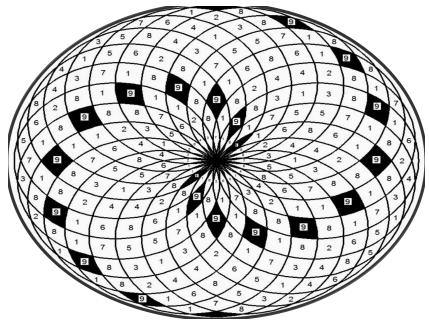
Homology groups of generalized polyomino type tilings

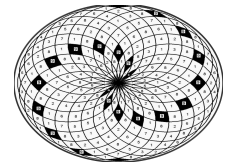
2nd Croatian Combinatorial Days
27 - 28 September
Zagreb

Edin Liđan
lidjan_edin@hotmail.com

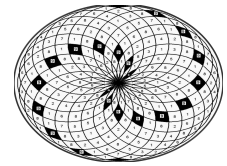
University of Bihać
Bosnia and Herzegovina



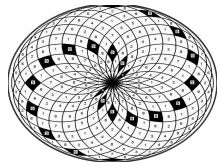
Contents



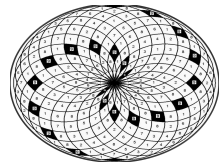
Contents



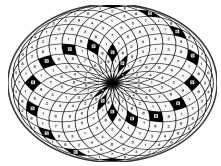
Contents



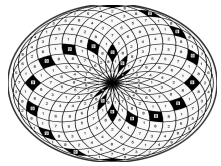
Contents



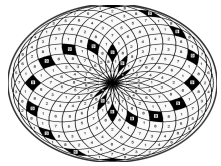
Contents



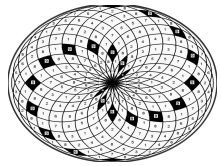
Contents



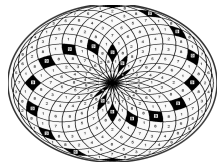
Contents



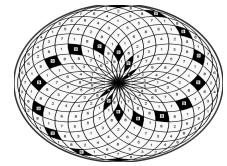
Contents



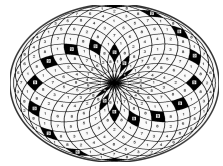
Contents



Introduction



Introduction



- Tiling, covering, packing

Introduction

- Tiling, covering, packing

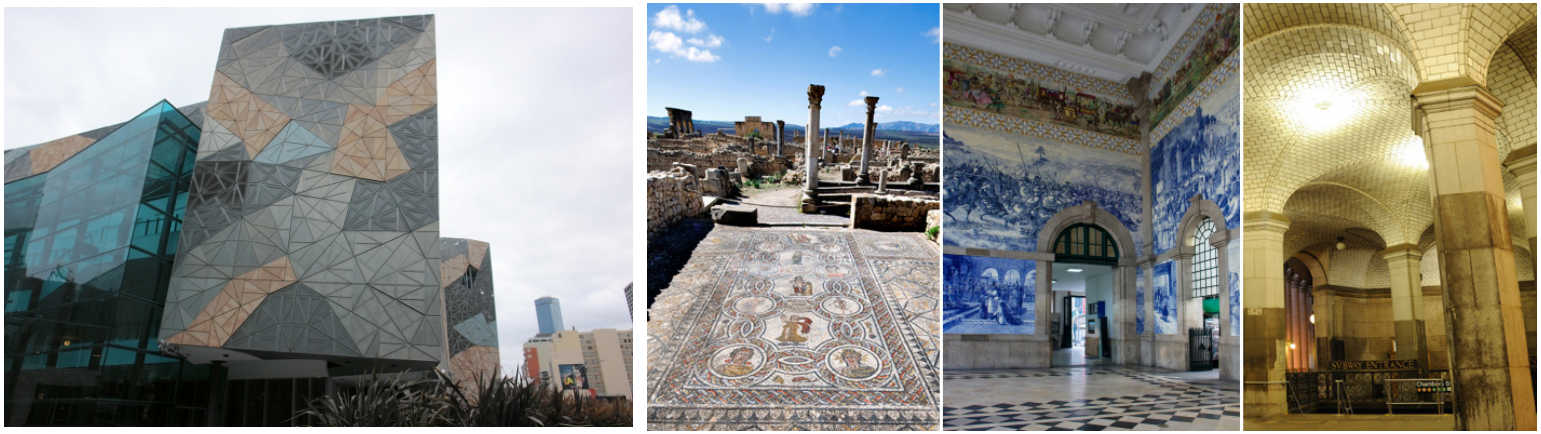
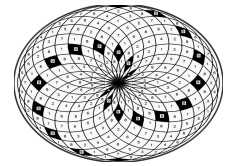
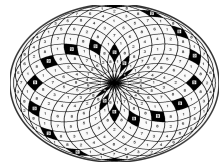


Figure: Tilings in arts and popular culture

Tiling problems

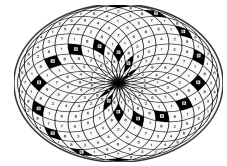


Tiling problems



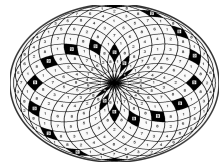
- Region for tiling

Tiling problems



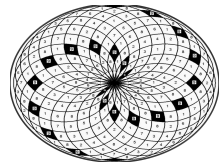
- Region for tiling
  - Finite region

Tiling problems



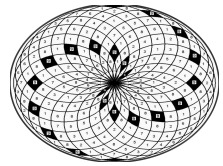
- Region for tiling
  - Finite region
  - Plane

Tiling problems



- Region for tiling
  - Finite region
  - Plane
  - Surface

Tiling problems

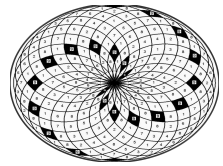


- Region for tiling
  - Finite region
  - Plane
  - Surface
- A finite set Σ of tiles

Definition

- A region M and finite set Σ of tile
- Does Σ tile tiles M?

Tiling problems

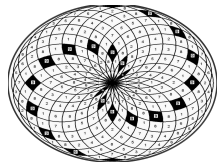


- Region for tiling
  - Finite region
  - Plane
  - Surface
- A finite set Σ of tiles

Definition

- A region M and finite set Σ of tile
- Does Σ tile tiles M?
- Is there a tiling?

Tiling problems



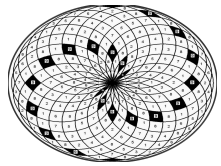
- Region for tiling
  - Finite region
  - Plane
  - Surface
- A finite set Σ of tiles

Definition

- A region M and finite set Σ of tile
- Does Σ tile tiles M?

- Is there a tiling?
- How many different tilings are there?

Tiling problems



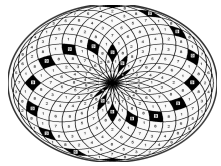
- Region for tiling
  - Finite region
  - Plane
  - Surface
- A finite set Σ of tiles

Definition

- A region M and finite set Σ of tile
- Does Σ tile tiles M?

- Is there a tiling?
- How many different tilings are there?
- Is a tiling easy to find?

Tiling problems

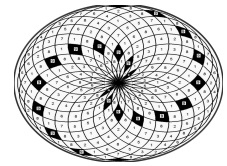


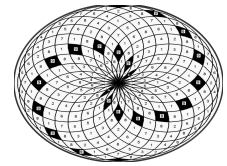
- Region for tiling
  - Finite region
  - Plane
  - Surface
- A finite set Σ of tiles

Definition

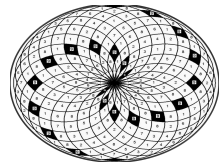
- A region M and finite set Σ of tile
- Does Σ tile tiles M?

- Is there a tiling?
- How many different tilings are there?
- Is a tiling easy to find?
- Is it easy to prove a tiling doesn't exist?





- Shapes



- Shapes

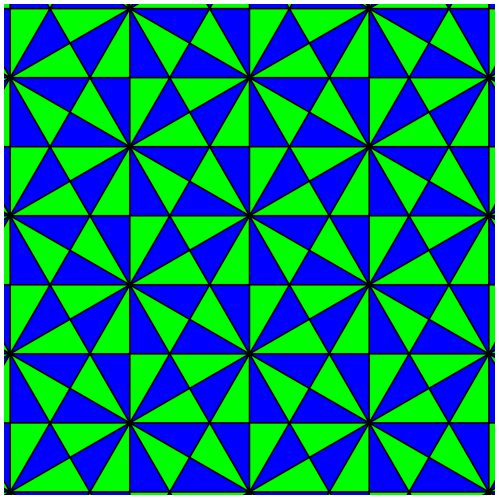
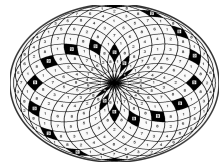


Figure: Triangular lattice (Polyamonds)



- Shapes

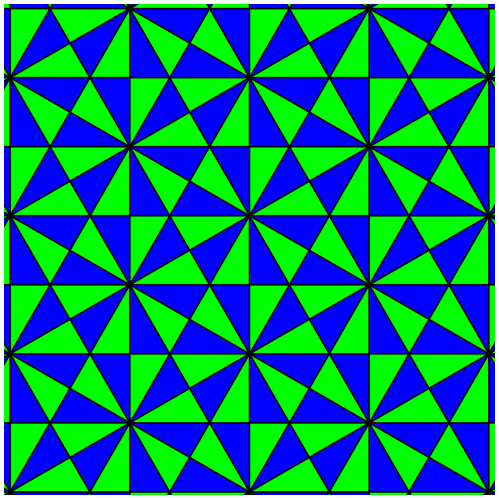


Figure: Triangular lattice (Polyamonds)

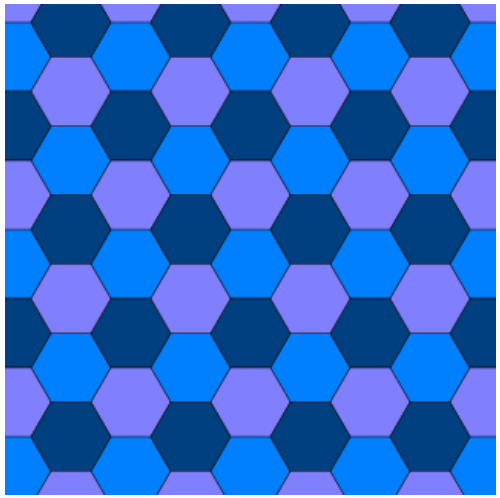


Figure: Hexagonal lattice (Polyhes)

- Shapes

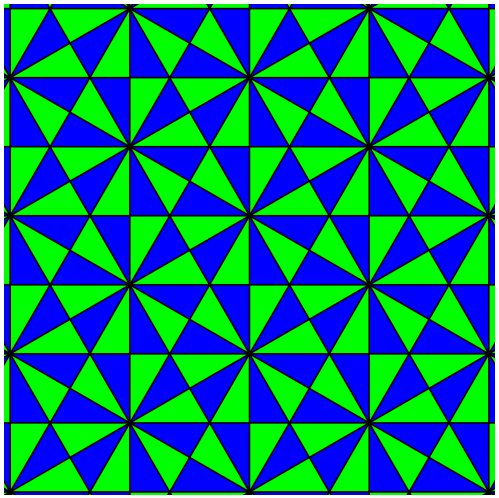


Figure: Triangular lattice (Polyamonds)

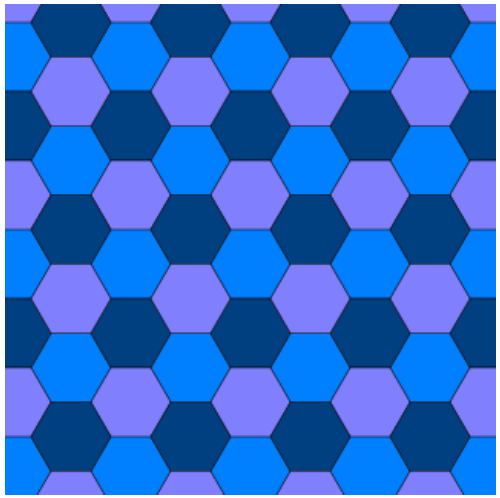


Figure: Hexagonal lattice (Polyhes)

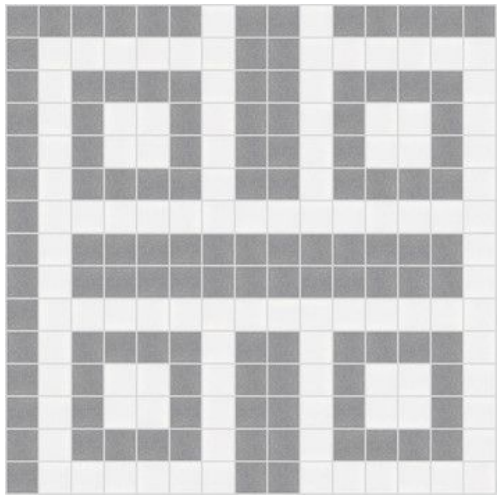


Figure: Square lattice (Polyominoes)

- Shapes

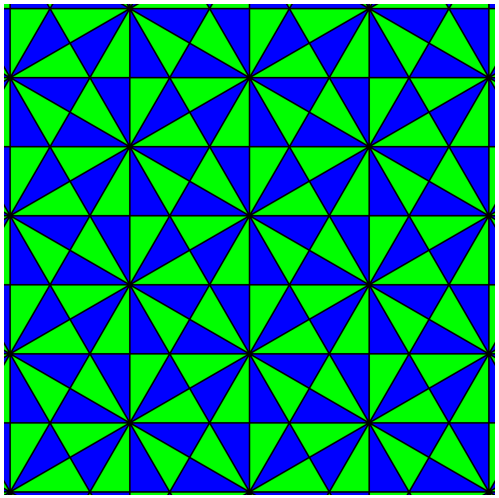


Figure: Triangular lattice (Polyamonds)

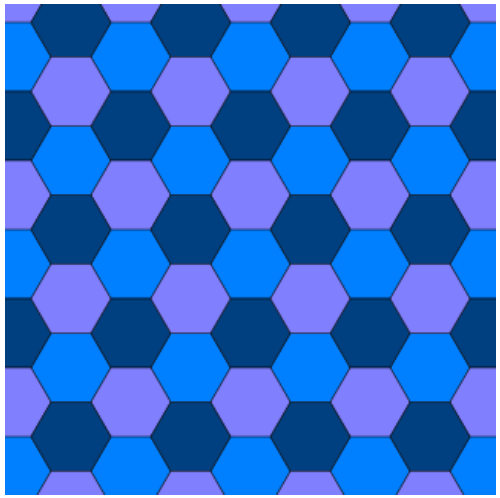


Figure: Hexagonal lattice (Polyhes)

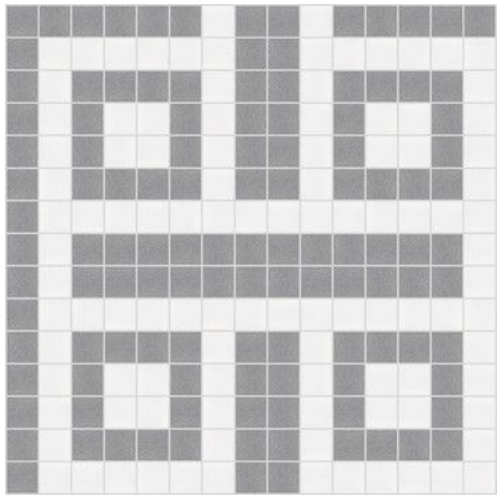


Figure: Square lattice (Polyominoes)

- periodic

- Shapes

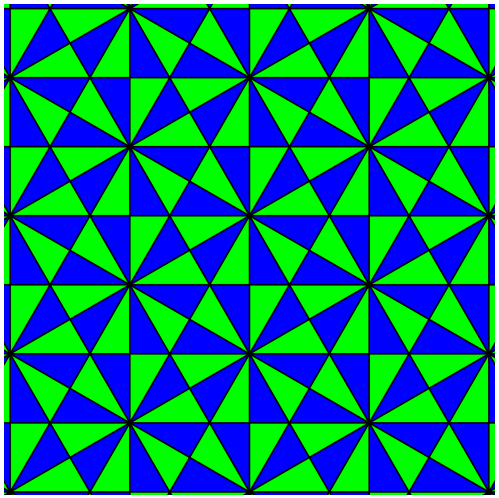


Figure: Triangular lattice (Polyamonds)

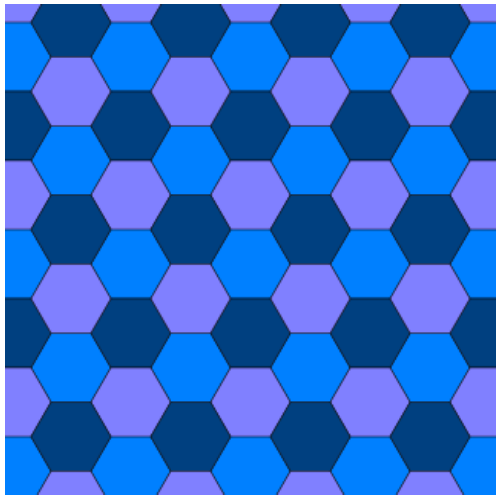


Figure: Hexagonal lattice (Polyhes)

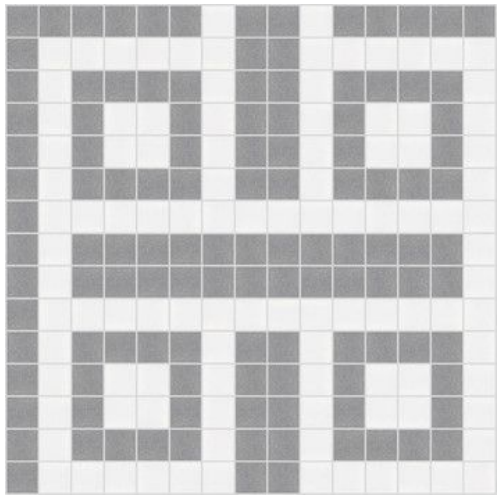
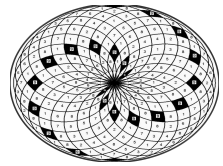


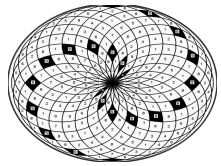
Figure: Square lattice (Polyominoes)

- periodic
- aperiodic

Polyomino



Polyomino



- Polyomino

Polyomino

- Polyomino

Figure: Polyomino

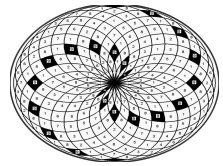
Polyomino

- Polyomino

Figure: Polyomino

Figure: Not a polyomino

Polyomino



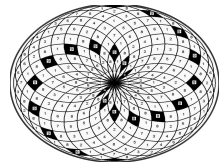
- Polyomino

Figure: Polyomino

Figure: Not a polyomino

- Solomon W. Golomb (1965.)

Polyomino



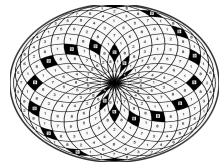
- Polyomino

Figure: Polyomino

Figure: Not a polyomino

- Solomon W. Golomb (1965.)
- Martin Gardner Scientific American, "Mathematical Games"

Classification of polyominoes



Classification of polyominoes

Figure:
Monomino

Classification of polyominoes

Figure:
Monomino

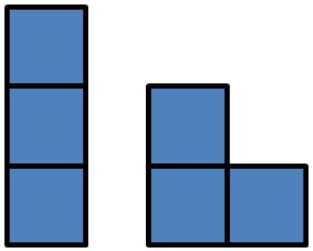


Figure:
Trominoes

Figure:
Domino

Classification of polyominoes

Figure: Monomino

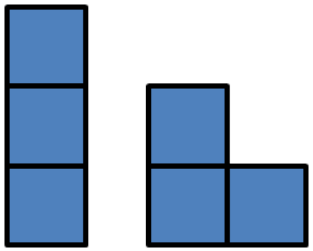


Figure: Trominoes

Figure: Domino

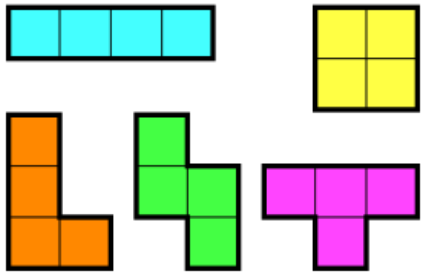


Figure: Tetrominoes

Classification of polyominoes

Figure: Monomino

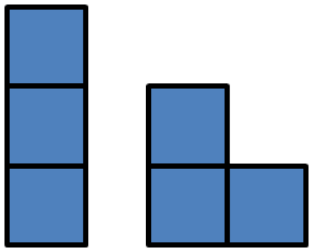


Figure: Trominoes

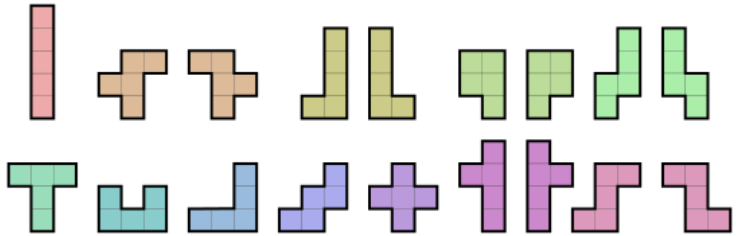


Figure: Pentominoes

Figure: Domino

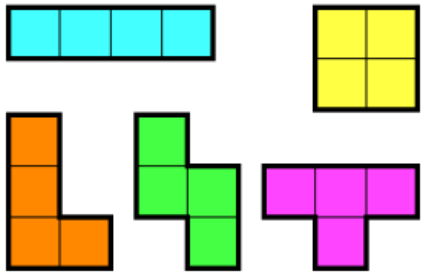


Figure: Tetrominoes

Classification of polyominoes

Figure: Monomino

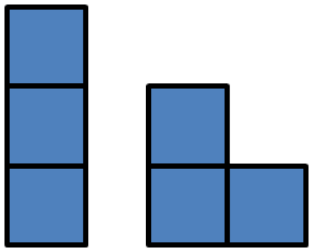


Figure: Trominoes

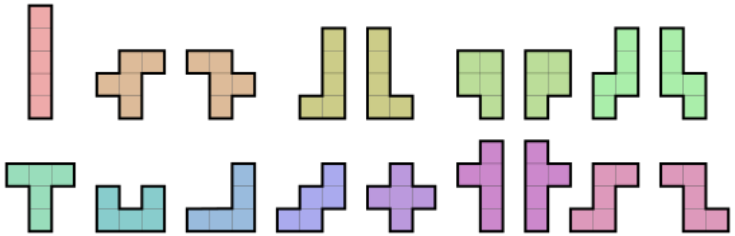


Figure: Pentominoes

Figure: Domino

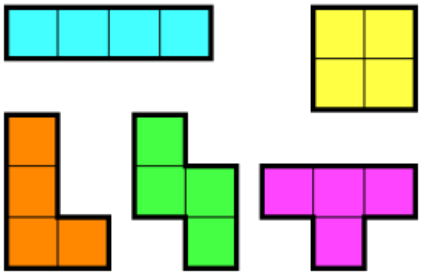


Figure: Tetrominoes

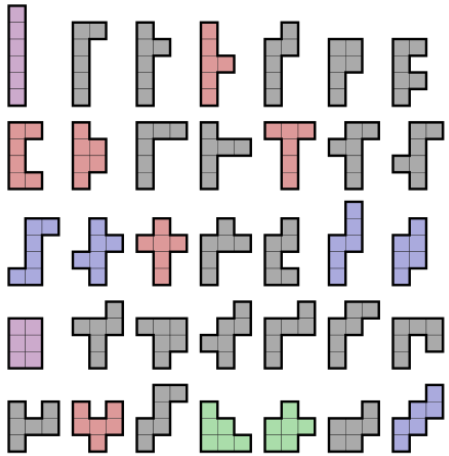
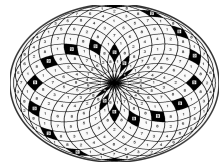
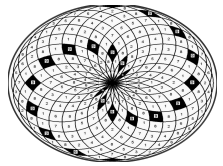


Figure: Hexominoes

Tilings with polyominoes. Homology group.

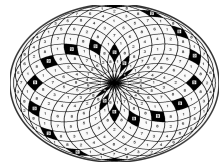


Tilings with polyominoes. Homology group.



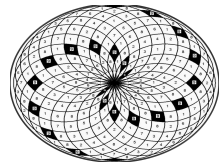
- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)

Tilings with polyominoes. Homology group.



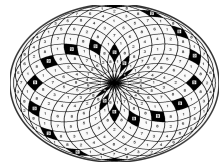
- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)
- Michael Reid, Tile homotopy group (2003)

Tilings with polyominoes. Homology group.



- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)
- Michael Reid, Tile homotopy group (2003)
- Homology group

Tilings with polyominoes. Homology group.

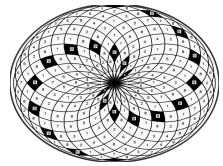


- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)
- Michael Reid, Tile homotopy group (2003)
- Homology group

Definition (Homology group)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

Tilings with polyominoes. Homology group.



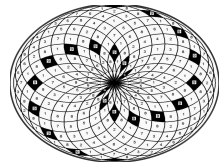
- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)
- Michael Reid, Tile homotopy group (2003)
- Homology group

Definition (Homology group)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ

Tilings with polyominoes. Homology group.



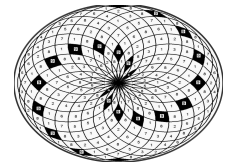
- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)
- Michael Reid, Tile homotopy group (2003)
- Homology group

Definition (Homology group)

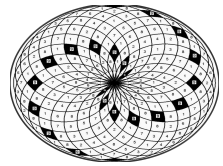
The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ
- A is the free abelian group (on all the cells of the square lattice).

Homology groups of tilings

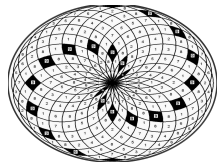


Homology groups of tilings



- We consider whether exists a proper tiling of given region M (surface, surface with the boundary, etc.) subdivided into "cells" like grid with a tiles from a given set Σ.

Homology groups of tilings

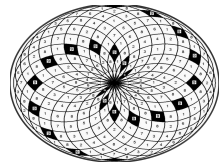


- We consider whether exists a proper tiling of given region M (surface, surface with the boundary, etc.) subdivided into "cells" like grid with a tiles from a given set Σ.

Definition (Homology group of tilings)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

Homology groups of tilings



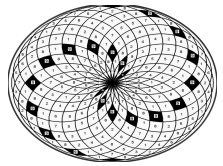
- We consider whether exists a proper tiling of given region M (surface, surface with the boundary, etc.) subdivided into "cells" like grid with a tiles from a given set Σ.

Definition (Homology group of tilings)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ

Homology groups of tilings



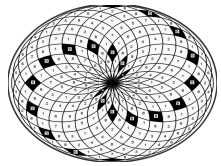
- We consider whether exists a proper tiling of given region M (surface, surface with the boundary, etc.) subdivided into "cells" like grid with a tiles from a given set Σ.

Definition (Homology group of tilings)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ
- A is free Abelian group on all the cells of given region M.

Homology groups of tilings



- We consider whether exists a proper tiling of given region M (surface, surface with the boundary, etc.) subdivided into "cells" like grid with a tiles from a given set Σ.

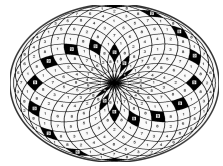
Definition (Homology group of tilings)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

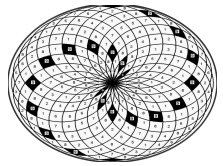
- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ
- A is free Abelian group on all the cells of given region M.

- A necessary condition for existence of a proper tiling is that the element corresponding to the sum of all cells of M is trivial in the homology group of tilings Σ.

Homology groups of generalized polyomino type tillings



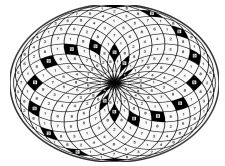
Homology groups of generalized polyomino type tillings



Example

Is it possible to tile torus chessboard 6×6 with tiles 1×4 (all orientation are allowed)?

Homology groups of generalized polyomino type tillings



Example

Is it possible to tile torus chessboard 6×6 with tiles 1×4 (all orientation are allowed)?

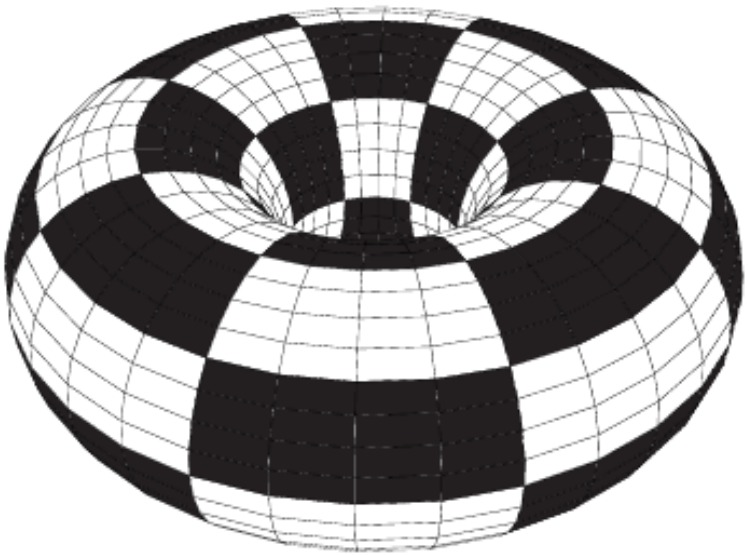
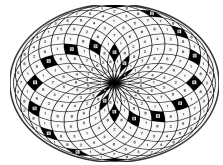


Figure: Torus Chessboard

Homology groups of generalized polyomino type tillings



Example

Is it possible to tile torus chessboard 6×6 with tiles 1×4 (all orientation are allowed)?

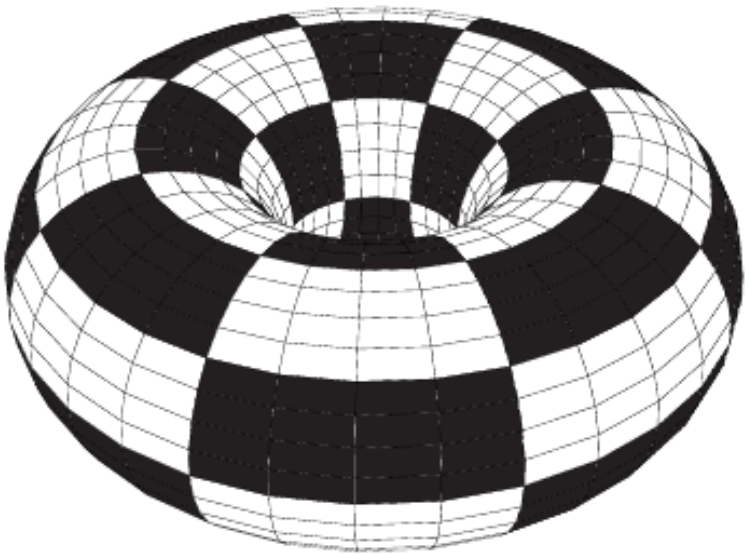


Figure: Torus Chessboard

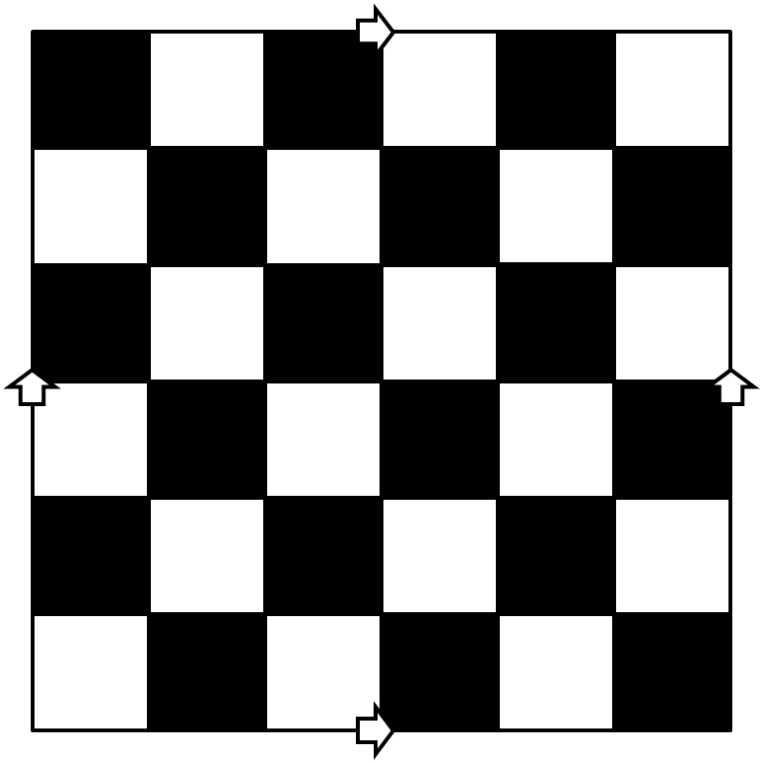
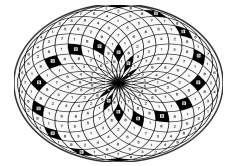


Figure: In torus plane model



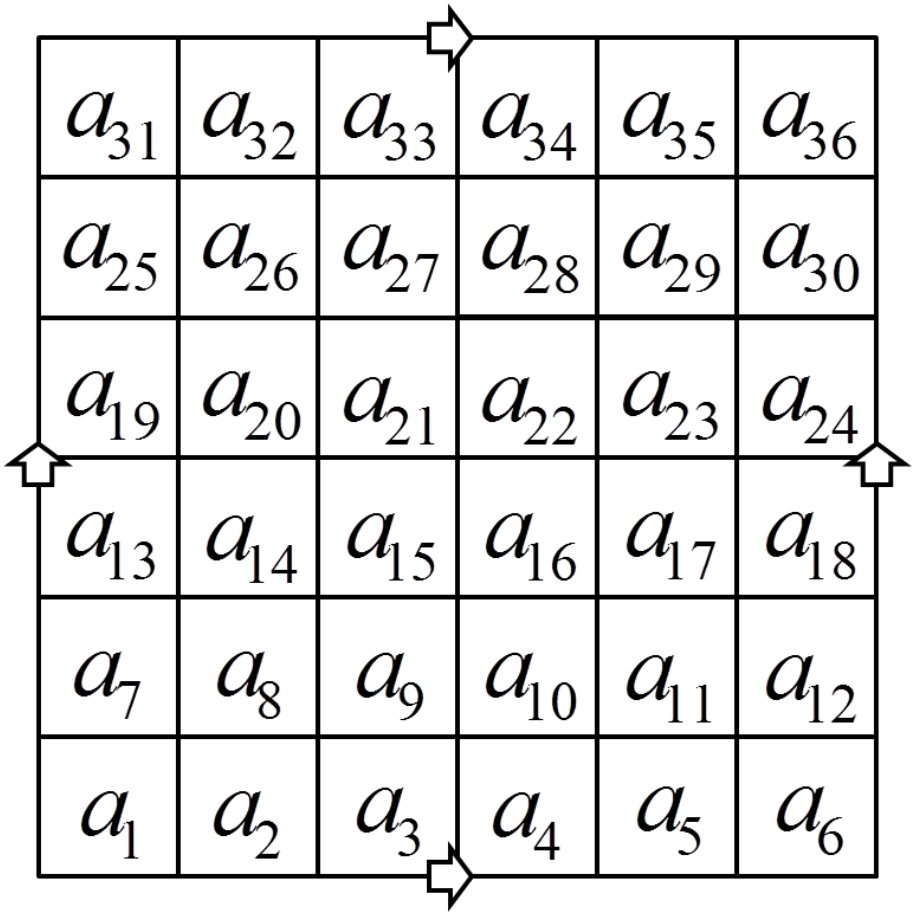


Figure: Naming cells

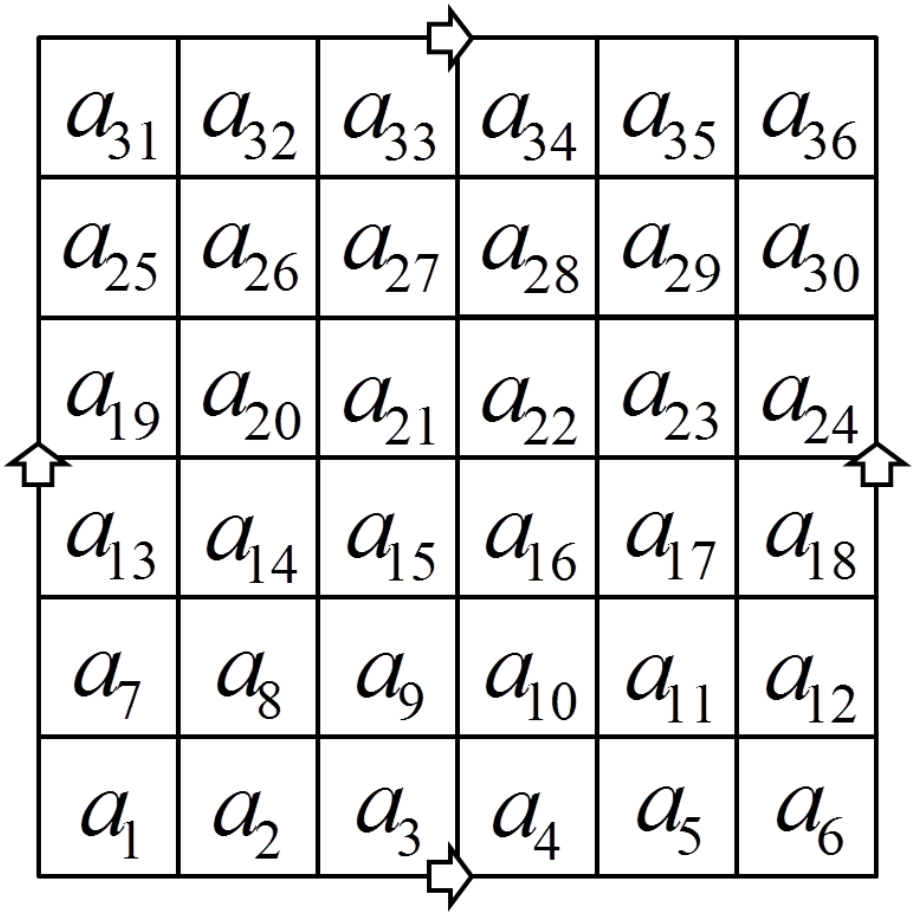


Figure: Naming cells

$$a_1 + a_2 + a_3 + a_4 = 0$$

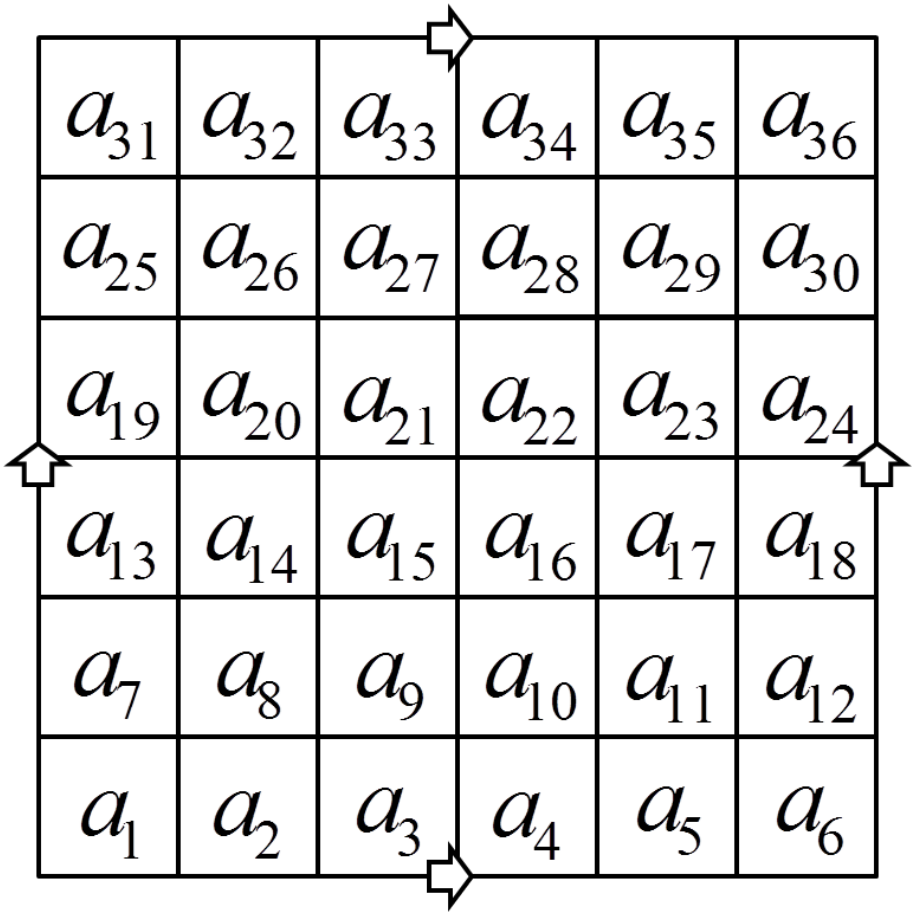


Figure: Naming cells

$$a_1 + a_2 + a_3 + a_4 = 0$$
$$a_2 + a_3 + a_4 + a_5 = 0$$

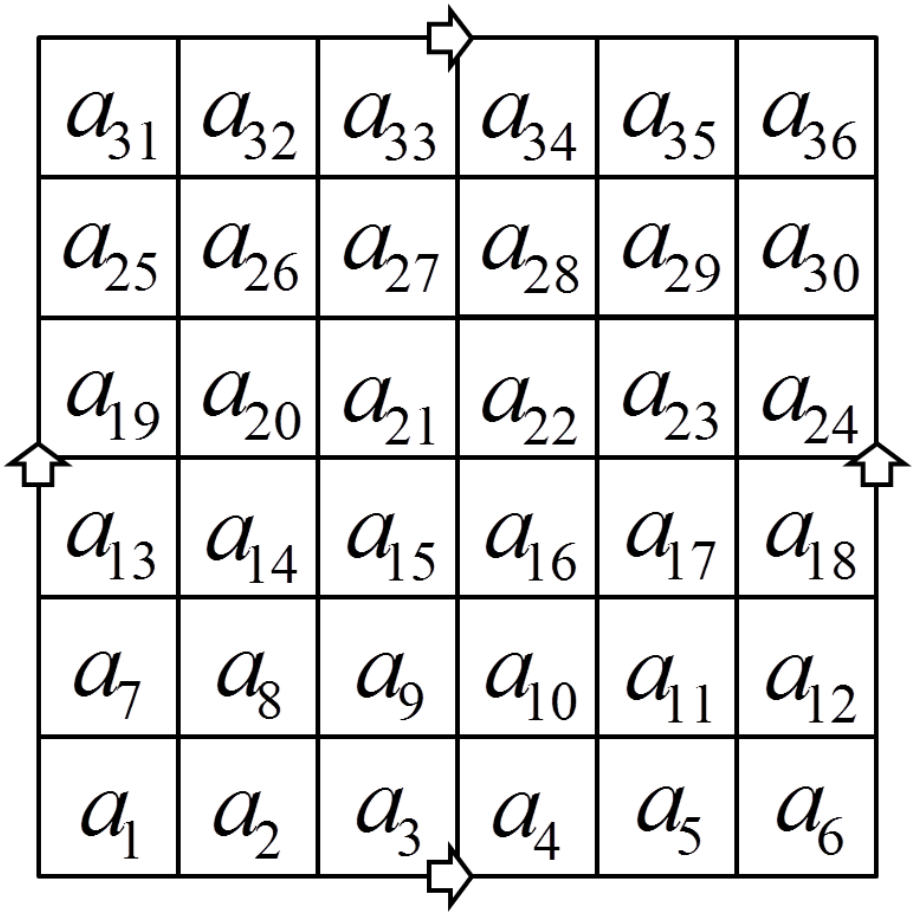


Figure: Naming cells

$$a_1 + a_2 + a_3 + a_4 = 0$$
$$a_2 + a_3 + a_4 + a_5 = 0$$
$$a_3 + a_4 + a_5 + a_6 = 0$$

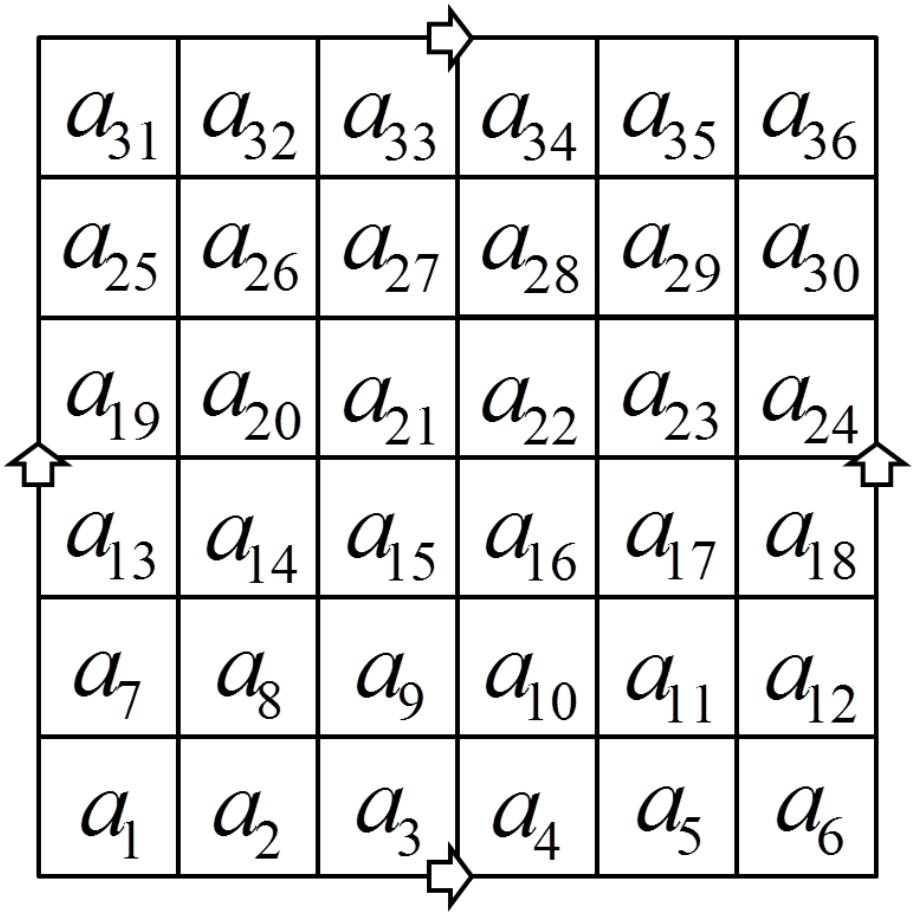


Figure: Naming cells

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0 \\
a_3 + a_4 + a_5 + a_6 &= 0 \\
a_4 + a_5 + a_6 + a_1 &= 0
\end{aligned}
$$

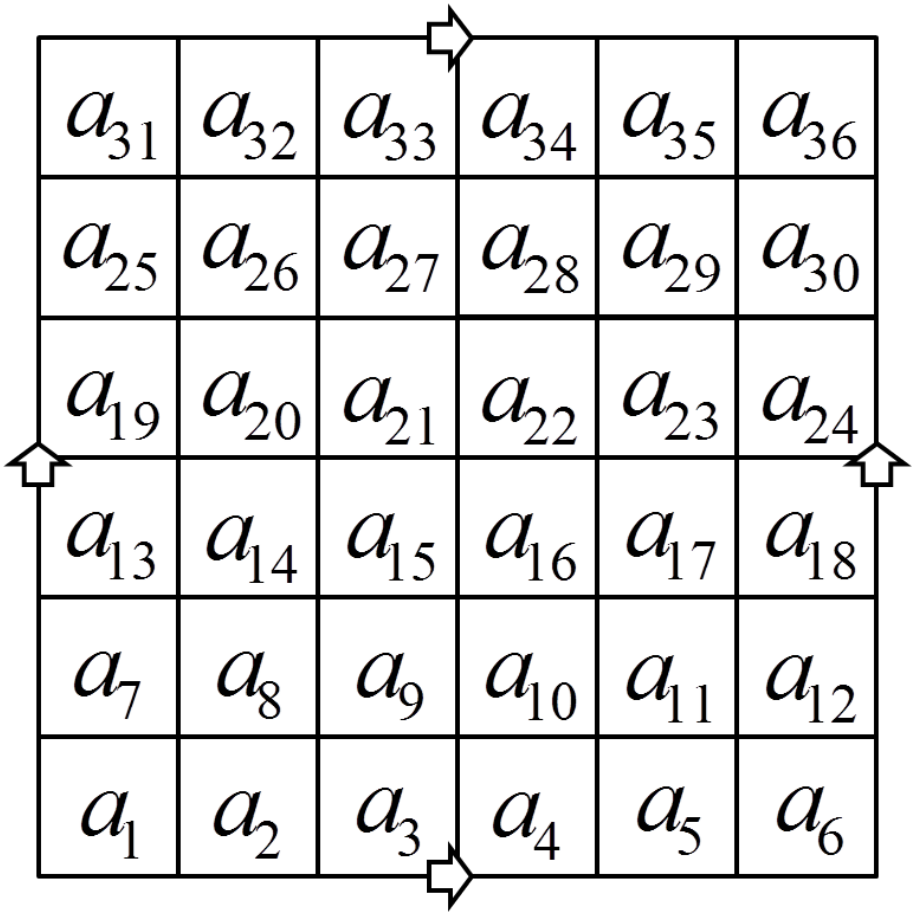


Figure: Naming cells

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0 \\
a_3 + a_4 + a_5 + a_6 &= 0 \\
a_4 + a_5 + a_6 + a_1 &= 0 \\
a_5 + a_6 + a_1 + a_2 &= 0
\end{aligned}
$$

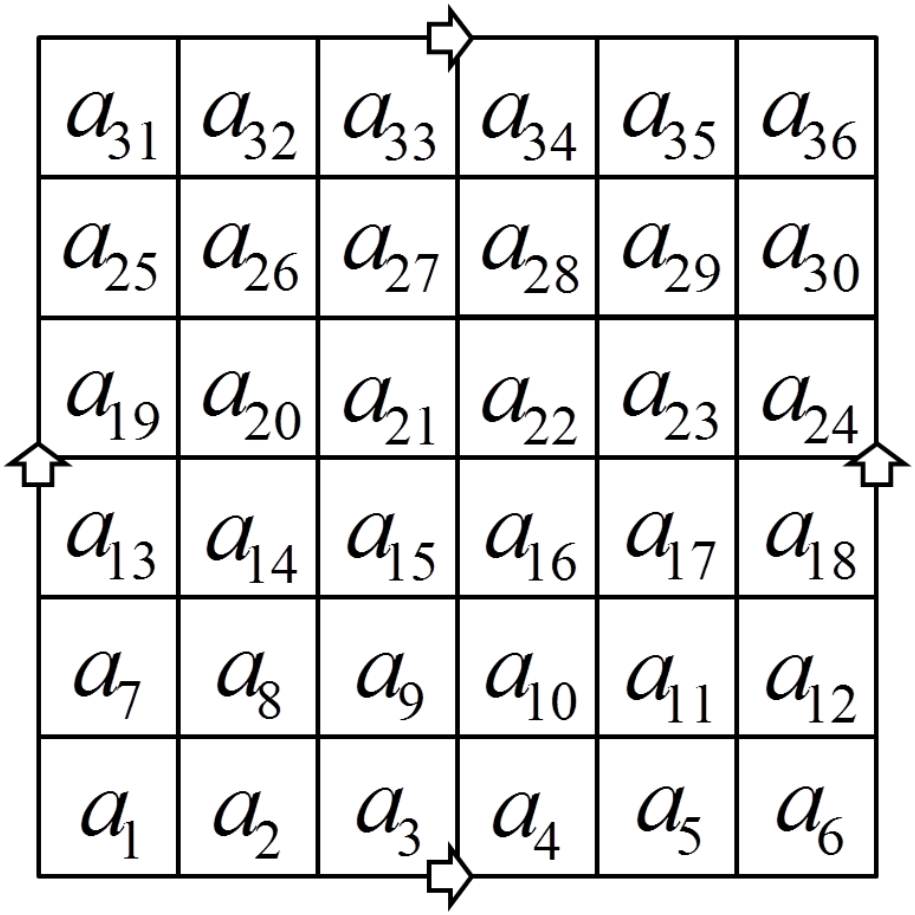


Figure: Naming cells

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0 \\
a_3 + a_4 + a_5 + a_6 &= 0 \\
a_4 + a_5 + a_6 + a_1 &= 0 \\
a_5 + a_6 + a_1 + a_2 &= 0
\end{aligned}
$$

- relation in finite group

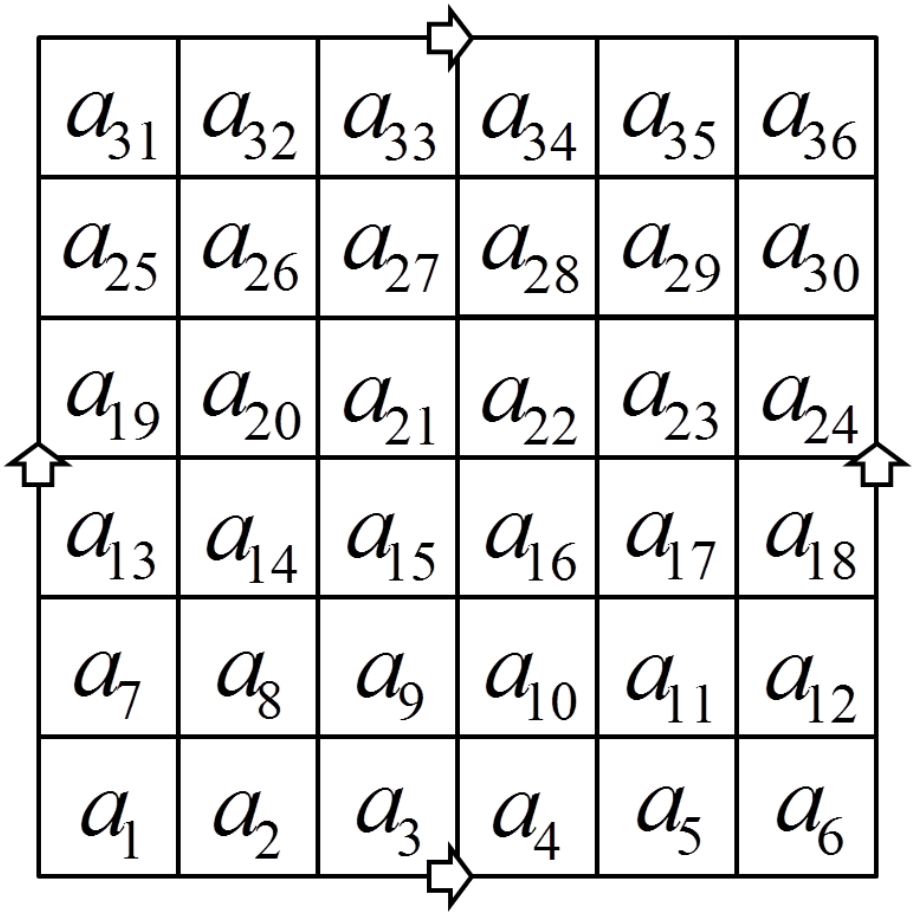


Figure: Naming cells

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0 \\
a_3 + a_4 + a_5 + a_6 &= 0 \\
a_4 + a_5 + a_6 + a_1 &= 0 \\
a_5 + a_6 + a_1 + a_2 &= 0
\end{aligned}
$$

- relation in finite group

$$a_1 = a_5 = a_3$$

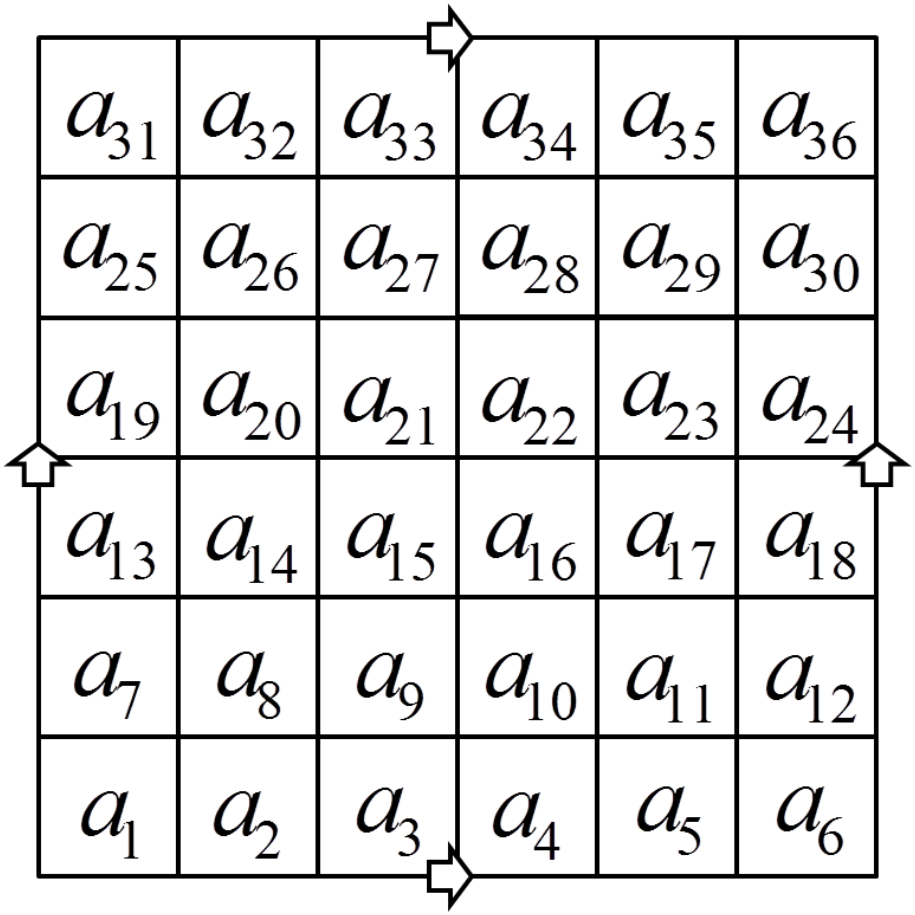
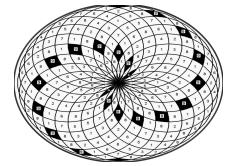


Figure: Naming cells

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0 \\
a_3 + a_4 + a_5 + a_6 &= 0 \\
a_4 + a_5 + a_6 + a_1 &= 0 \\
a_5 + a_6 + a_1 + a_2 &= 0
\end{aligned}
$$

- relation in finite group

$$a_1 = a_5 = a_3$$
$$a_2 = a_6 = a_4$$



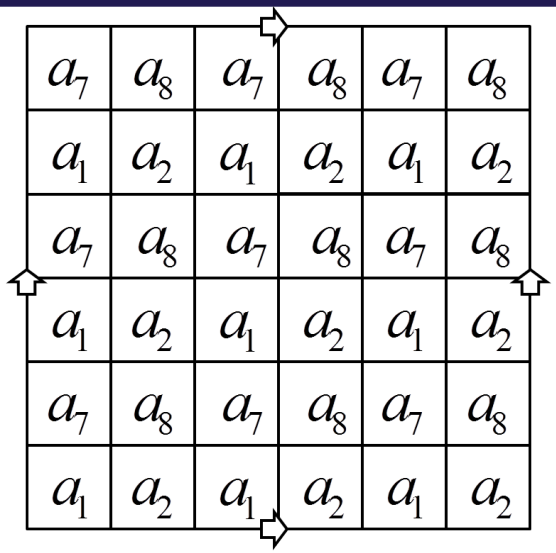


Figure: Equivalent cells

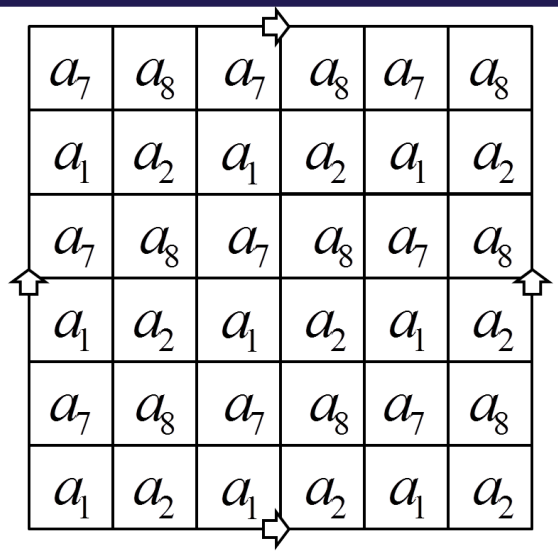


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

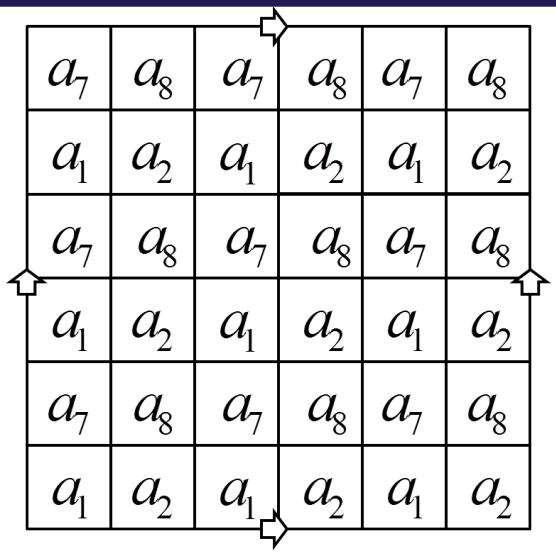


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0$$

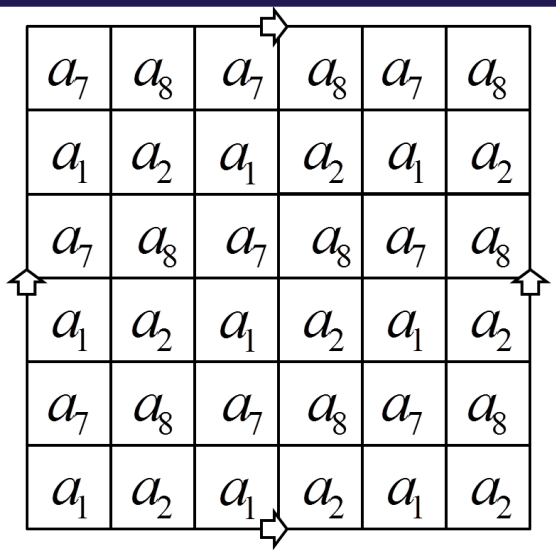


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0$$
$$2a_7 + 2a_8 = 0$$

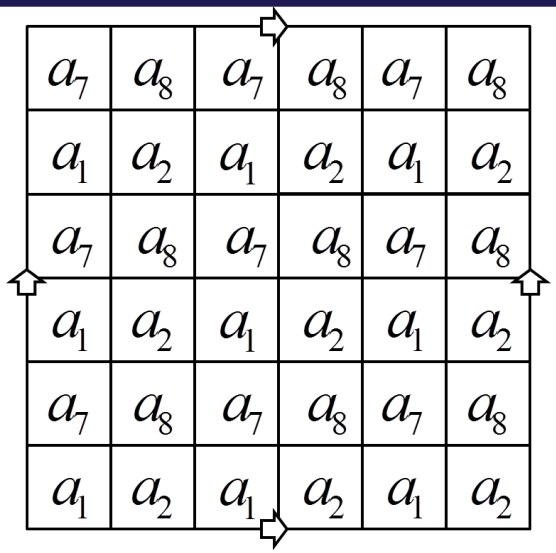


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0$$

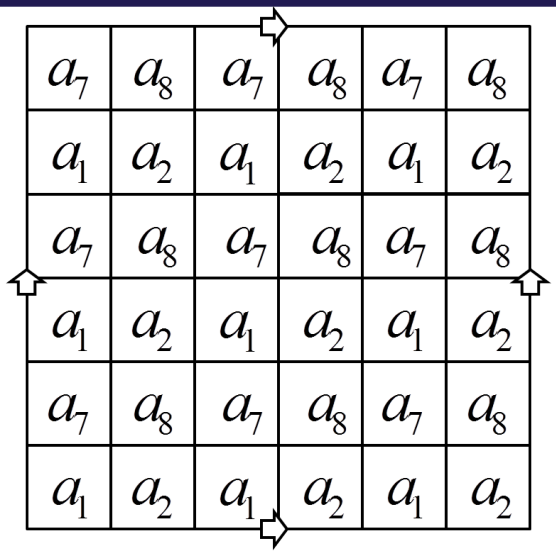


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

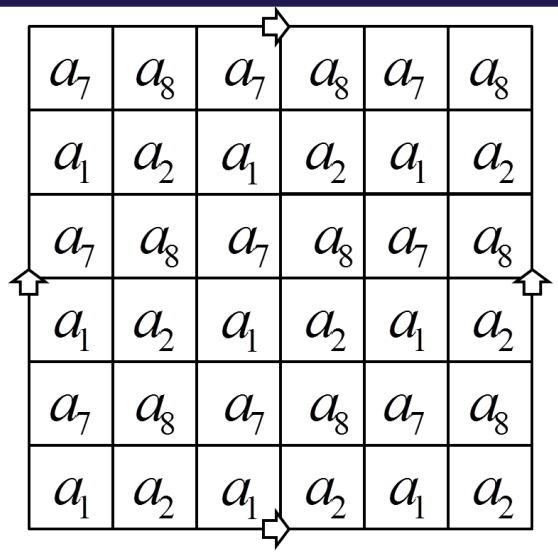


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

- 4 generators a_1, a_2, a_7, a_8

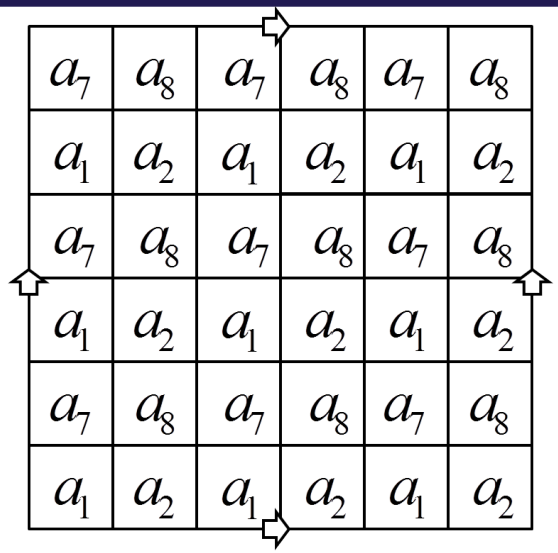


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

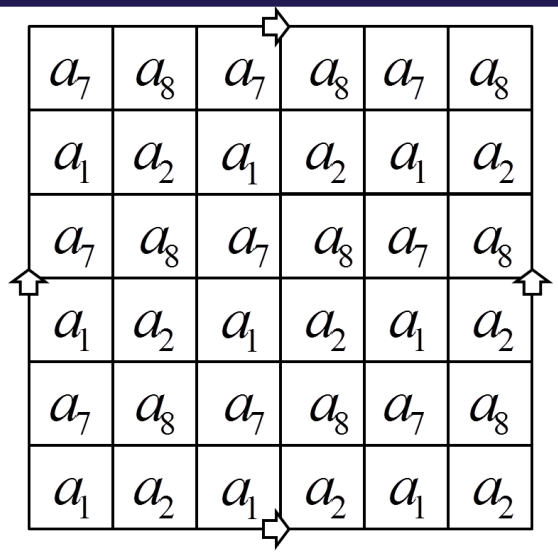


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1,

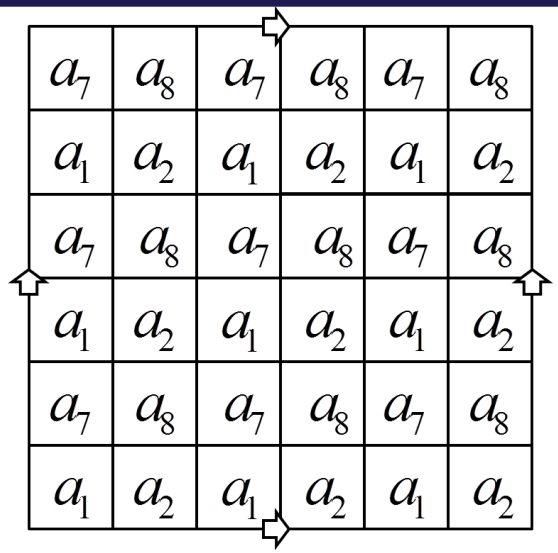


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1, 9 cells a_2,

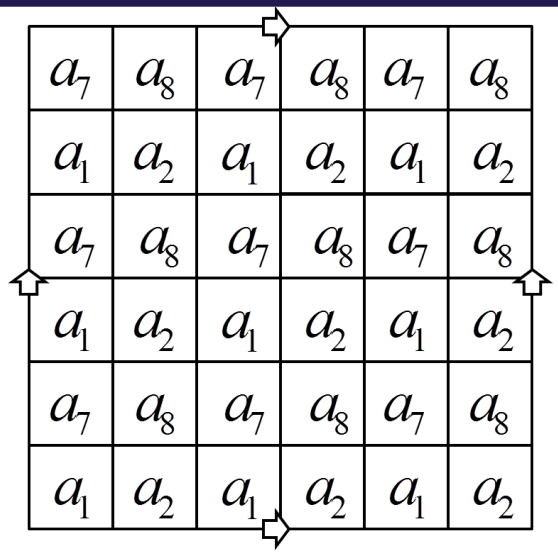


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1, 9 cells a_2, 9 cells a_7,

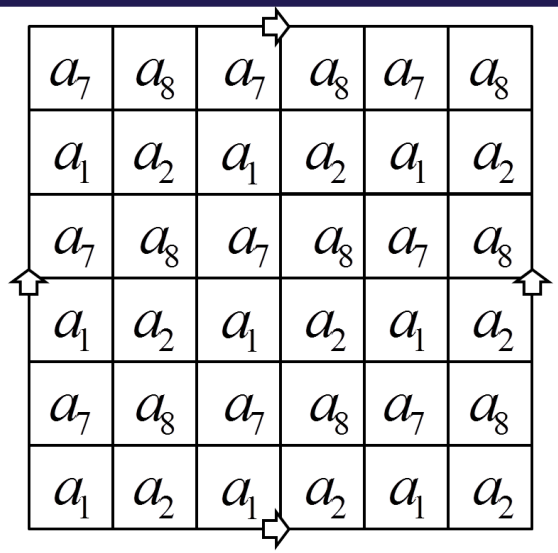


Figure: Equivalent cells

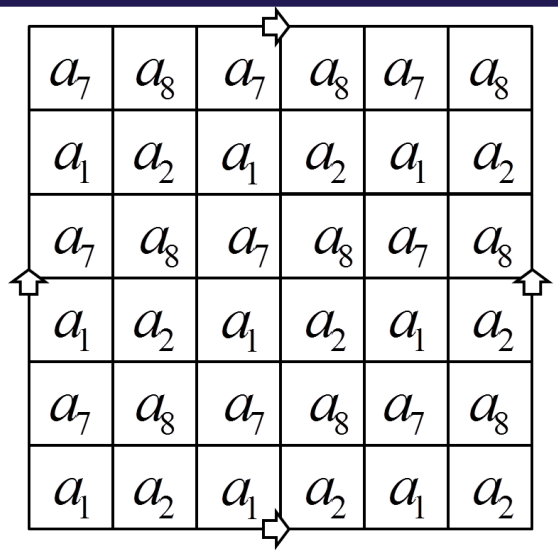
- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8



- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

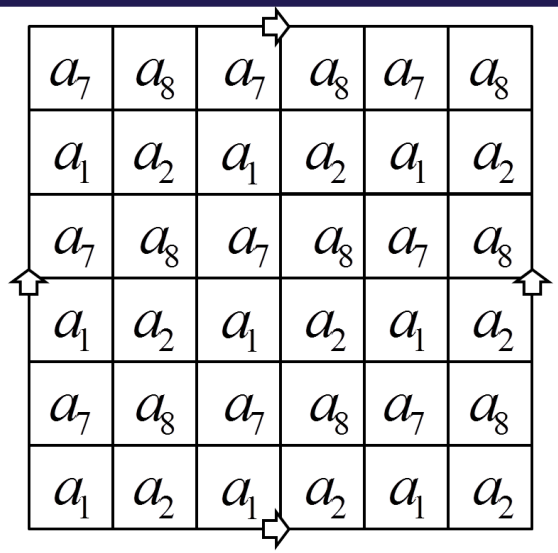
Figure: Equivalent cells

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8

$$a_1 + a_2 + a_7 + a_8$$



- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

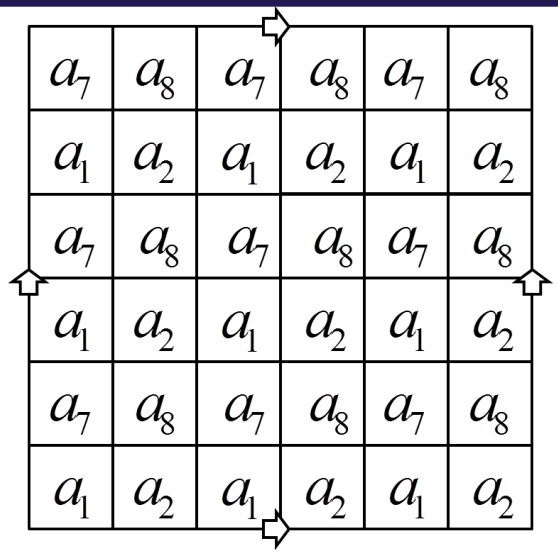
Figure: Equivalent cells

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8

$$a_1 + a_2 + a_7 + a_8$$



- if we put now tile 1×4 on our chessboard

$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
$$2a_7 + 2a_8 = 0 \qquad 2a_2 + 2a_8 = 0$$

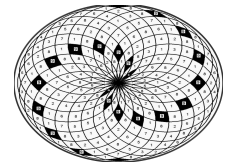
Figure: Equivalent cells

- 4 generators a_1, a_2, a_7, a_8
- Homology groups

$$< G(a_1, a_2, a_7, a_8 | 2a_1+2a_2, 2a_7+2a_8, 2a_1+2a_7, 2a_2+2a_8) >$$

- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8

$$a_1 + a_2 + a_7 + a_8$$



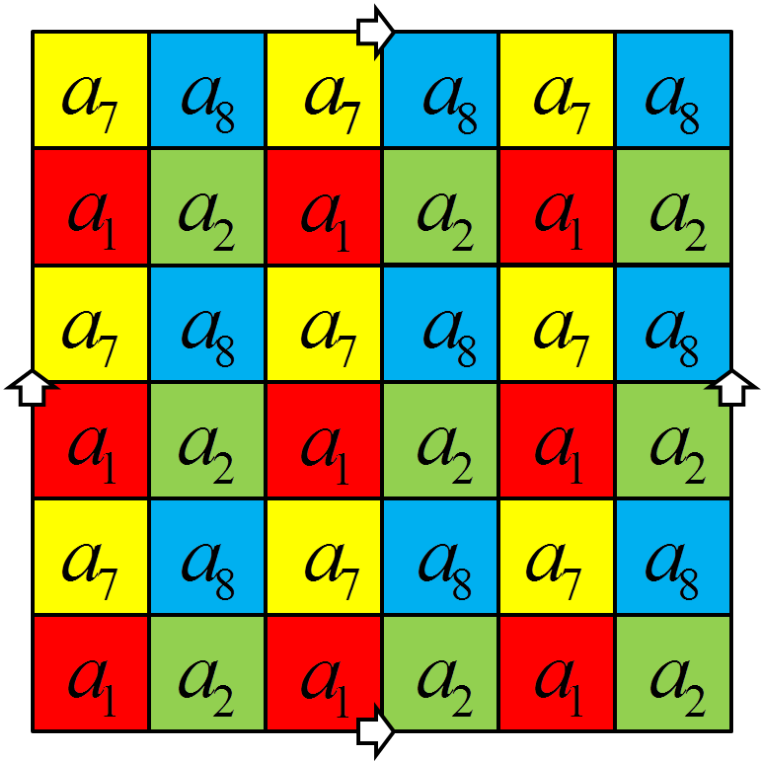


Figure: Coloring the Chessboard

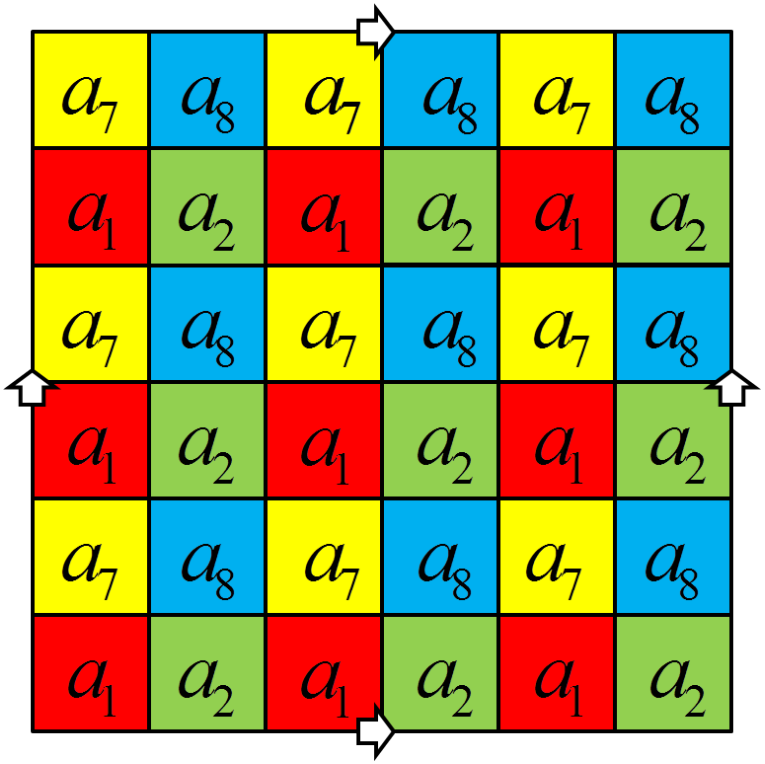


Figure: Coloring the Chessboard

- 9 red cells

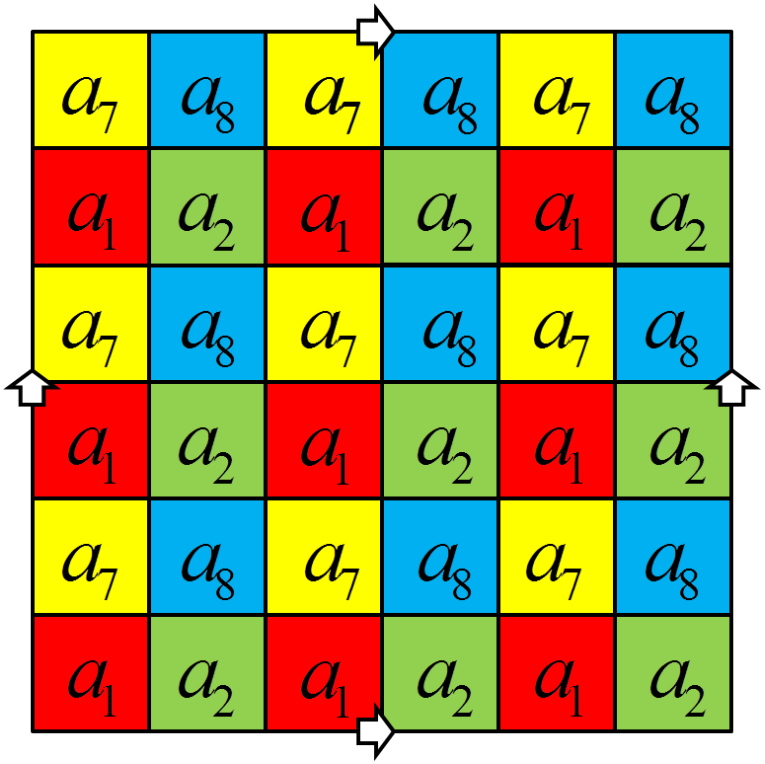


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells

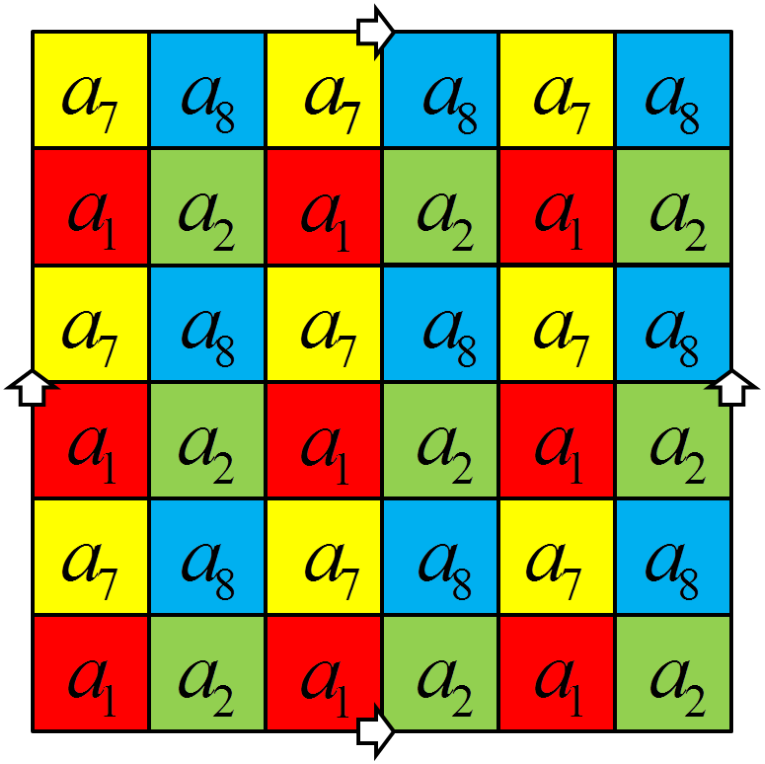


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells

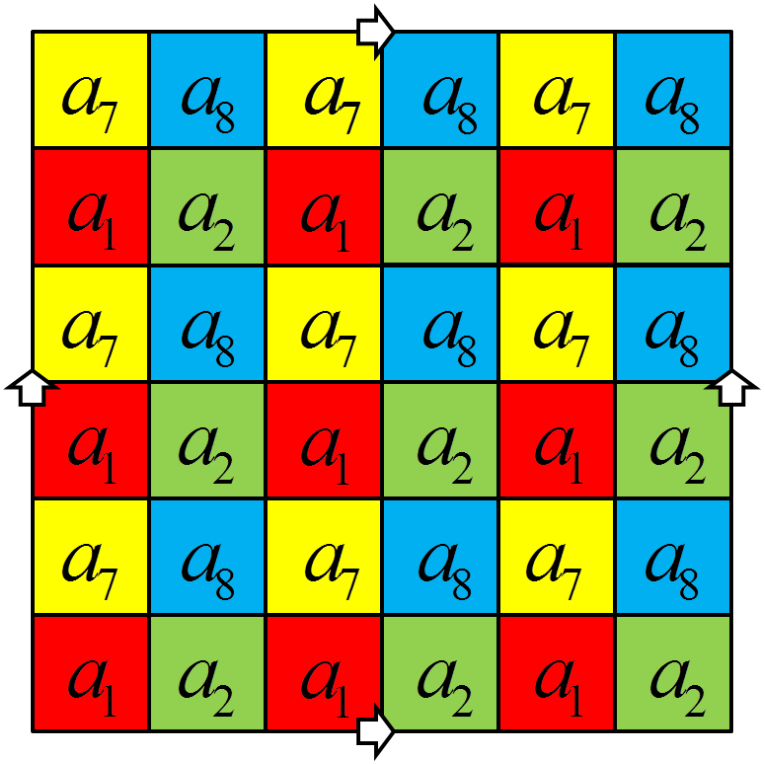


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells

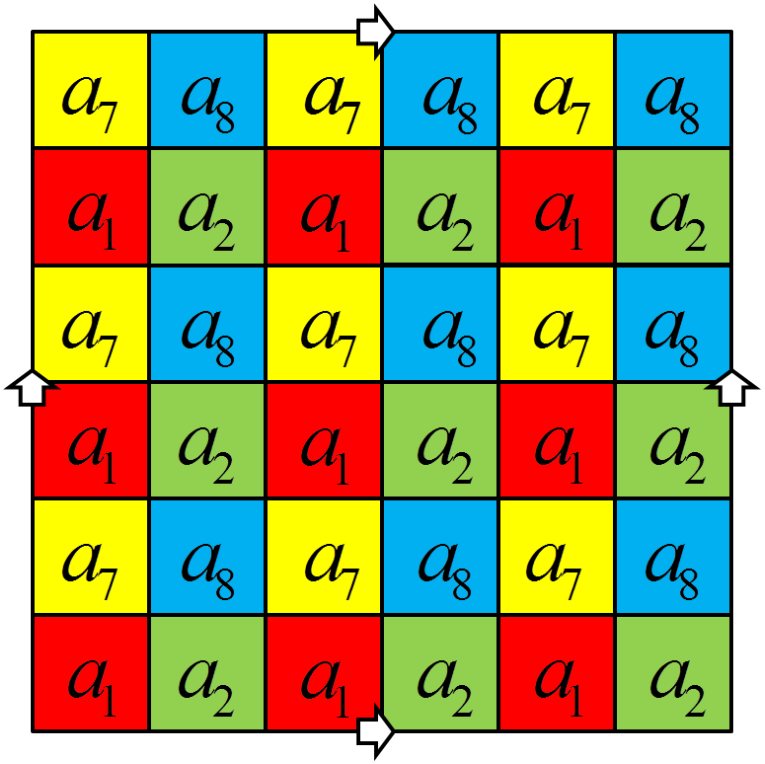


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering

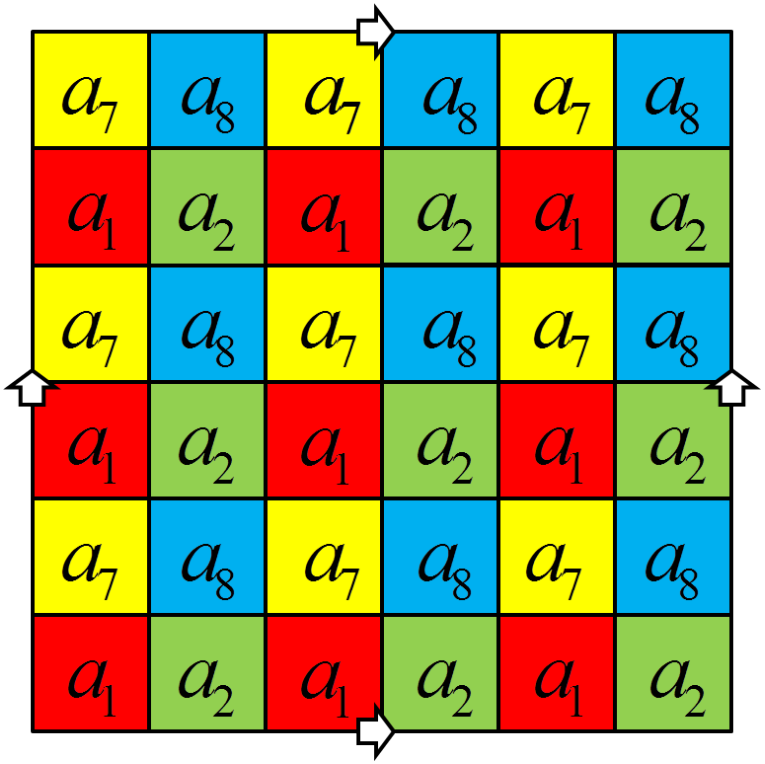


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green

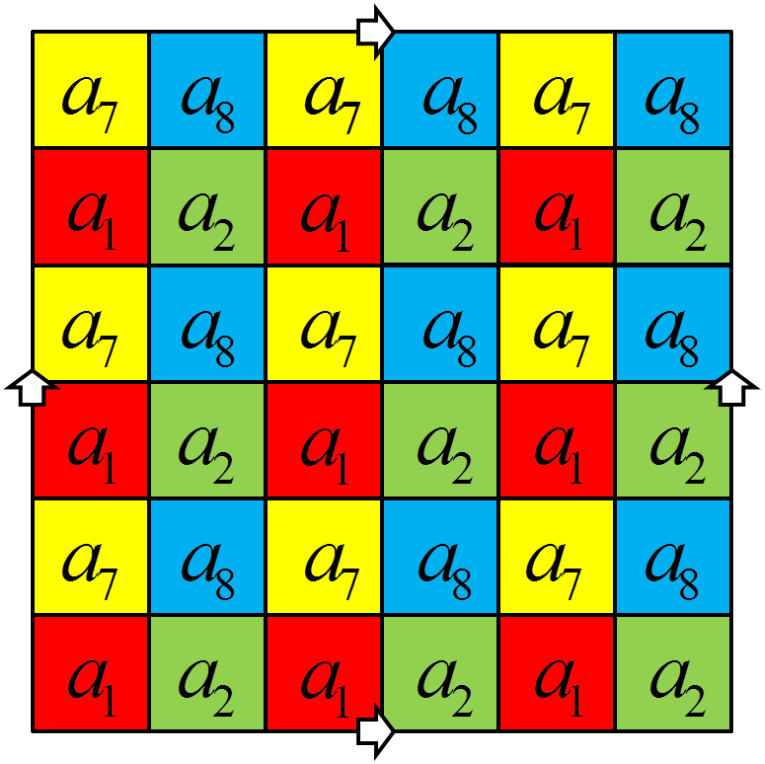


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue

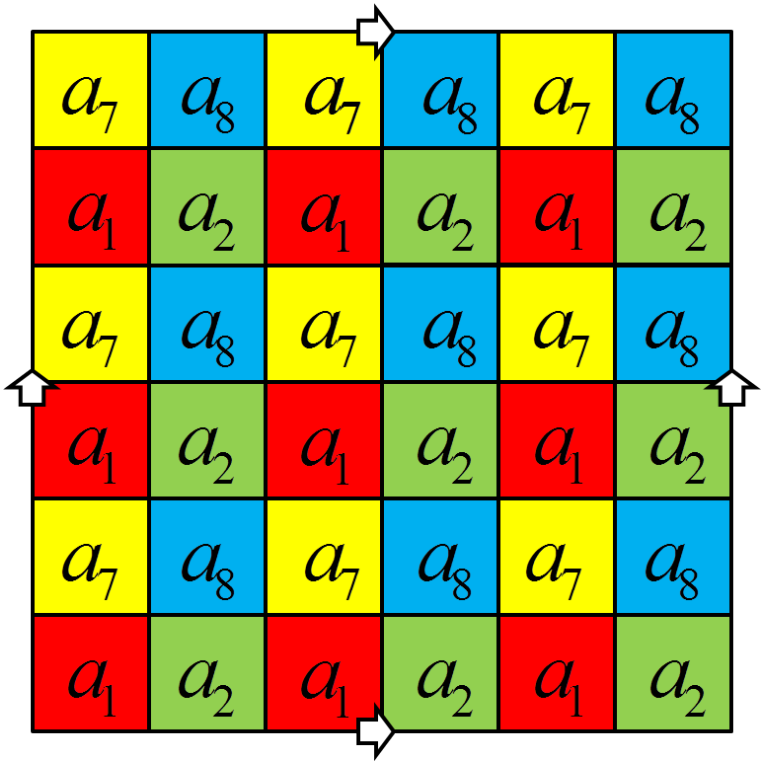


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue
  - 2 red and 2 yellow

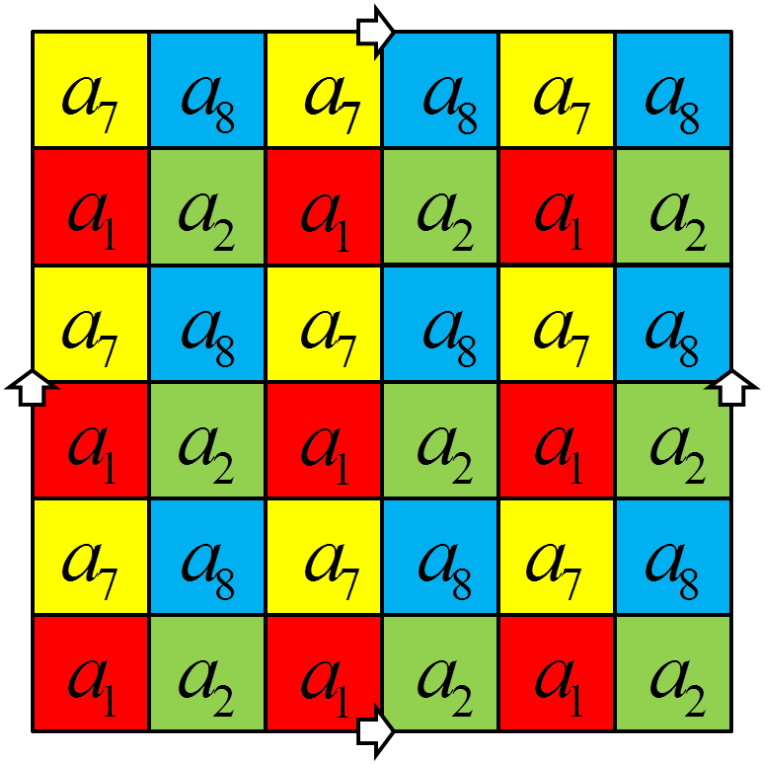


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue
  - 2 red and 2 yellow
  - 2 green and 2 blue

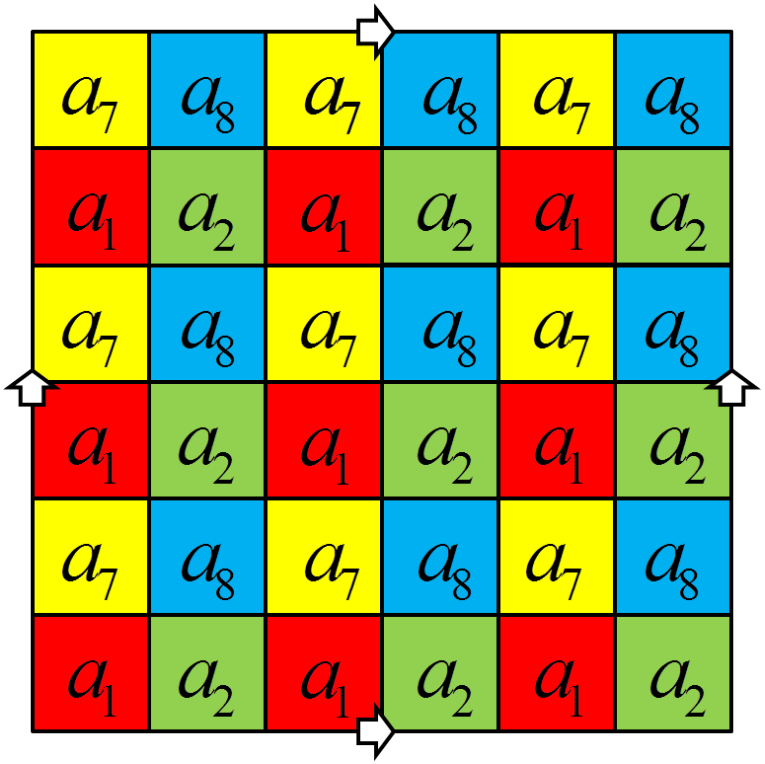
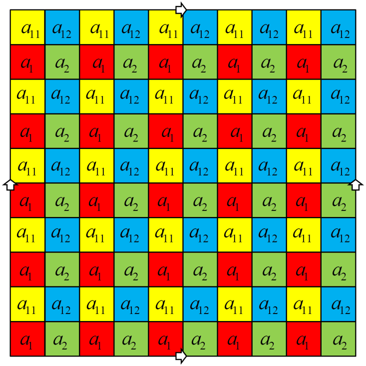


Figure: Coloring the Chessboard

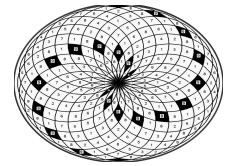
- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue
  - 2 red and 2 yellow
  - 2 green and 2 blue
- tiling is not possible

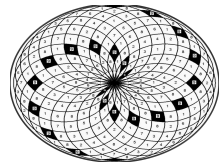
□



Theorem

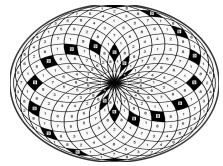
The torus chessboard of dimension $(4k+2) \times (4k+2)$ can be not tiling with the tile 1×4.





Example

Is it possible to tile torus chessboard 10×10 with T - tetrominoes? (all orientation are allowed)



Example

Is it possible to tile torus chessboard 10×10 with T - tetrominoes? (all orientation are allowed)

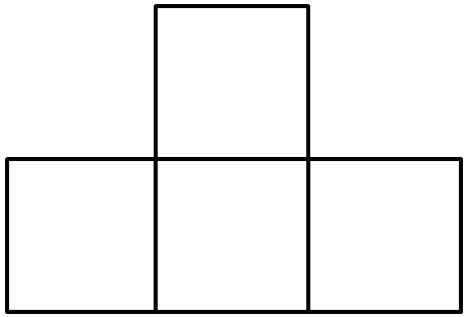
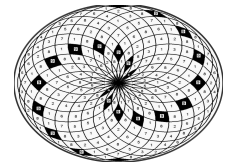


Figure: T – tetramino



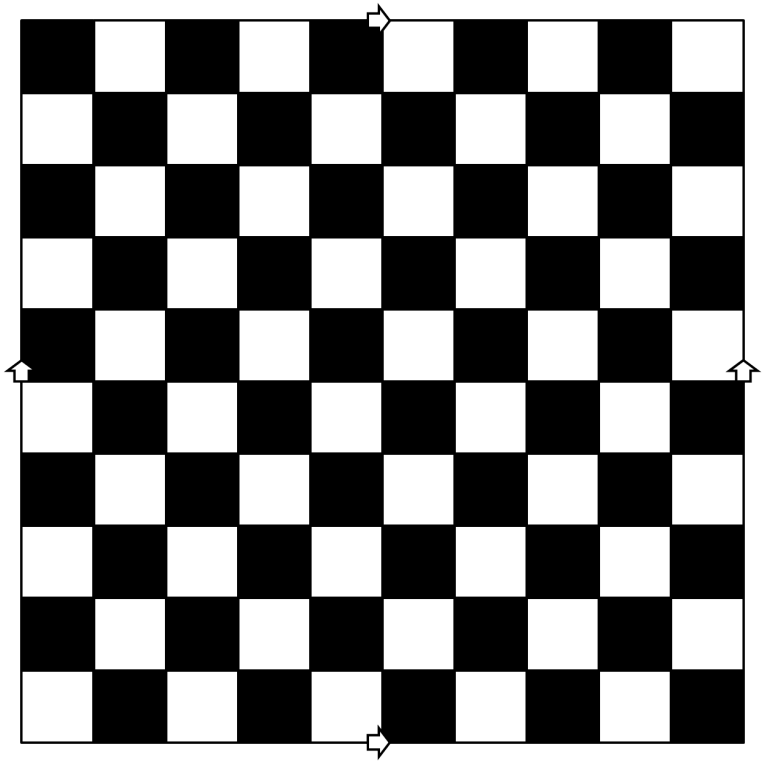


Figure: In torus plane model

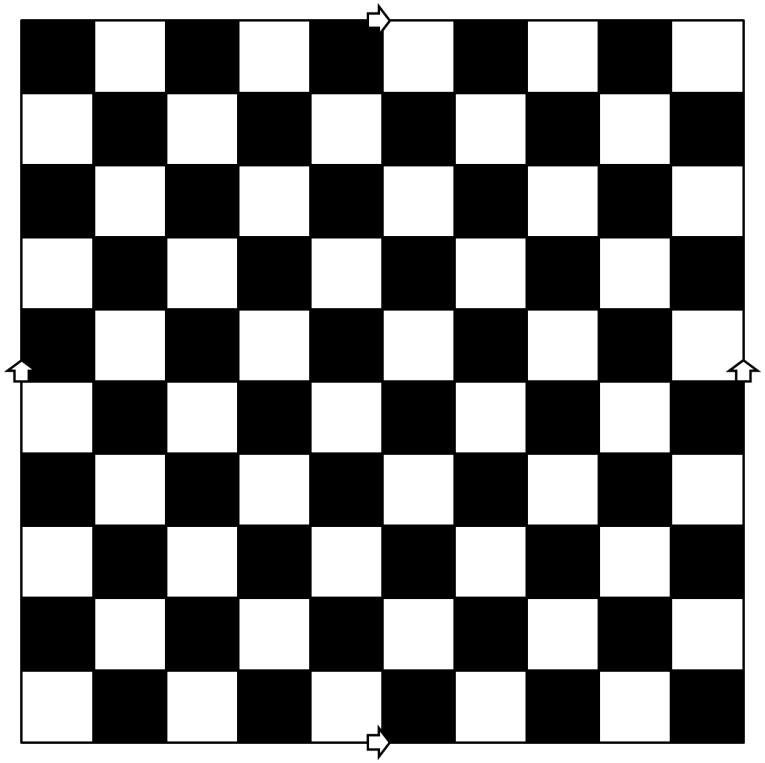
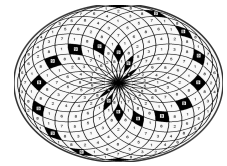
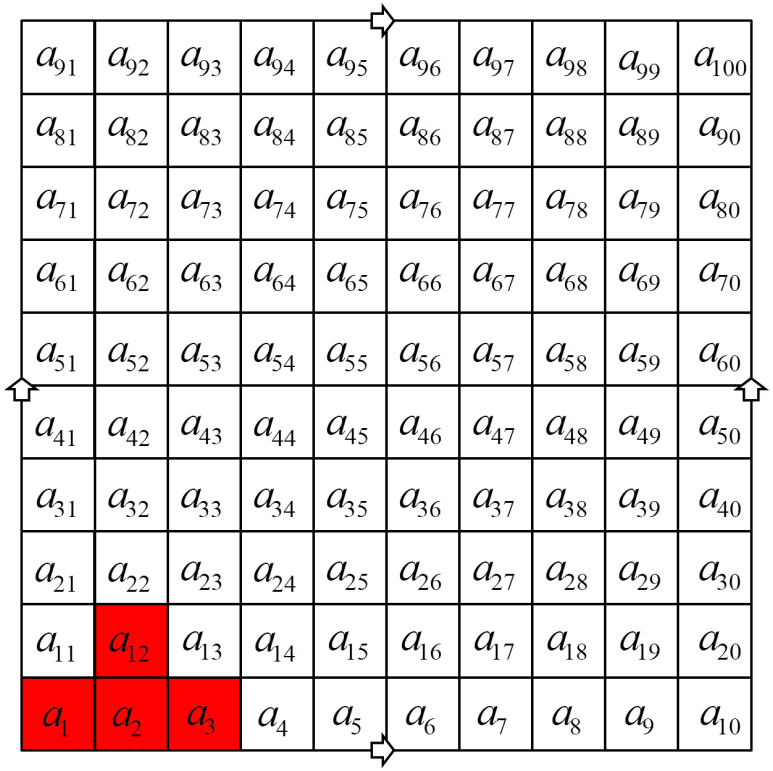


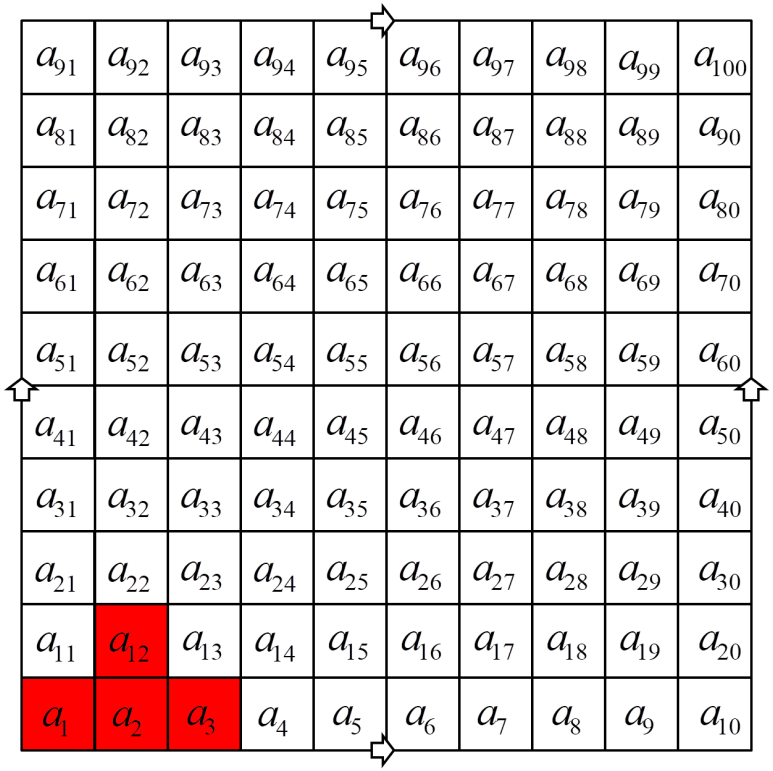
Figure: In torus plane model

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}

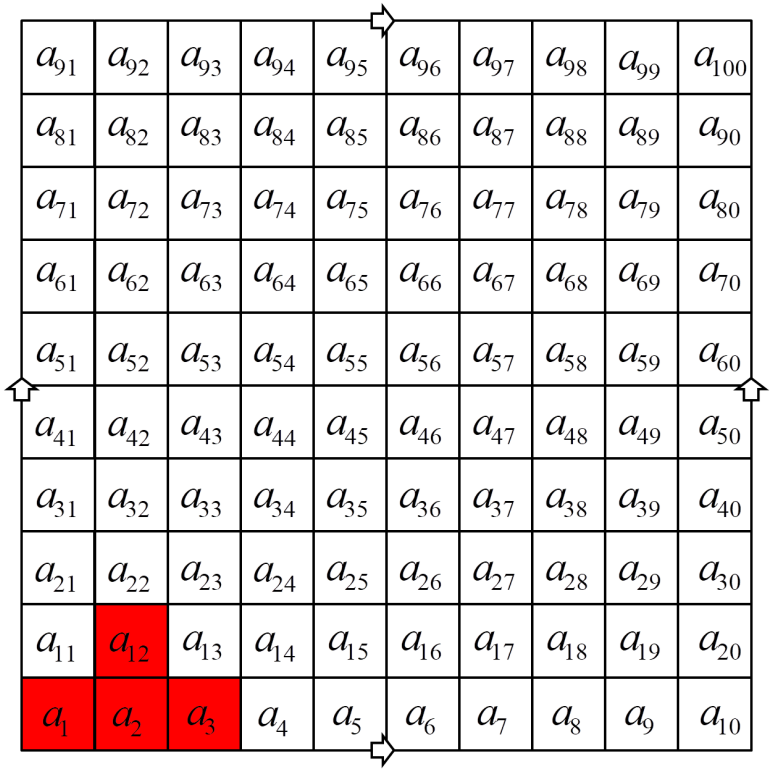
Figure: Naming cells



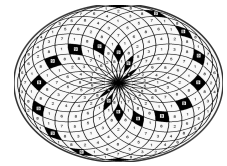

a_{91} a_{92} a_{93} a_{94} a_{95} a_{96} a_{97} a_{98} a_{99} a_{100}
a_{81} a_{82} a_{83} a_{84} a_{85} a_{86} a_{87} a_{88} a_{89} a_{90}
a_{71} a_{72} a_{73} a_{74} a_{75} a_{76} a_{77} a_{78} a_{79} a_{80}
a_{61} a_{62} a_{63} a_{64} a_{65} a_{66} a_{67} a_{68} a_{69} a_{70}
a_{51} a_{52} a_{53} a_{54} a_{55} a_{56} a_{57} a_{58} a_{59} a_{60}
a_{41} a_{42} a_{43} a_{44} a_{45} a_{46} a_{47} a_{48} a_{49} a_{50}
a_{31} a_{32} a_{33} a_{34} a_{35} a_{36} a_{37} a_{38} a_{39} a_{40}
a_{21} a_{22} a_{23} a_{24} a_{25} a_{26} a_{27} a_{28} a_{29} a_{30}
a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18} a_{19} a_{20}
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10}

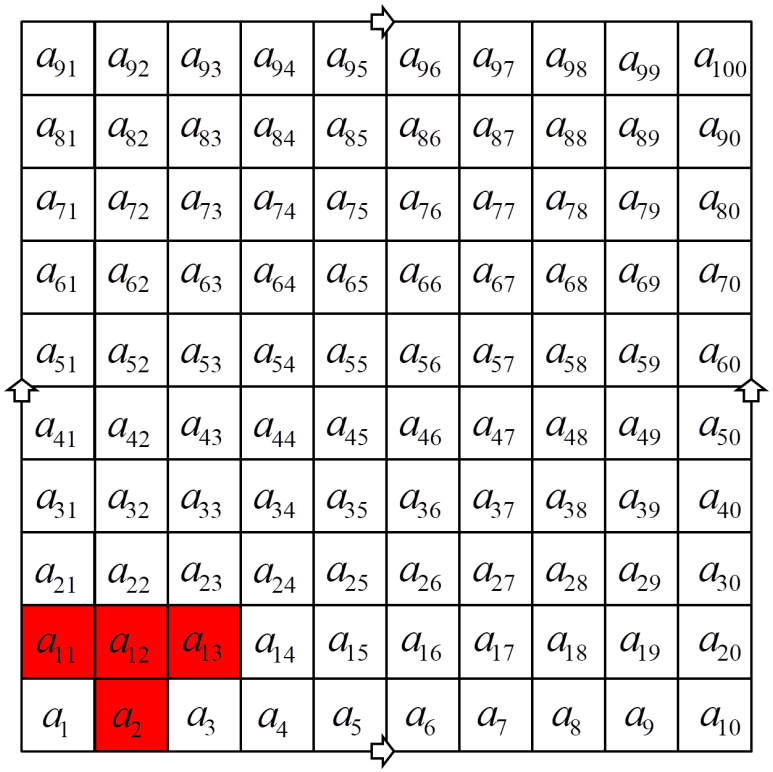

a_{91} a_{92} a_{93} a_{94} a_{95} a_{96} a_{97} a_{98} a_{99} a_{100}
a_{81} a_{82} a_{83} a_{84} a_{85} a_{86} a_{87} a_{88} a_{89} a_{90}
a_{71} a_{72} a_{73} a_{74} a_{75} a_{76} a_{77} a_{78} a_{79} a_{80}
a_{61} a_{62} a_{63} a_{64} a_{65} a_{66} a_{67} a_{68} a_{69} a_{70}
a_{51} a_{52} a_{53} a_{54} a_{55} a_{56} a_{57} a_{58} a_{59} a_{60}
a_{41} a_{42} a_{43} a_{44} a_{45} a_{46} a_{47} a_{48} a_{49} a_{50}
a_{31} a_{32} a_{33} a_{34} a_{35} a_{36} a_{37} a_{38} a_{39} a_{40}
a_{21} a_{22} a_{23} a_{24} a_{25} a_{26} a_{27} a_{28} a_{29} a_{30}
a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18} a_{19} a_{20}
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10}

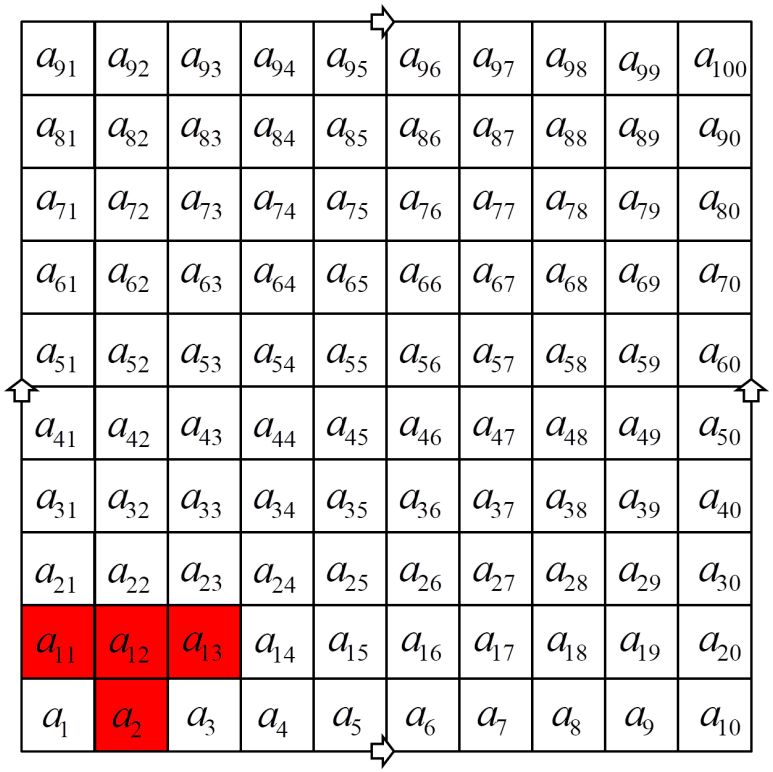
$$a_1 + a_2 + a_3 + a_{12} = 0$$



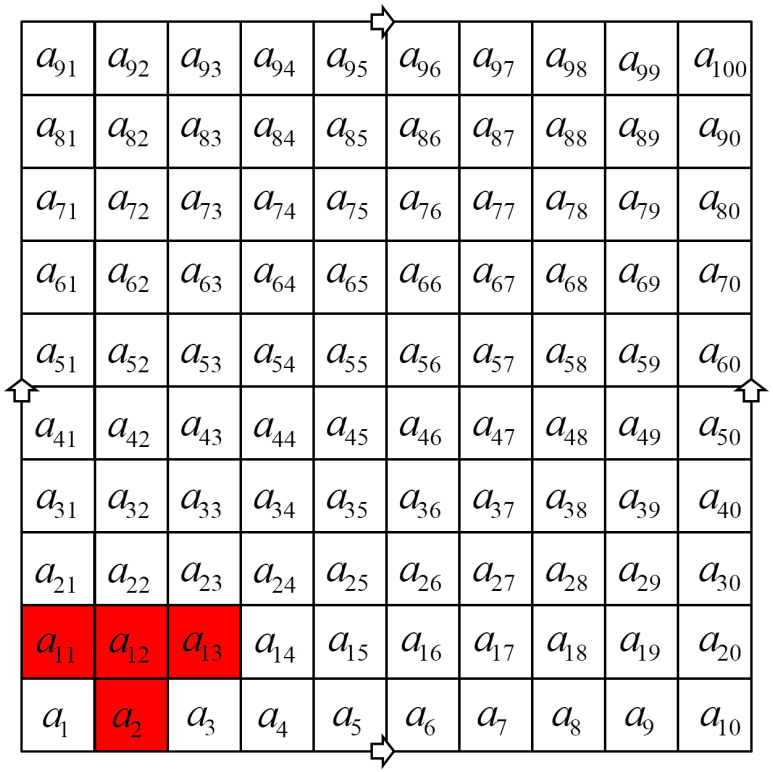
$$a_1 + a_2 + a_3 + a_{12} = 0$$
$$a_2 + a_3 + a_4 + a_{13} = 0$$
$$a_3 + a_4 + a_5 + a_{14} = 0$$
$$a_4 + a_5 + a_6 + a_{15} = 0$$
$$a_5 + a_6 + a_7 + a_{16} = 0$$
$$a_6 + a_7 + a_8 + a_{17} = 0$$
$$a_7 + a_8 + a_9 + a_{18} = 0$$
$$a_8 + a_9 + a_{10} + a_{19} = 0$$
$$a_9 + a_{10} + a_1 + a_{20} = 0$$
$$a_{10} + a_1 + a_2 + a_{11} = 0$$




a_{91} a_{92} a_{93} a_{94} a_{95} a_{96} a_{97} a_{98} a_{99} a_{100}
a_{81} a_{82} a_{83} a_{84} a_{85} a_{86} a_{87} a_{88} a_{89} a_{90}
a_{71} a_{72} a_{73} a_{74} a_{75} a_{76} a_{77} a_{78} a_{79} a_{80}
a_{61} a_{62} a_{63} a_{64} a_{65} a_{66} a_{67} a_{68} a_{69} a_{70}
a_{51} a_{52} a_{53} a_{54} a_{55} a_{56} a_{57} a_{58} a_{59} a_{60}
a_{41} a_{42} a_{43} a_{44} a_{45} a_{46} a_{47} a_{48} a_{49} a_{50}
a_{31} a_{32} a_{33} a_{34} a_{35} a_{36} a_{37} a_{38} a_{39} a_{40}
a_{21} a_{22} a_{23} a_{24} a_{25} a_{26} a_{27} a_{28} a_{29} a_{30}
a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18} a_{19} a_{20}
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10}

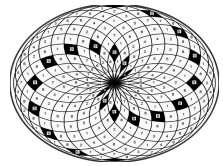


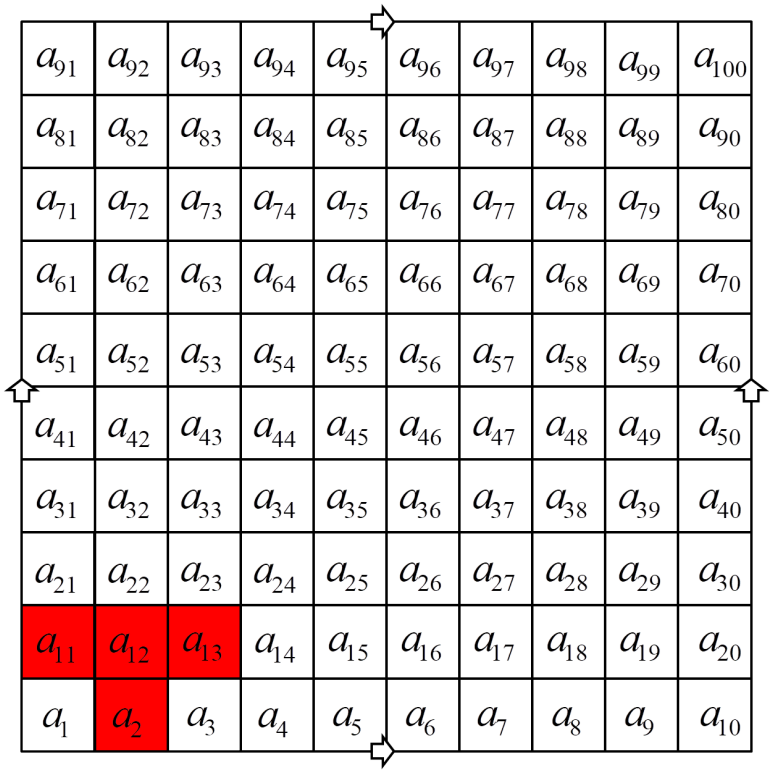
$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$



$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

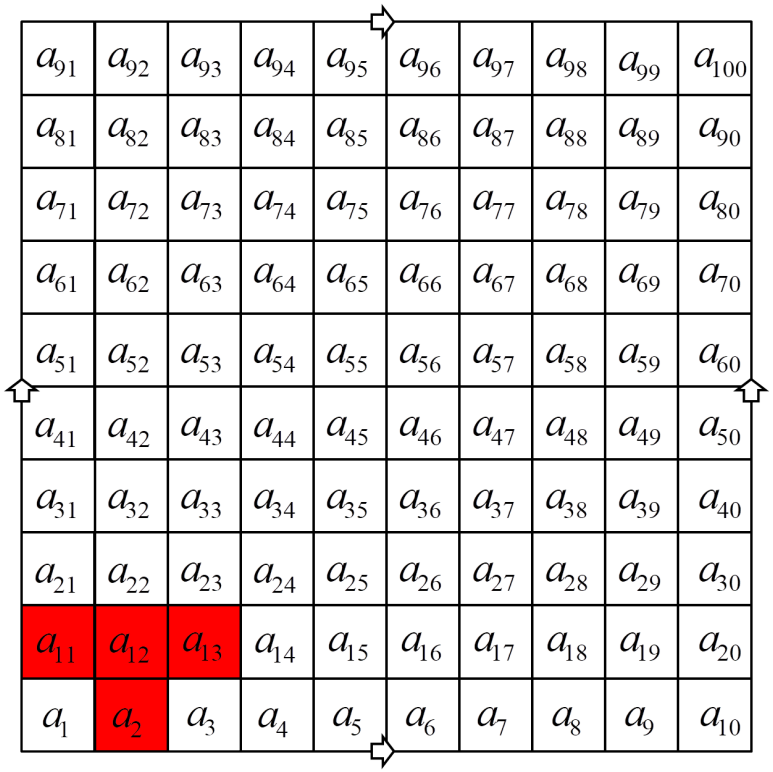
$$a_{11} + a_{12} + a_{13} + a_2 = 0$$





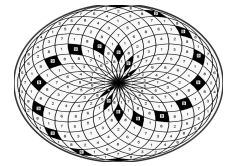
$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

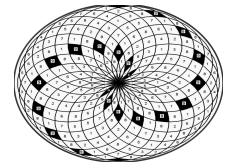
$$\begin{aligned} a_{11} + a_{12} + a_{13} + a_2 &= 0 \\ a_{11} + a_{12} + a_{13} + a_{22} &= 0 \end{aligned}$$



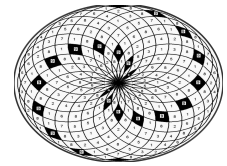
$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

$$\begin{aligned} a_{11} + a_{12} + a_{13} + a_2 &= 0 \\ a_{11} + a_{12} + a_{13} + a_{22} &= 0 \\ a_2 &= a_{22} \end{aligned}$$





$$a_1 = a_3 = a_5 = a_7 = a_9 = a_{12} = a_{14} = a_{16} = a_{18} = a_{20}$$
$$a_2 = a_6 = a_8 = a_{10} = a_{11} = a_{13} = a_{15} = a_{17} = a_{19}$$



$$a_1 = a_3 = a_5 = a_7 = a_9 = a_{12} = a_{14} = a_{16} = a_{18} = a_{20}$$
$$a_2 = a_6 = a_8 = a_{10} = a_{11} = a_{13} = a_{15} = a_{17} = a_{19}$$

Analogue

$$a_{21} = a_{23} = a_{25} = a_{27} = a_{29} = a_{32} = a_{34} = a_{36} = a_{38} = a_{40}$$
$$a_{41} = a_{43} = a_{45} = a_{47} = a_{49} = a_{42} = a_{44} = a_{46} = a_{48} = a_{60}$$
$$a_{61} = a_{63} = a_{65} = a_{67} = a_{69} = a_{62} = a_{64} = a_{66} = a_{68} = a_{80}$$
$$a_{81} = a_{83} = a_{85} = a_{87} = a_{89} = a_{82} = a_{84} = a_{86} = a_{88} = a_{100}$$
$$a_{22} = a_{24} = a_{26} = a_{28} = a_{30} = a_{31} = a_{33} = a_{35} = a_{37} = a_{39}$$
$$a_{42} = a_{44} = a_{46} = a_{48} = a_{50} = a_{51} = a_{53} = a_{55} = a_{57} = a_{59}$$
$$a_{62} = a_{64} = a_{66} = a_{68} = a_{70} = a_{71} = a_{73} = a_{75} = a_{77} = a_{79}$$
$$a_{82} = a_{84} = a_{86} = a_{88} = a_{90} = a_{91} = a_{93} = a_{95} = a_{97} = a_{99}$$

a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2

Figure: Equivalent cells

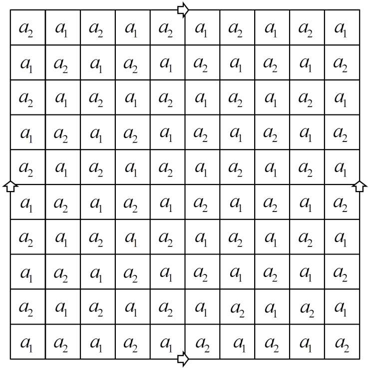


Figure: Equivalent cells

$$3a_1 + a_2 = 0$$

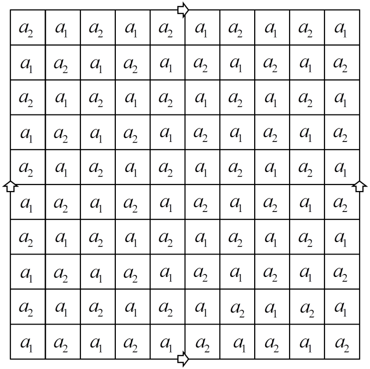


Figure: Equivalent cells

$$
\begin{aligned}
3a_1 + a_2 &= 0 \\
3a_2 + a_1 &= 0
\end{aligned}
$$

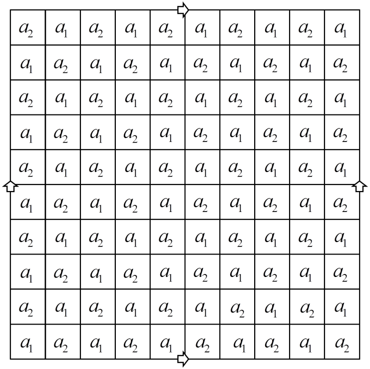


Figure: Equivalent cells

$$\begin{aligned} 3a_1 + a_2 &= 0 \\ 3a_2 + a_1 &= 0 \end{aligned}$$

- $< a_1, a_2 | 3a_1+a_2, 3a_2+a_1 >$

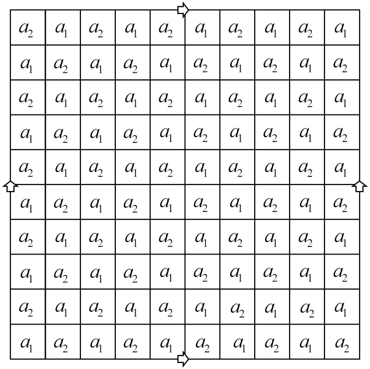


Figure: Equivalent cells

$$\begin{aligned} 3a_1 + a_2 &= 0 \\ 3a_2 + a_1 &= 0 \end{aligned}$$

- $< a_1, a_2 |_{3a_1+a_2, 3a_2+a_1} >$
- $=< a_1 |_{8a_1=0} >= \mathbb{Z}_8$

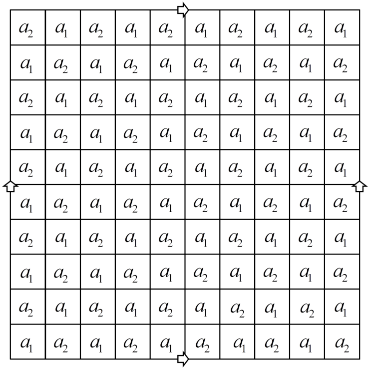


Figure: Equivalent cells

$$\begin{aligned} 3a_1 + a_2 &= 0 \\ 3a_2 + a_1 &= 0 \end{aligned}$$

- $< a_1, a_2 | _{3a_1+a_2, 3a_2+a_1} >$
- $=< a_1 | _{8a_1=0} >= \mathbb{Z}_8$

$$50a_1 + 50a_2 = -100a_1 = 4a_1$$

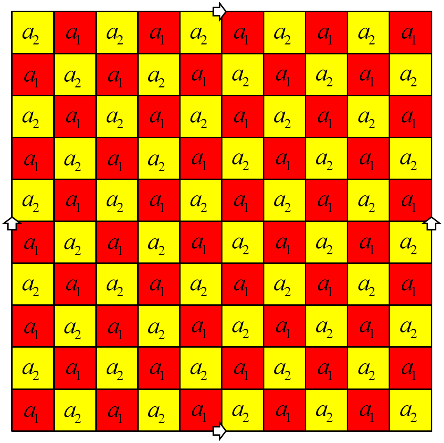


Figure: Coloring the Chessboard

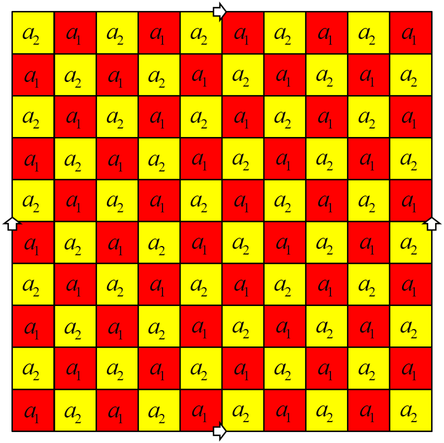


Figure: Coloring the Chessboard

- 50 red cells

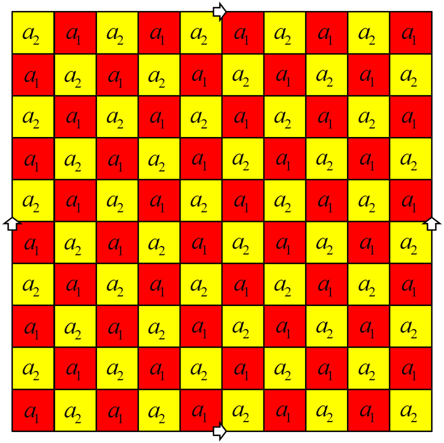


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells

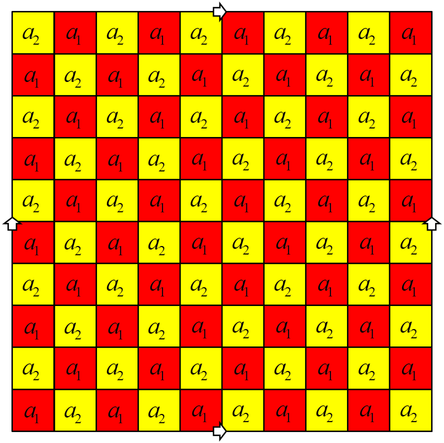


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells
- every T – tetramino cover

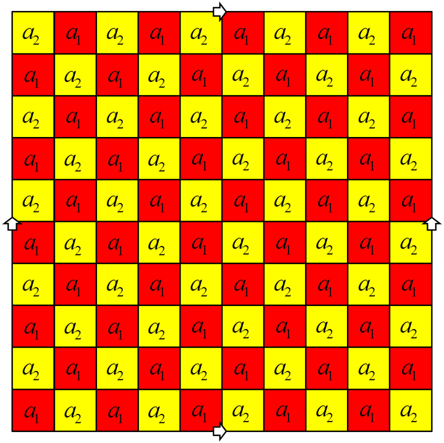


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells
- every T – tetramino cover
  - 3 red and 1 yellow

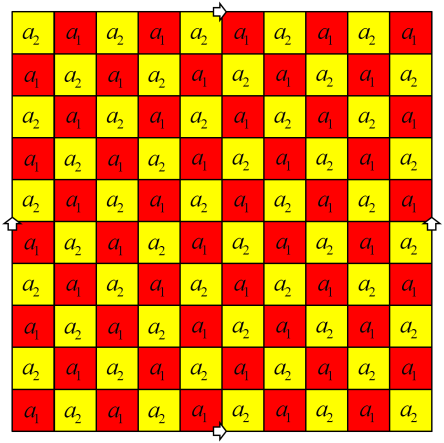


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells
- every T – tetramino cover
  - 3 red and 1 yellow
  - 3 yellow and 1 red

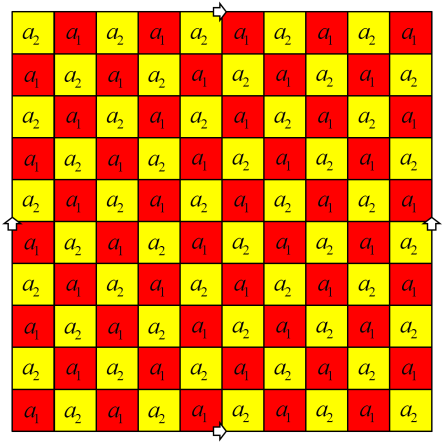
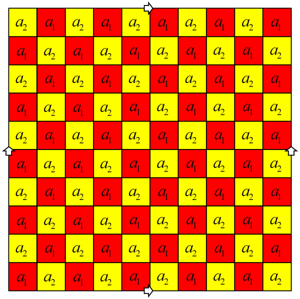


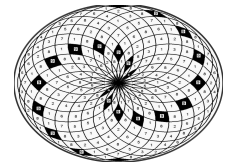
Figure: Coloring the Chessboard

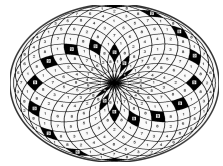
- 50 red cells
- 50 yellow cells
- every T – tetramino cover
  - 3 red and 1 yellow
  - 3 yellow and 1 red
- tiling is not possible



Theorem

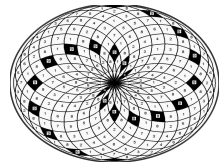
The torus chessboard of dimension $(4k+2) \times (4k+2)$ can be not tiling with T – tetrominoes.





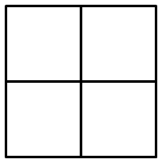
Example

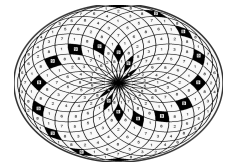
Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?



Example

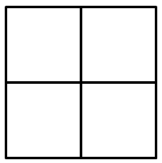
Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?

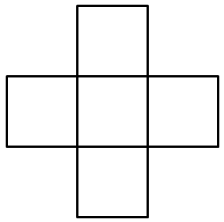




Example

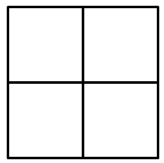
Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?

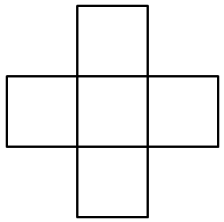




Example

Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?





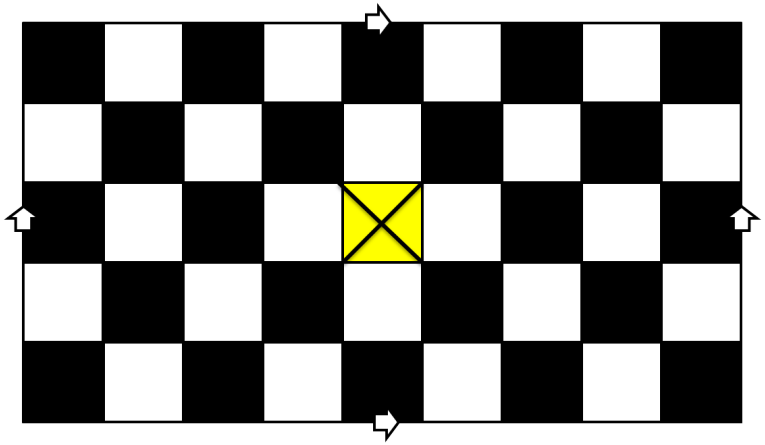
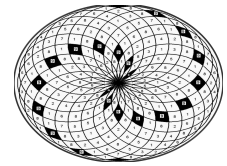
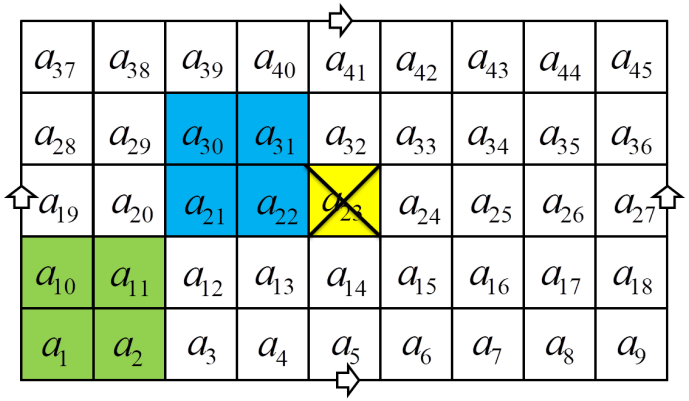


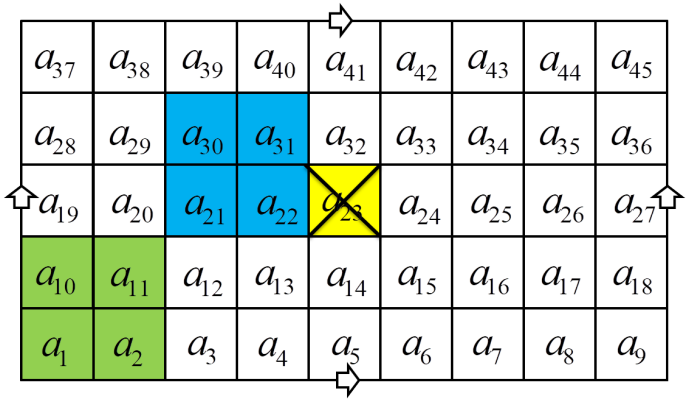
Figure: In torus plane model

a_{37}	a_{38}	a_{39}	a_{40}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}
a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}
a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9

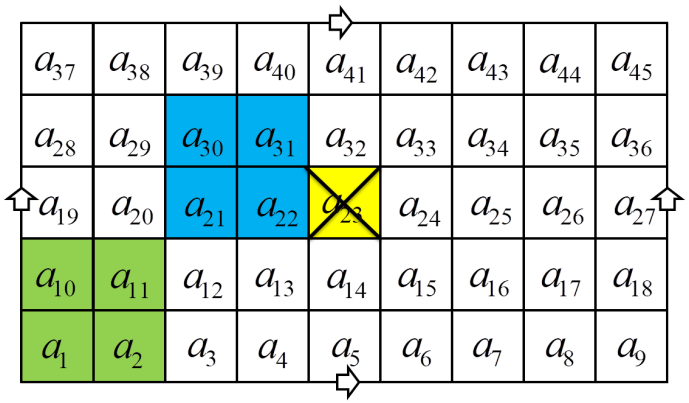
Figure: Naming cell




a_{37} a_{38} a_{39} a_{40} a_{41} a_{42} a_{43} a_{44} a_{45}
a_{28} a_{29} a_{30} a_{31} a_{32} a_{33} a_{34} a_{35} a_{36}
a_{19} a_{20} a_{21} a_{22} a_{23} a_{24} a_{25} a_{26} a_{27}
a_{10} a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18}
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9

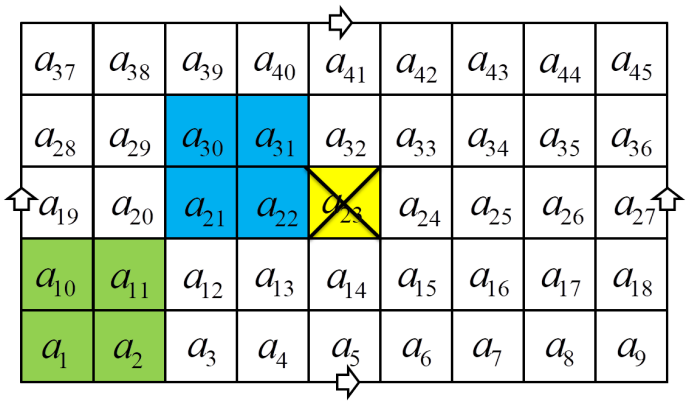


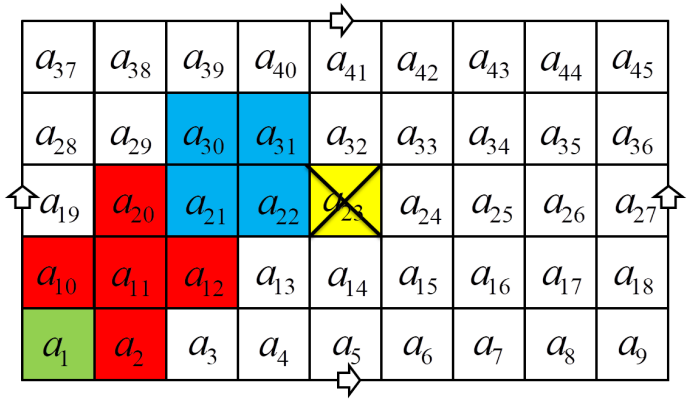
$$a_1 + a_2 + a_{10} + a_{11} = 0$$



$$a_1 + a_2 + a_{10} + a_{11} = 0$$

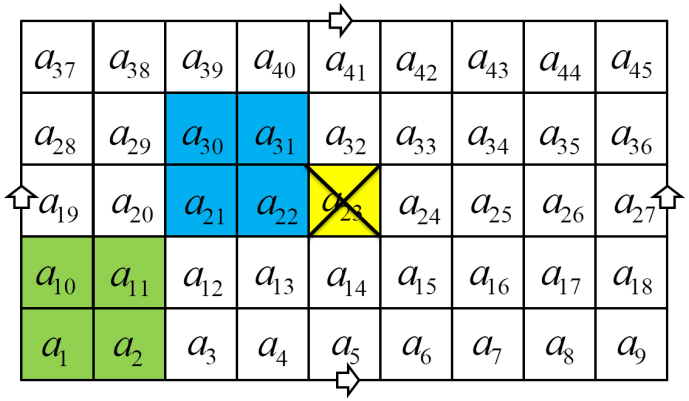
$$a_{21} + a_{30} + a_{22} + a_{31} = 0$$



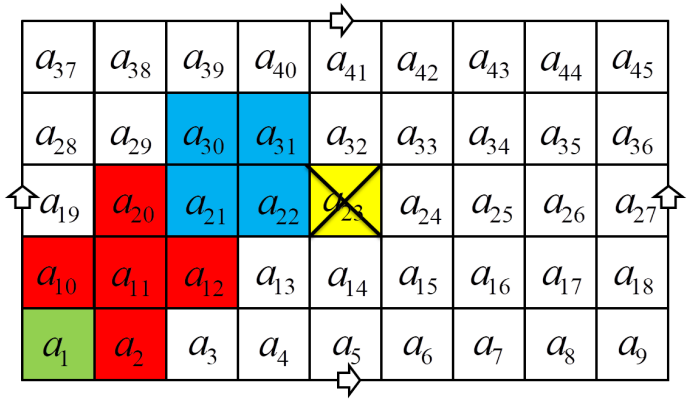


$$a_1 + a_2 + a_{10} + a_{11} = 0$$

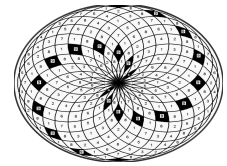
$$a_{21} + a_{30} + a_{22} + a_{31} = 0$$

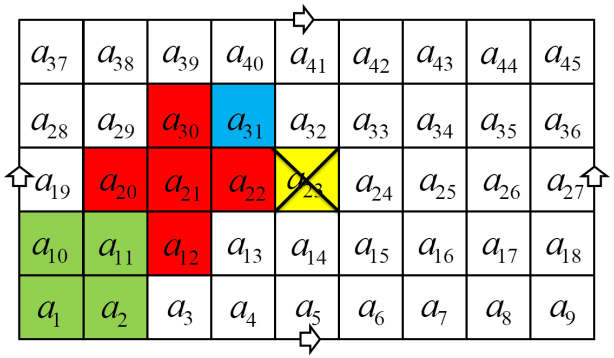


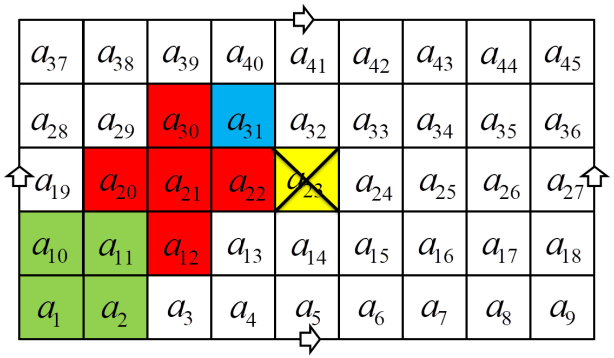
$$a_1 + a_2 + a_{10} + a_{11} = 0$$
$$a_{21} + a_{30} + a_{22} + a_{31} = 0$$



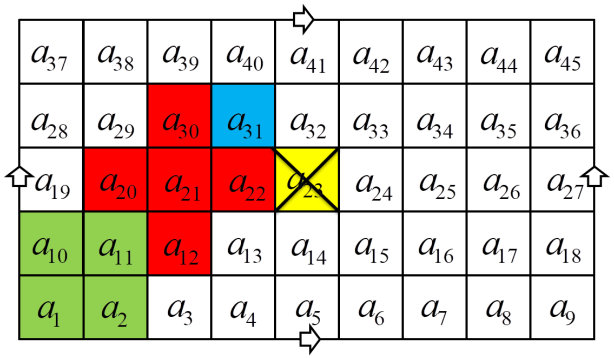
$$a_{20} + a_{10} + a_{11} + a_{12} + a_2 = 0$$



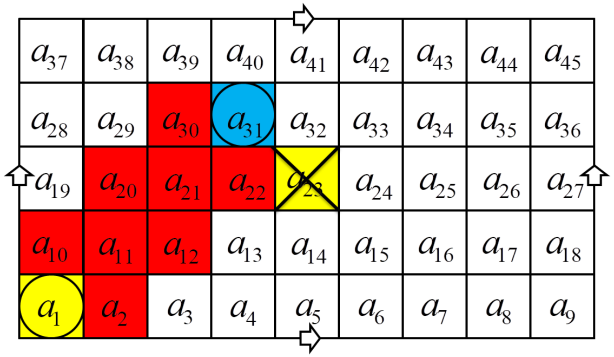

a_{37} a_{38} a_{39} a_{40} a_{41} a_{42} a_{43} a_{44} a_{45}
a_{28} a_{29} a_{30} a_{31} a_{32} a_{33} a_{34} a_{35} a_{36}
a_{19} a_{20} a_{21} a_{22} a_{23} a_{24} a_{25} a_{26} a_{27}
a_{10} a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18}
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9

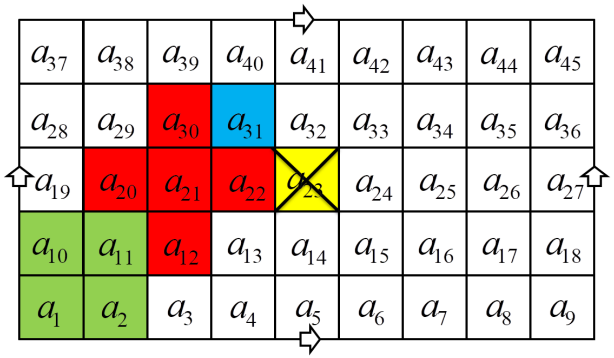


$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$

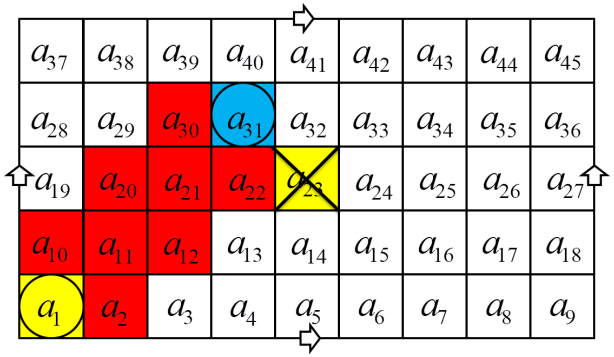


$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$



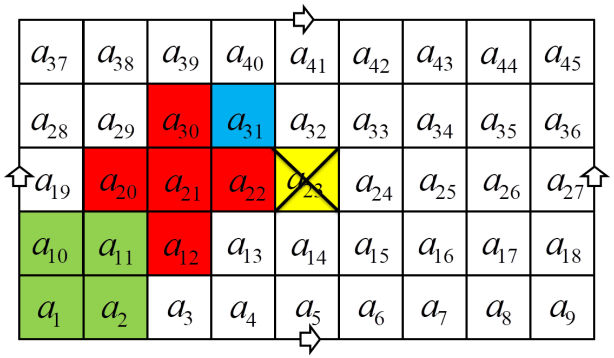


$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$

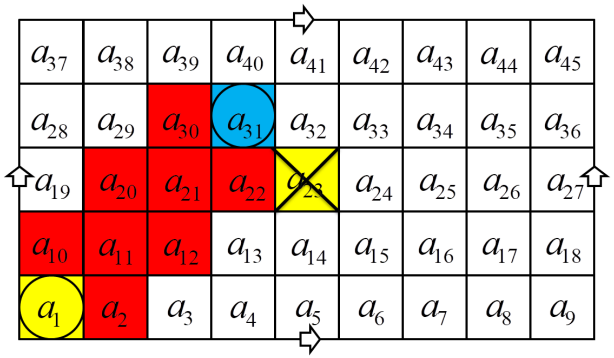


$$a_1 = a_{31}$$

University of Bihać
Bosnia and Herzegovina



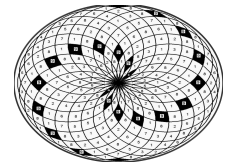
$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$



$$a_1 = a_{31}$$

Analogue

$$a_{31} = a_{16} = a_{37} = a_{22}$$
$$a_1 = a_{34} = a_{10} = a_{40} = a_{25}$$
$$a_{37} = a_{13} = a_{28}$$
$$a_{28} = a_4 = a_{19} = a_{43}$$
$$a_{28} = a_7$$



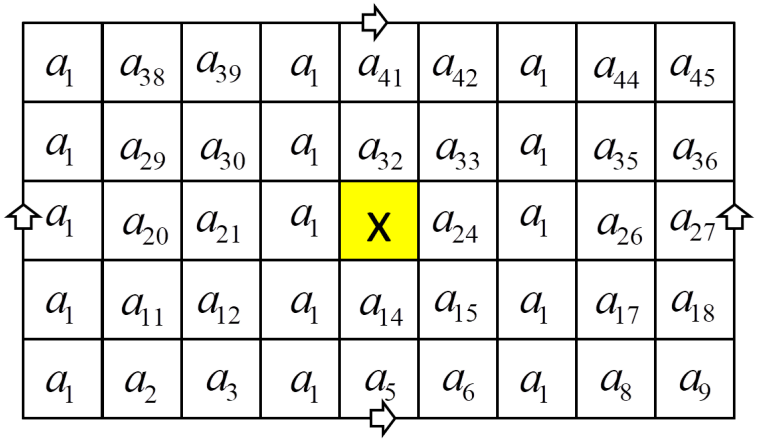


Figure: Cells generated with a_1

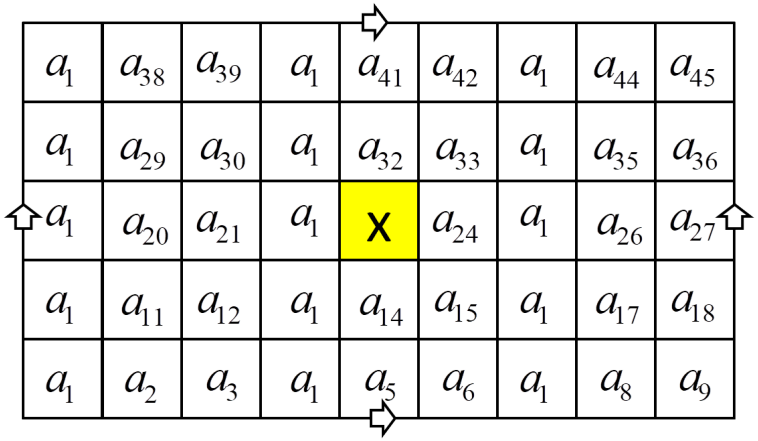


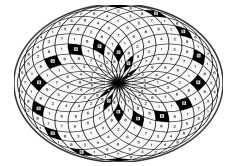
Figure: Cells generated with a_1

Analogue

$$a_2 = a_{34} = a_5, a_{34} = a_{11} = a_{41} = a_{26}$$

$$a_{11} = a_{44}, a_{20} = a_5, a_{29} = a_{14} = a_{34}$$

$$a_8 = a_{29}, a_{38} = a_{17} = a_{32}, a_{17} = a_{29}$$



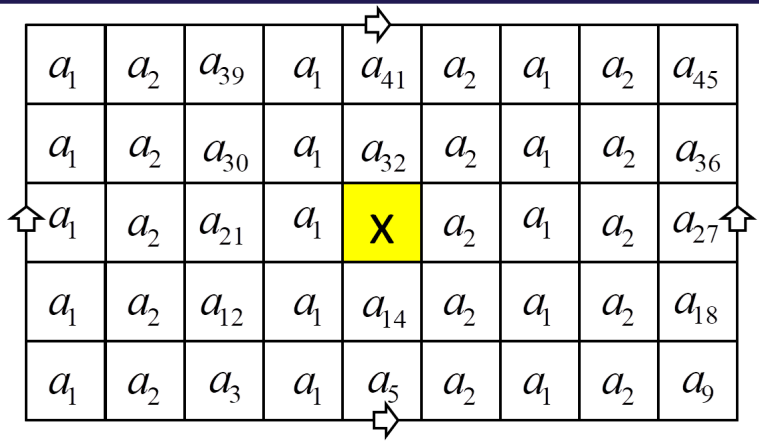


Figure: Cells generated with a_2

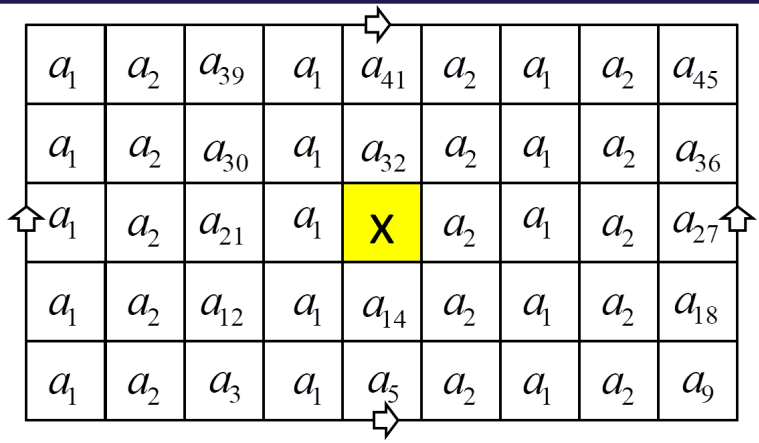


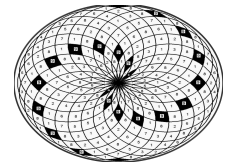
Figure: Cells generated with a_2

Analogue

$$a_3 = a_{36} = a_6 = a_{27}$$

$$a_{39} = a_{18} = a_{42} = a_{27}, a_{12} = a_{33}$$

$$a_{33} = a_9, a_{21}, a_{24} = a_9, a_{30} = a_{15}$$



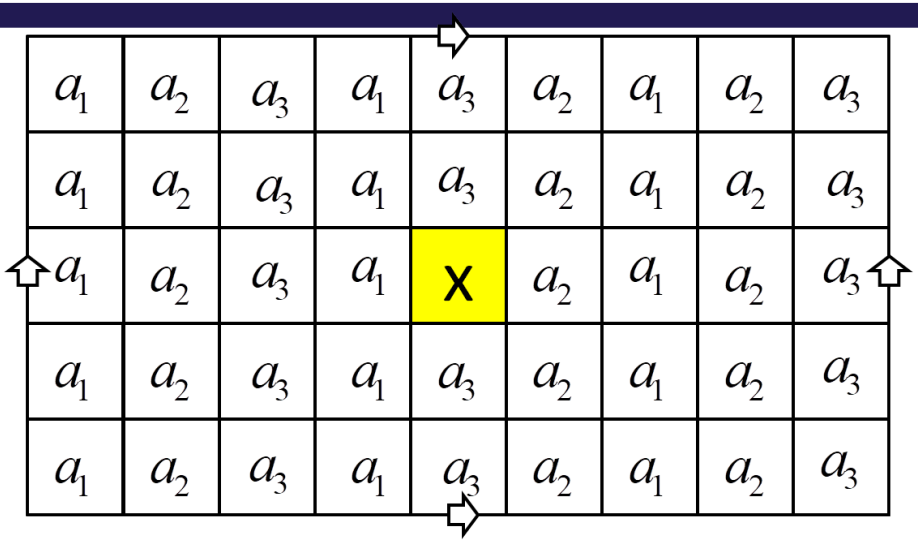


Figure: Cells generated with a_3

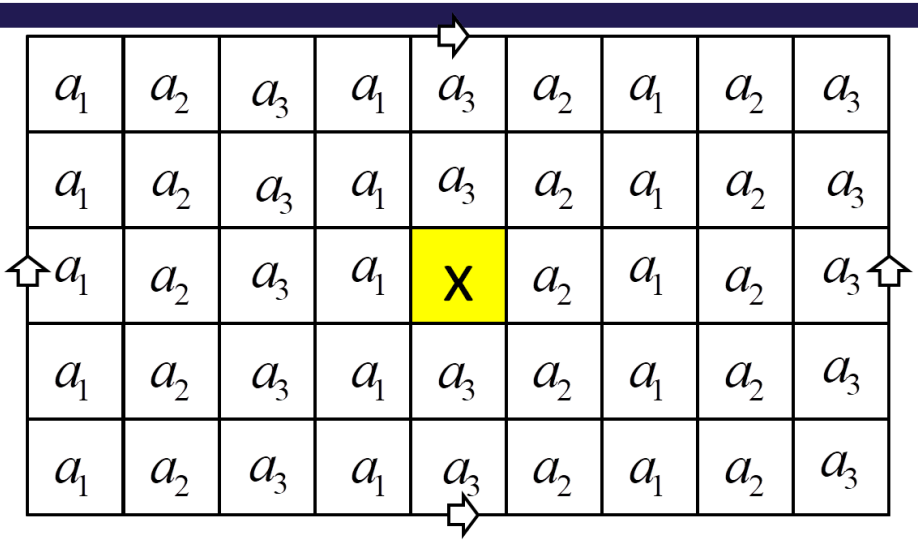


Figure: Cells generated with a_3

$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$

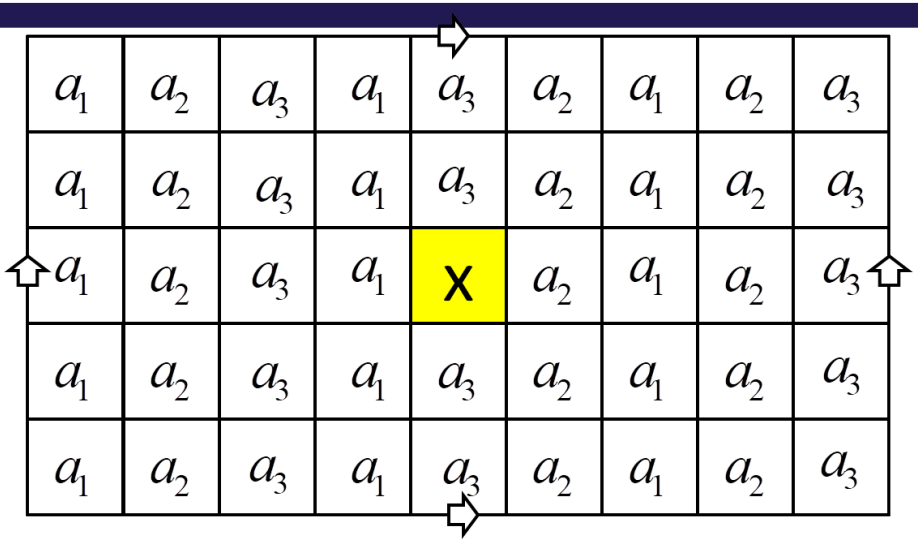
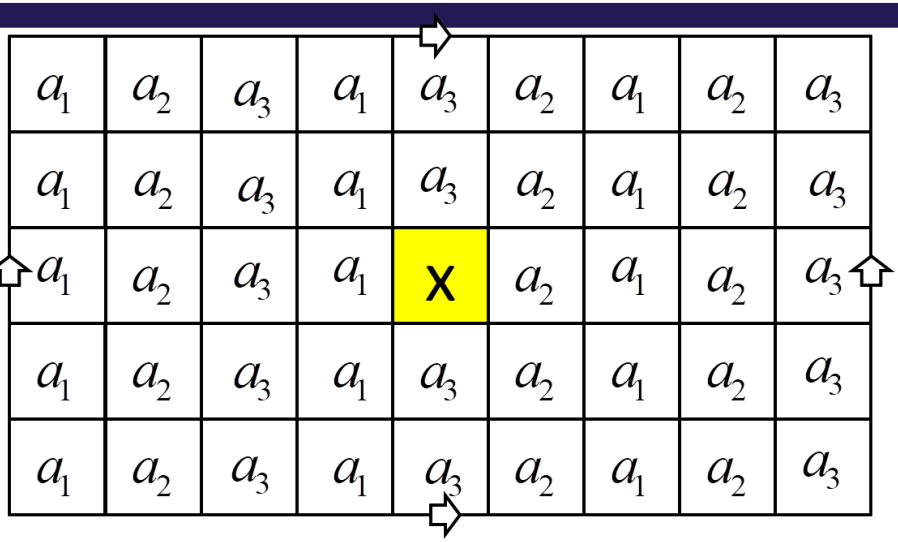


Figure: Cells generated with a_3

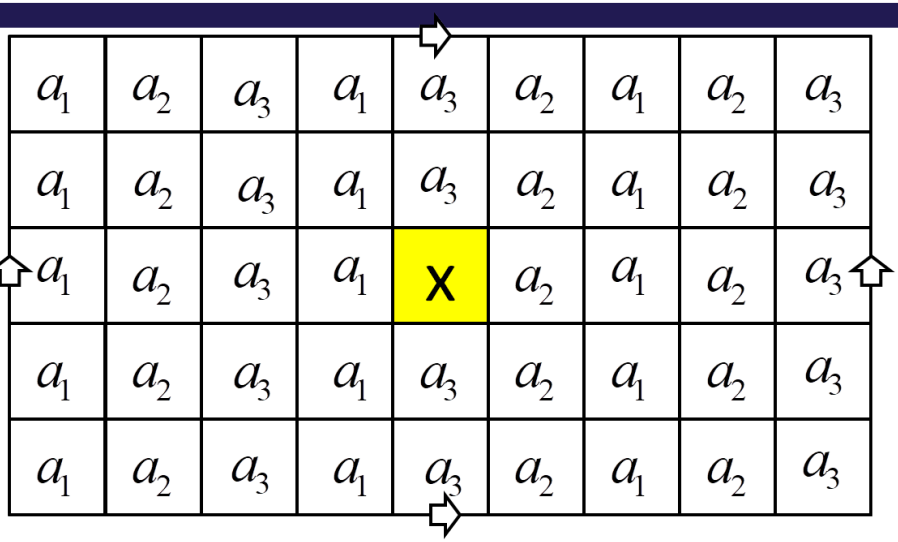
$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$
$$3a_2 + a_1 + a_3 = 0$$
$$3a_3 + a_1 + a_2 = 0$$
$$3a_1 + a_2 + a_3 = 0$$



$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$
$$3a_2 + a_1 + a_3 = 0$$
$$3a_3 + a_1 + a_2 = 0$$
$$3a_1 + a_2 + a_3 = 0$$

Figure: Cells generated with a_3

- $< a_1, a_2, a_3 | 2a_1 = 2a_2 = 2a_3 = a_1 + a_2 + a_3 = 0 >$

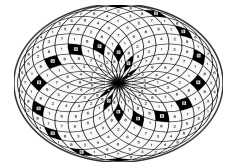


$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$
$$3a_2 + a_1 + a_3 = 0$$
$$3a_3 + a_1 + a_2 = 0$$
$$3a_1 + a_2 + a_3 = 0$$

Figure: Cells generated with a_3

- $< a_1, a_2, a_3 | 2a_1 = 2a_2 = 2a_3 = a_1 + a_2 + a_3 = 0 >$

$$15a_1 + 14a_2 + 15a_3 = a_1 + a_3$$



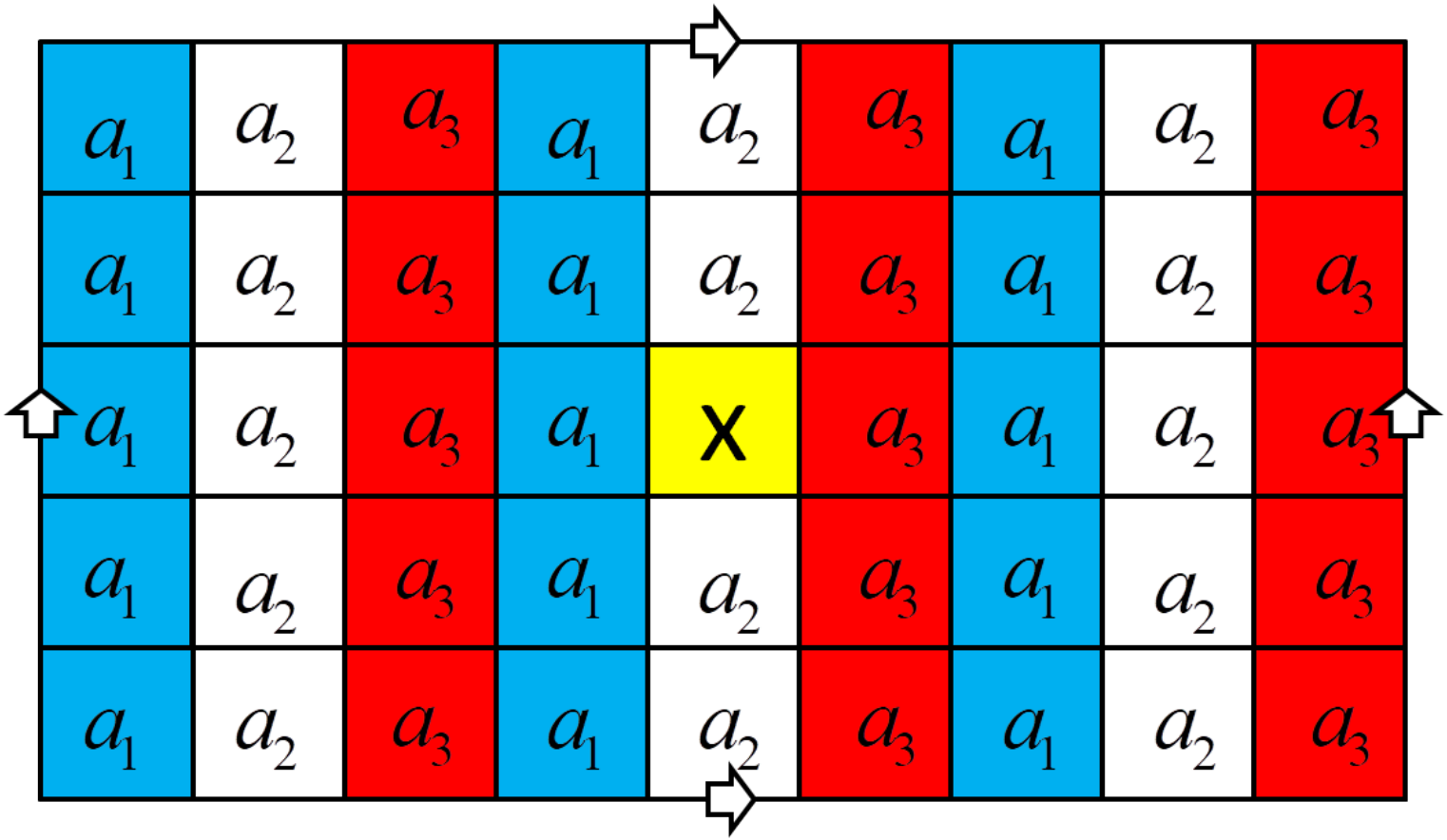
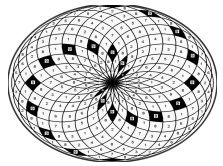
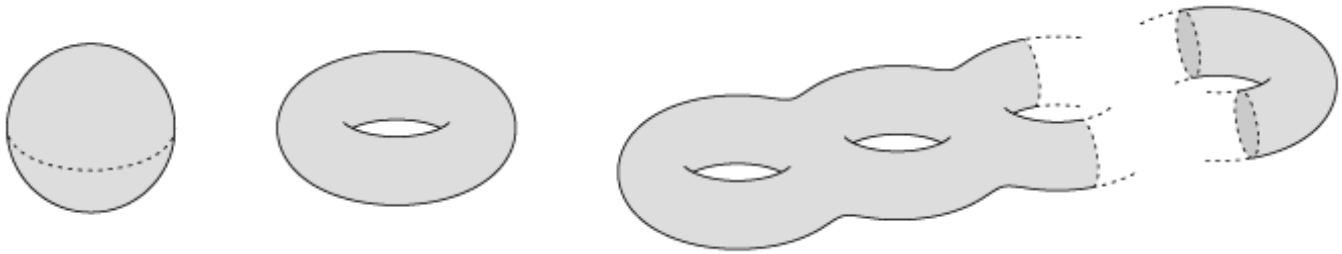


Figure: Coloring the chessboard

Conclusion



- The same idea can be used for studying tilings on surfaces of genus g. Which are subdivided in more general cells grids.

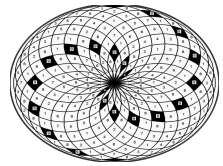


References

[1] J. H. Conway, J. C. Lagarias: *Tilings with polyominoes and combinatorial group theory*, Journal of Combinatorial Theory, Series A 53, (1990), 183 – 208.

[2] M. Reid: *Tile homotopy groups*, L'Enseignement Math'ematique 49 (2003), no. 1–2, pp. 123 – 155.

Contact Information



Thank you for your attention.

Edin Liđan
lidjan_edin@hotmail.com
University of Bihać
Bosnia and Herzegovina
www.unbi.ba

Questions?