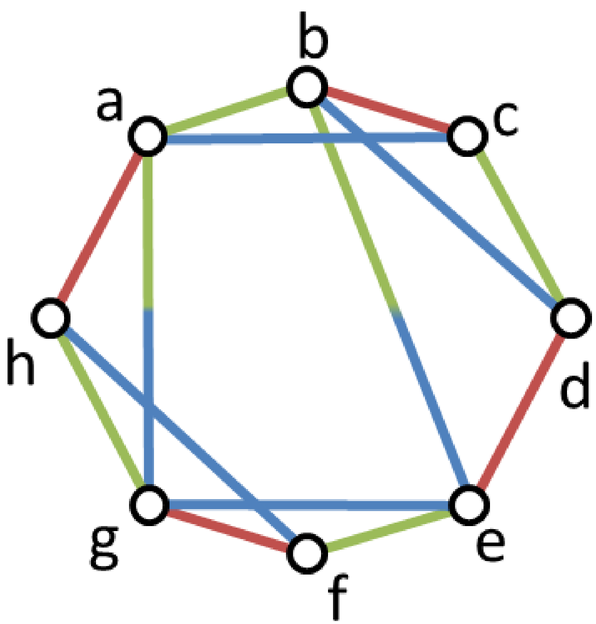
CroCoDays 2018, 28.09.2018

A conjecture about Perfect Matchings
motivated by Quantum Mechanics



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universität wien

IQI

ÖAW Austrian Academy of Sciences

VCQ Vienna Center for Quantum Science and Technology

High-dimensional multipartite entanglement

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n=2, d=2:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

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or

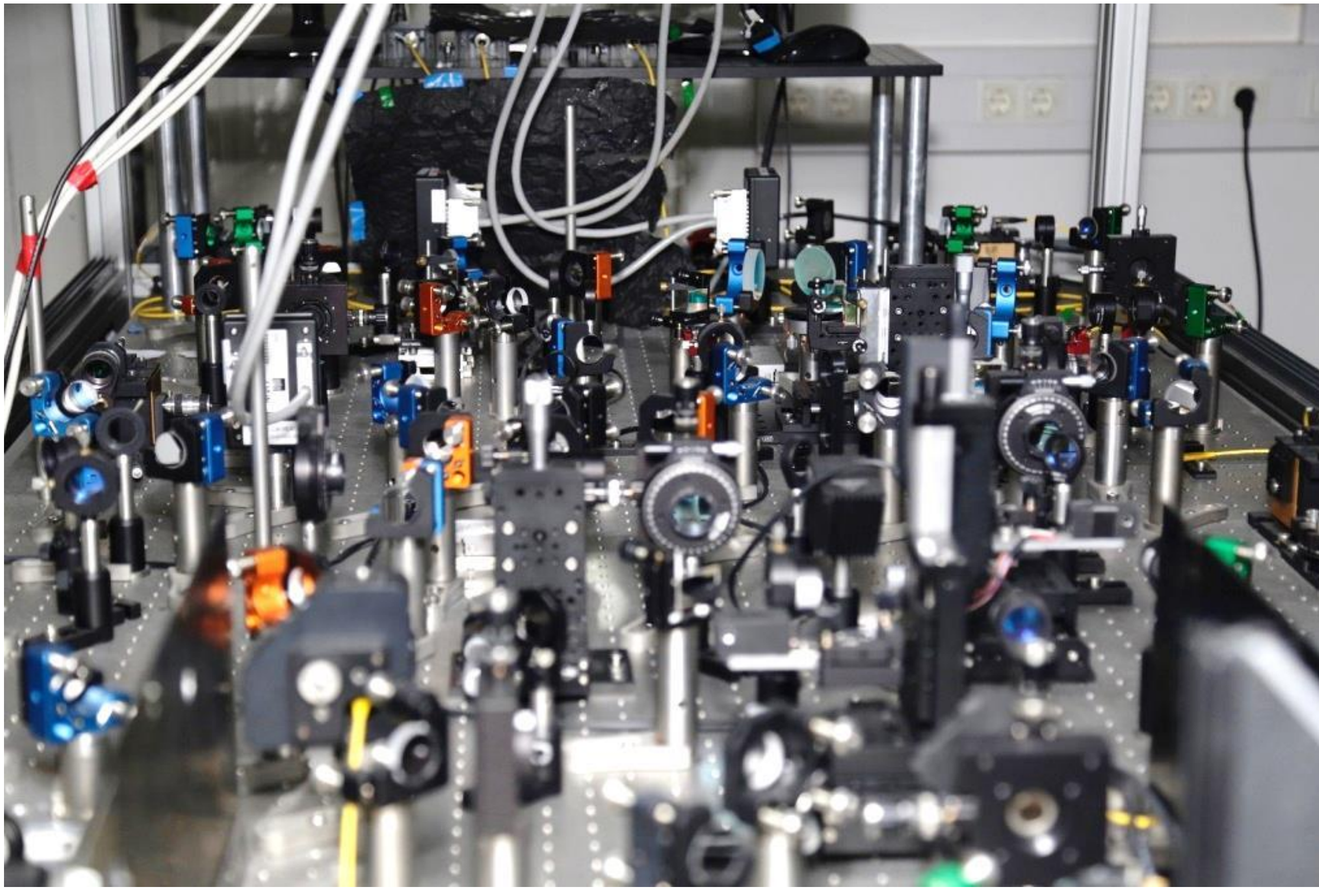
n=3, d=2: $$|\psi\rangle_{GHZ-2D} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

or

n=2, **d=3**: $$|\psi\rangle_{3D} = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$$

or or

High-dimensional multipartite entanglement



$+ |111\rangle)$

or

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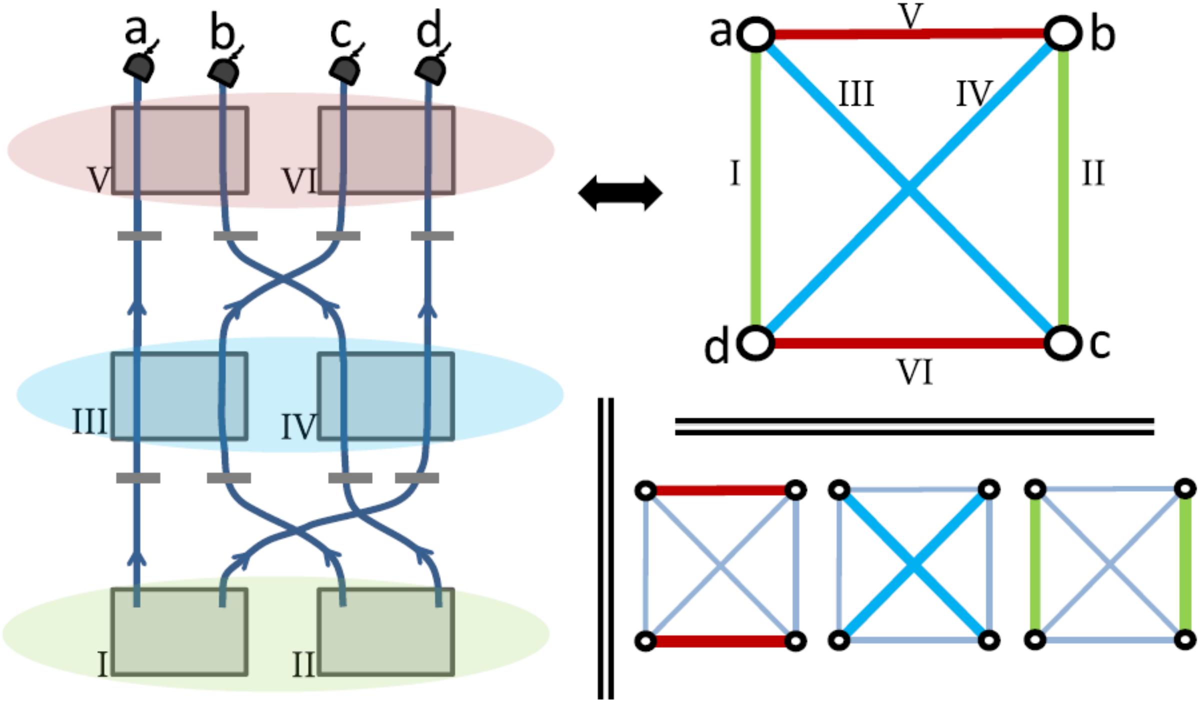
n=3, d=3:

$$|\psi\rangle_{GHZ-3D} = \frac{1}{\sqrt{3}}\left(|000\rangle + |111\rangle + |222\rangle\right)$$

or or

Quantum Experiment – Graph Theory link



[1] MK, Gu, Zeilinger, "Quantum Experiments and Graphs:
Multiparty States as coherent superpositions of Perfect Matchings“,
Phys. Rev. Lett. **119**, 240403 (2017)

Graph Theory – preparing the stage…

Definition:

A bi-colored weighted graph $G(V,E)$ is an undirected graph, where

- every edge is colored (monochromatic or bichromatic)
- has a complex weight associated to each edge

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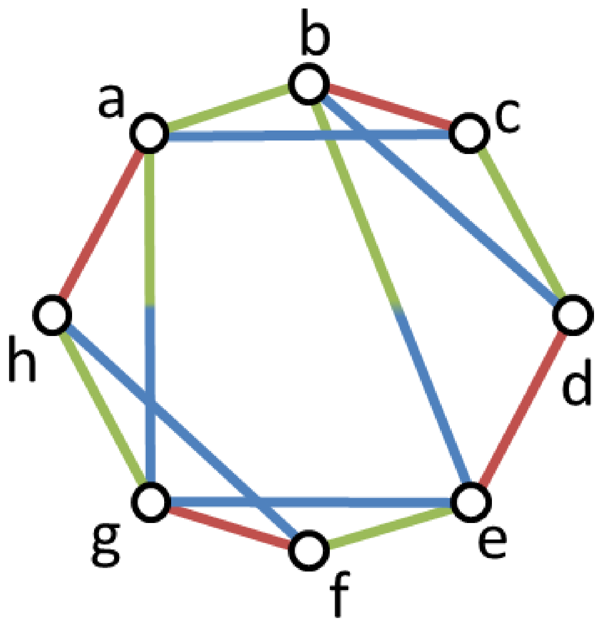
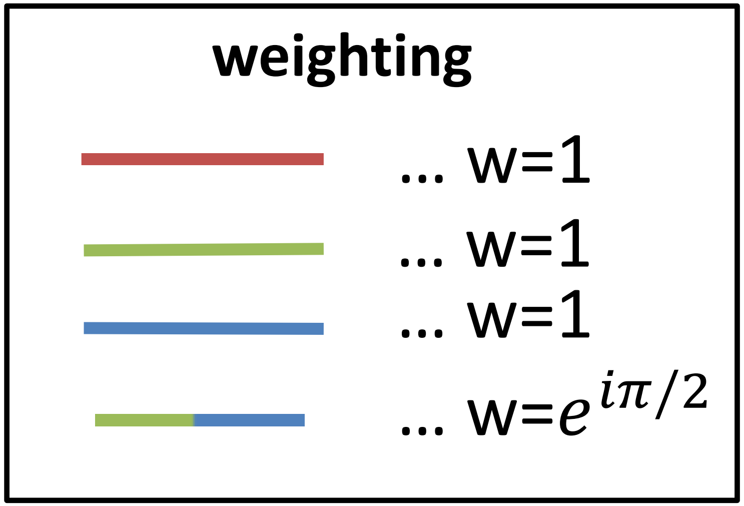
Example:

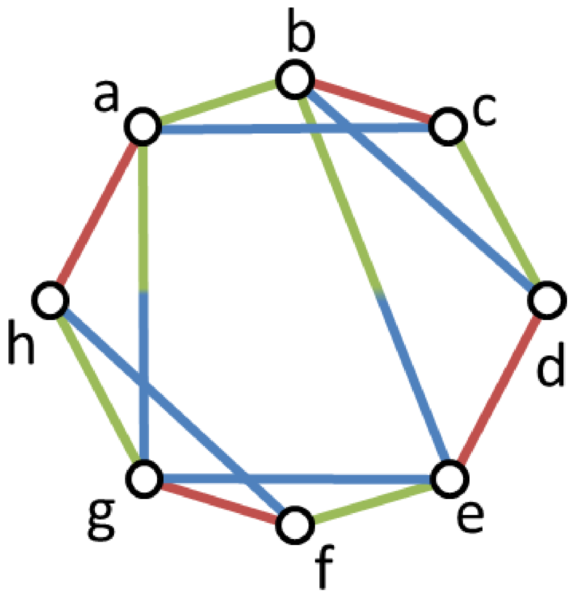


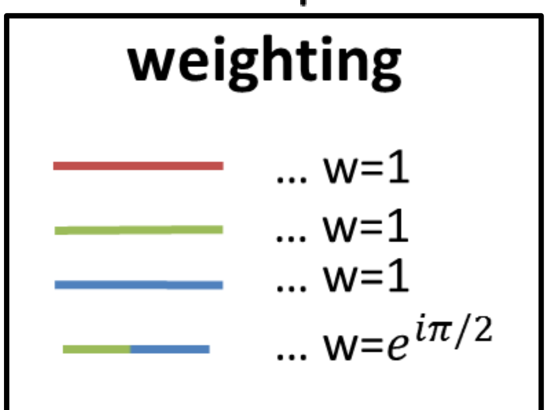


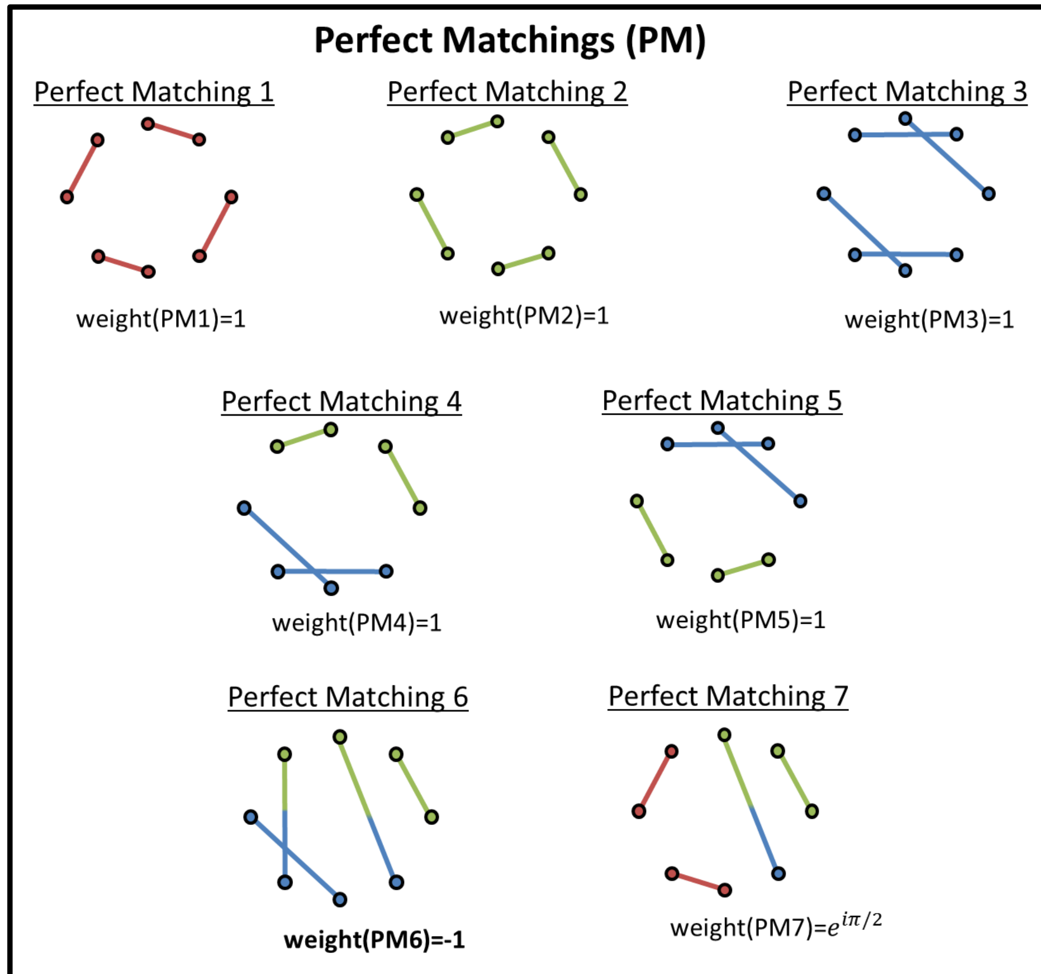
Graph Theory – preparing the stage…

Weight of Perfect Matching (PM): Product of weight of all edges in PM

Inherited Vertex Coloring (IVC): Every Vertex gets the color of the PM's incident edge







Graph Theory – preparing the stage…

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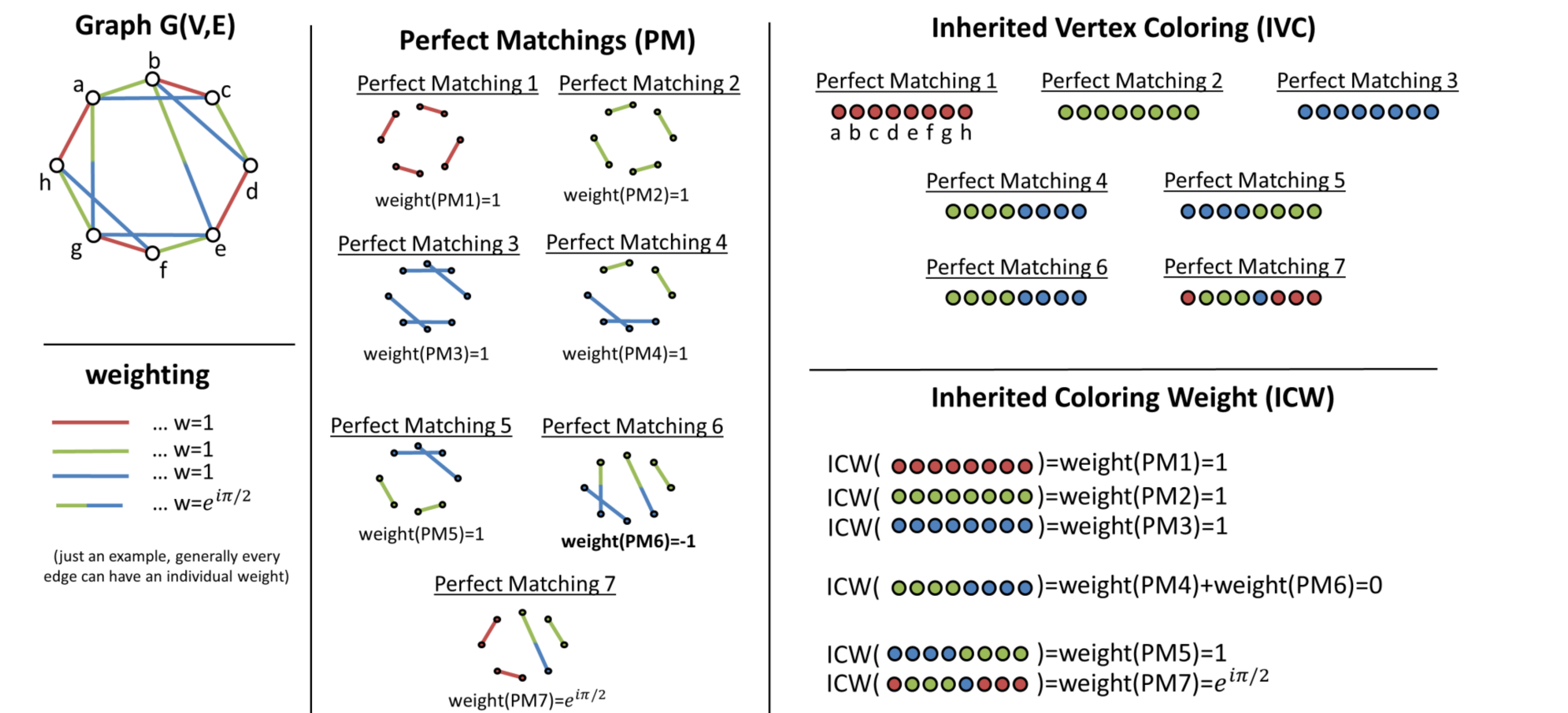
Inherited Vertex Coloring (IVC): Every Vertex gets the color of the PM‘s incident edge

Inherited Coloring Weight (ICW) for some coloring c is the sum of all weights of PMs with coloring c:

$$ICW(c) = \sum_i \prod_{|E_j| \in PM_i} w_{i,j}$$



Graph Theory – the Question

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Question: Is there a bi-colored weighted graph with $|V| > 4$ with three possible edge colors (red, blue, green) with the following properties:

1) For each of the three colors there is a monochromatic vertex coloring c_{mono} with non-zero weight, i.e. $|ICW(c_{mono})| > 0$

2) Every non-monochromatic vertex coloring c_{n-mono} has zero weight, i.e. $|ICW(c_{n-mono})| = 0$.

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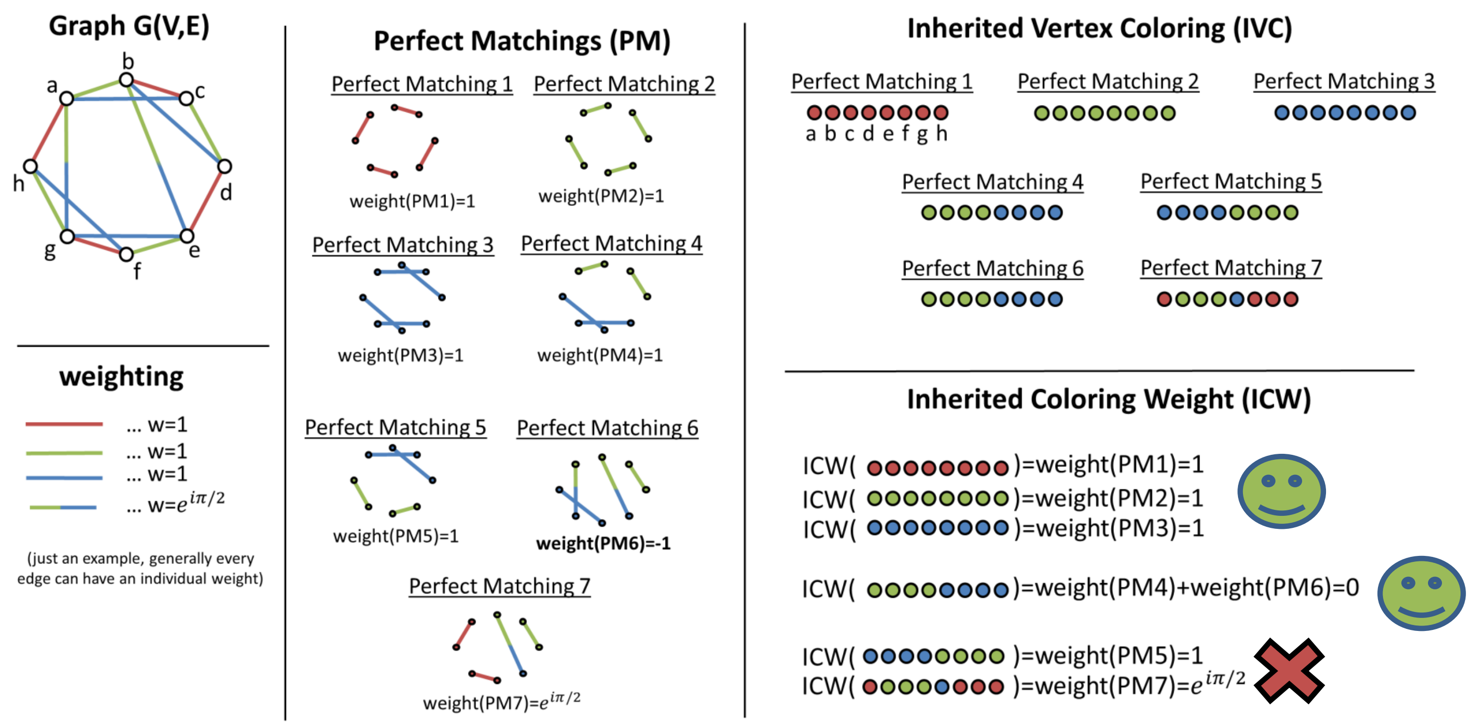

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Partial Solutions

- A weaker form of this question was solved by Ilya Bogdanov. There, the question was for graphs with only monochromatic vertex colorings. (That can only exist for |V|=4 with three colors)
- A related question has been solved by Gjergji Zaimi. He showed that graphs exist where each non-monochromatic coloring appears at least two times, which is a necessary condition for the $|ICW(c_{n-mono})| = 0$.

Graph Theory – the Conjecture

Question: Is there a bi-colored weighted graph with $|V| > 4$ with three possible edge colors (red, blue, green) with the following properties:

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2) Every non-monochromatic vertex coloring c_{n-mono} has zero weight, i.e. $|ICW(c_{n-mono})| = 0$.

My Conjecture: Such a graph does **not** exist.

Any thoughts or ideas:

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THANKS FOR LISTENING!!! ☺