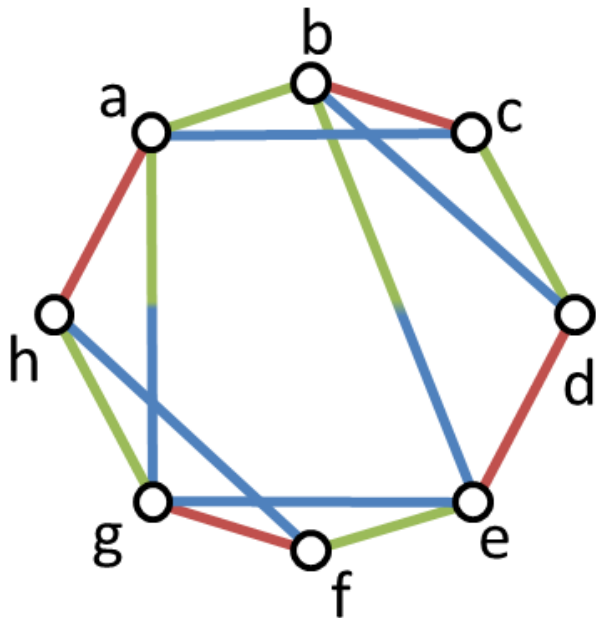


# A conjecture about Perfect Matchings motivated by Quantum Mechanics



Mario Krenn

*Group of Anton Zeilinger*

*University of Vienna*

*IQOQI Vienna / Austrian Academy of Sciences*

<http://mariokrenn.wordpress.com/>

[mario.krenn@univie.ac.at](mailto:mario.krenn@univie.ac.at)



universität  
wien



VCQ



Vienna Center for Quantum  
Science and Technology

# High-dimensional multipartite entanglement

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$n=2, d=2$ :



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

 or 



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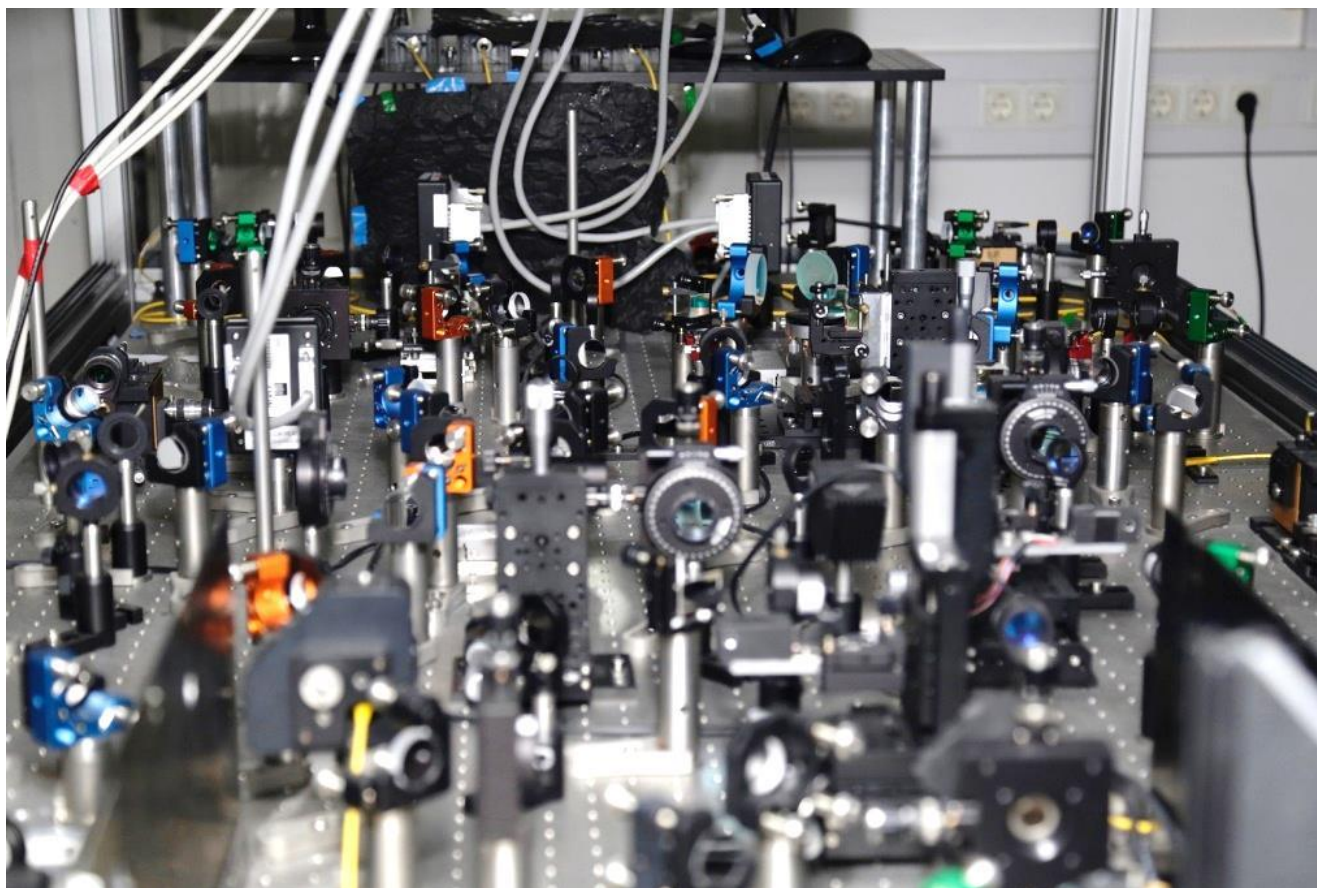
$n=3, d=2$ :  $|\psi\rangle_{GHZ-2D} = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$

 or 

$n=2, d=3$ :  $|\psi\rangle_{3D} = \frac{1}{\sqrt{3}} (|00\rangle + |11\rangle + |22\rangle)$

 or  or 

# High-dimensional multipartite entanglement



$+ |111\rangle$




or 

$+ |222\rangle$

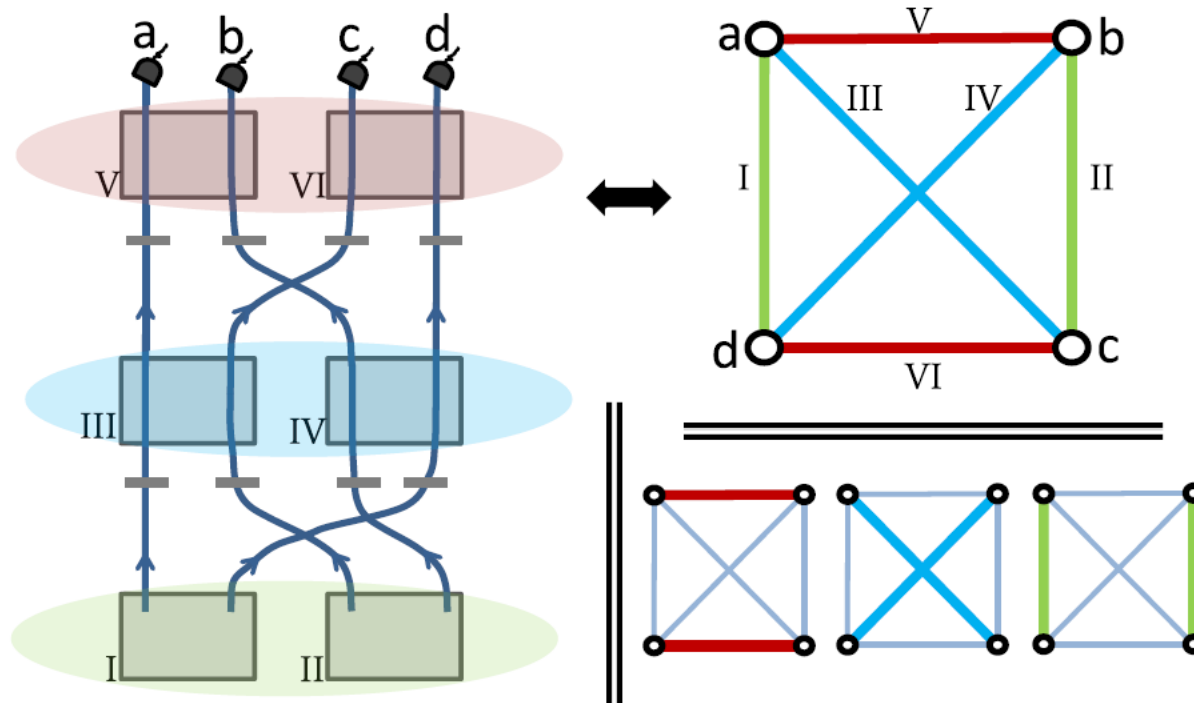
or 

$n=3, d=3:$

$$|\psi\rangle_{GHZ-3D} = \frac{1}{\sqrt{3}} (|000\rangle + |111\rangle + |222\rangle)$$

 or  or 

# Quantum Experiment – Graph Theory link



- [1] MK, Gu, Zeilinger, "Quantum Experiments and Graphs: Multipartite States as coherent superpositions of Perfect Matchings", *Phys. Rev. Lett.* **119**, 240403 (2017)

# Graph Theory – preparing the stage...

## **Definition:**

A bi-colored weighted graph  $G(V,E)$  is an undirected graph, where

- every edge is colored (monochromatic or bichromatic)
- has a complex weight associated to each edge

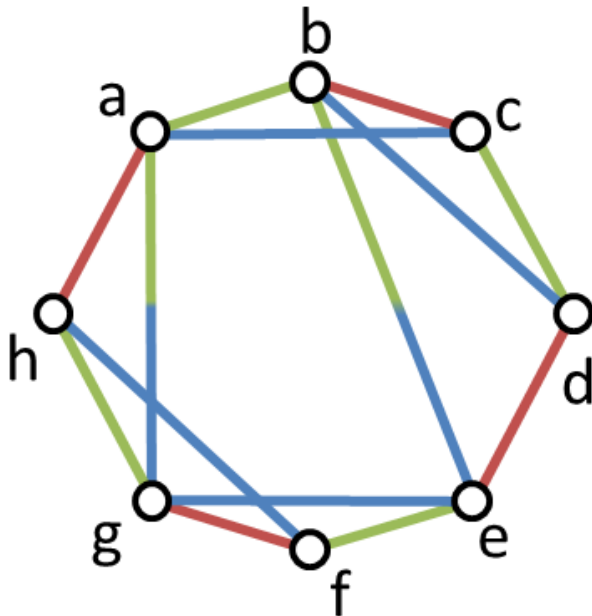
# Graph Theory – preparing the stage...





## Definition:

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## Example:



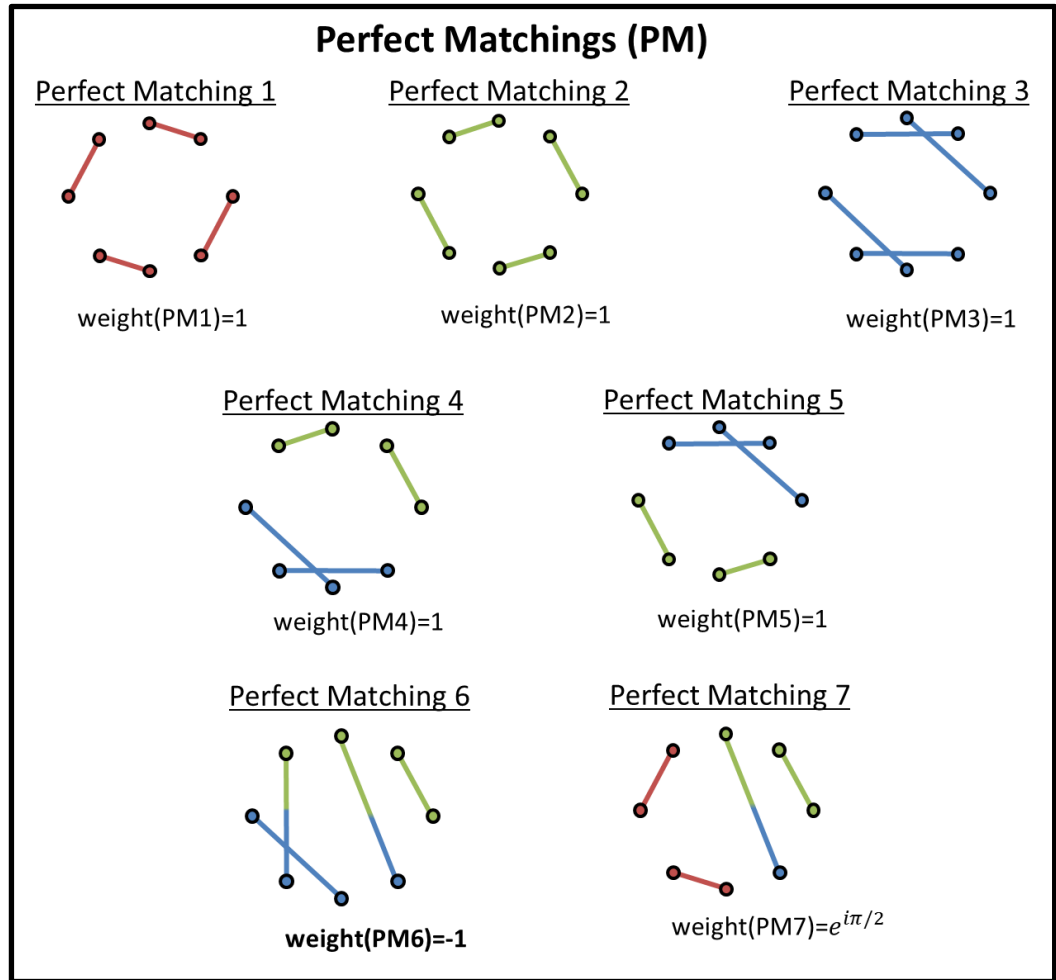
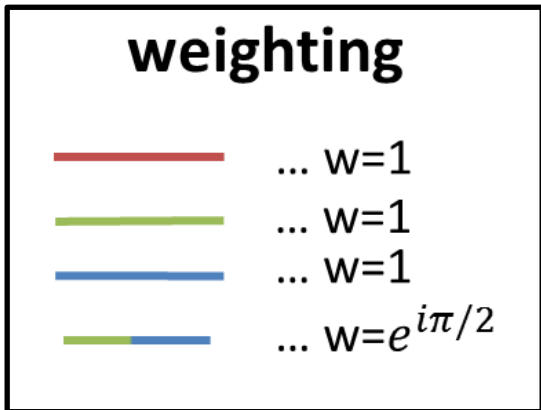
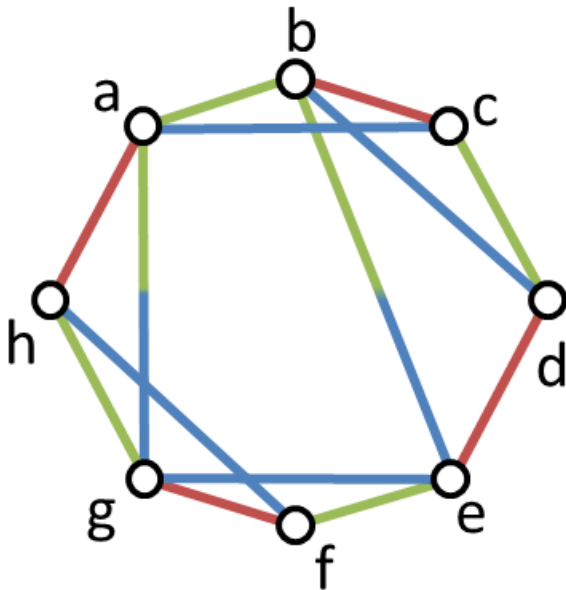
weighting	
	... $w=1$
	... $w=1$
	... $w=1$
	... $w=e^{i\pi/2}$



# Graph Theory – preparing the stage...

**Weight of Perfect Matching (PM):** Product of weight of all edges in PM

**Inherited Vertex Coloring (IVC):** Every Vertex gets the color of the PM's incident edge



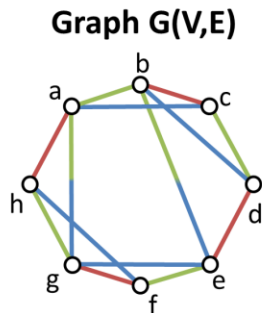
# Graph Theory – preparing the stage...

**Weight of Perfect Matching (PM):** Product of weight of all edges in PM

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**Inherited Coloring Weight (ICW)** for some coloring  $c$  is the sum

of all weights of PMs with coloring  $c$ : 
$$ICW(c) = \sum_i \prod_{|E_j| \in PM_i} w_{i,j}$$



**weighting**

- ...  $w=1$
- ...  $w=1$
- ...  $w=1$
- ...  $w=e^{i\pi/2}$

(just an example, generally every edge can have an individual weight)

**Perfect Matchings (PM)**

Perfect Matching 1      Perfect Matching 2



weight(PM1)=1



weight(PM2)=1

Perfect Matching 3



weight(PM3)=1

Perfect Matching 4



weight(PM4)=1

Perfect Matching 5



weight(PM5)=1

Perfect Matching 6



weight(PM6)=-1

Perfect Matching 7



weight(PM7)= $e^{i\pi/2}$

**Inherited Vertex Coloring (IVC)**

Perfect Matching 1



Perfect Matching 2



Perfect Matching 3



Perfect Matching 4



Perfect Matching 5



Perfect Matching 6



Perfect Matching 7



**Inherited Coloring Weight (ICW)**

$ICW(\text{red dots}) = \text{weight}(\text{PM1}) = 1$

$ICW(\text{green dots}) = \text{weight}(\text{PM2}) = 1$

$ICW(\text{blue dots}) = \text{weight}(\text{PM3}) = 1$

$ICW(\text{light blue dots}) = \text{weight}(\text{PM4}) + \text{weight}(\text{PM6}) = 0$

$ICW(\text{light blue dots}) = \text{weight}(\text{PM5}) = 1$

$ICW(\text{red dots}) = \text{weight}(\text{PM7}) = e^{i\pi/2}$

# Graph Theory – the Question

**Weight of Perfect Matching (PM):** Product of weight of all edges in PM

**Inherited Vertex Coloring (IVC):** Every Vertex gets the color of the PM's incident edge

**Inherited Coloring Weight (ICW)** for some coloring  $c$  is the sum

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**Question:** Is there a bi-colored weighted graph with  $|V| > 4$  with three possible edge colors (red, blue, green) with the following properties:

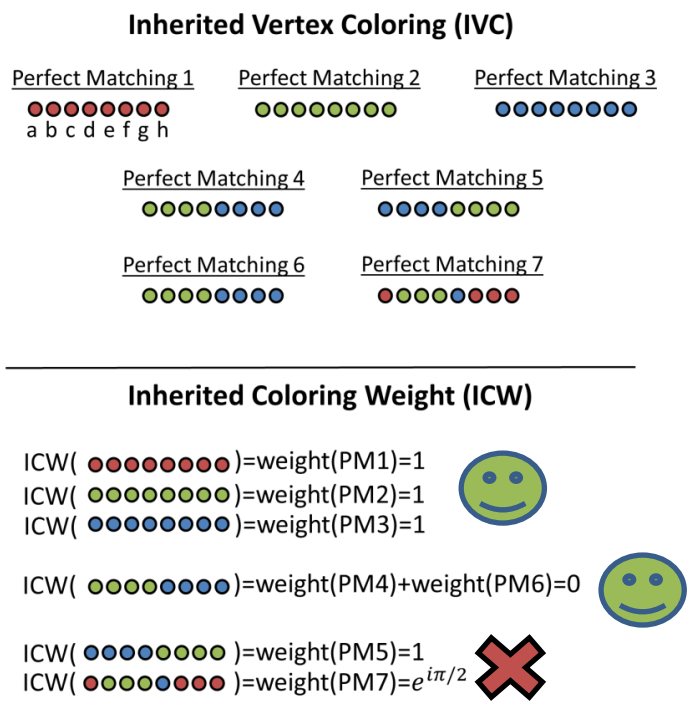
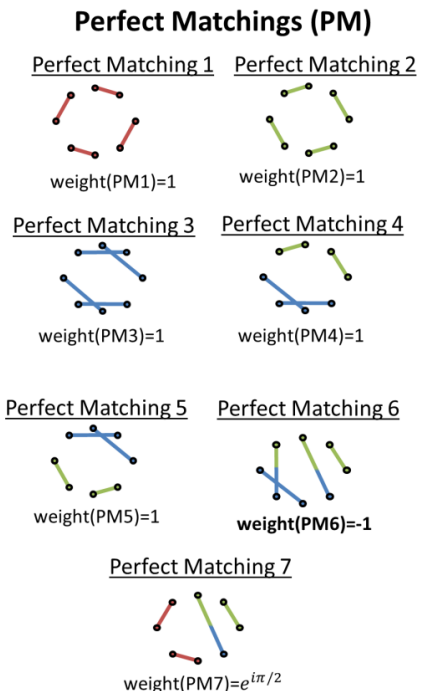
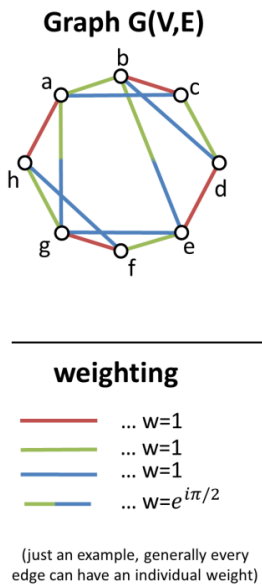
1) For each of the three colors there is a monochromatic vertex coloring  $c_{mono}$  with non-zero weight, i.e.  $|ICW(c_{mono})| > 0$

2) Every non-monochromatic vertex coloring  $c_{n-mono}$  has zero weight, i.e.  $|ICW(c_{n-mono})| = 0$ .

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## Partial Solutions

- A weaker form of this question was solved by [Ilya Bogdanov](#). There, the question was for graphs with only monochromatic vertex colorings. (That can only exist for  $|V|=4$  with three colors)
- A related question has been solved by [Gjergji Zaimi](#). He showed that graphs exist where each non-monochromatic coloring appears at least two times, which is a necessary condition for the  $|ICW(c_{n-mono})| = 0$ .

# Graph Theory – the Conjecture

**Question:** Is there a bi-colored weighted graph with  $|V| > 4$  with three possible edge colors (red, blue, green) with the following properties:

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**My Conjecture:** Such a graph does **not** exist.

Any thoughts or ideas:

*mario.krenn@univie.ac.at*

**THANKS FOR LISTENING!!!** 😊