

Prescribed Automorphism Groups: A GAP Package^{*}

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University of Zagreb, Croatia

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* This work has been supported by the Croatian Science Foundation under the project 9752.

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Complete lists of GAP packages:

<https://www.gap-system.org/Packages/packages.html>

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- The current installation files (version 0.1.4) can be obtained from the authors. Please write to vedran.krcadinac@math.hr.

PAG in action: Constructing block designs

A t - (v, k, λ) design is a v -element set V of points and a family \mathcal{B} of k -subsets of V called blocks such that every t -subset of V is contained in exactly λ blocks.

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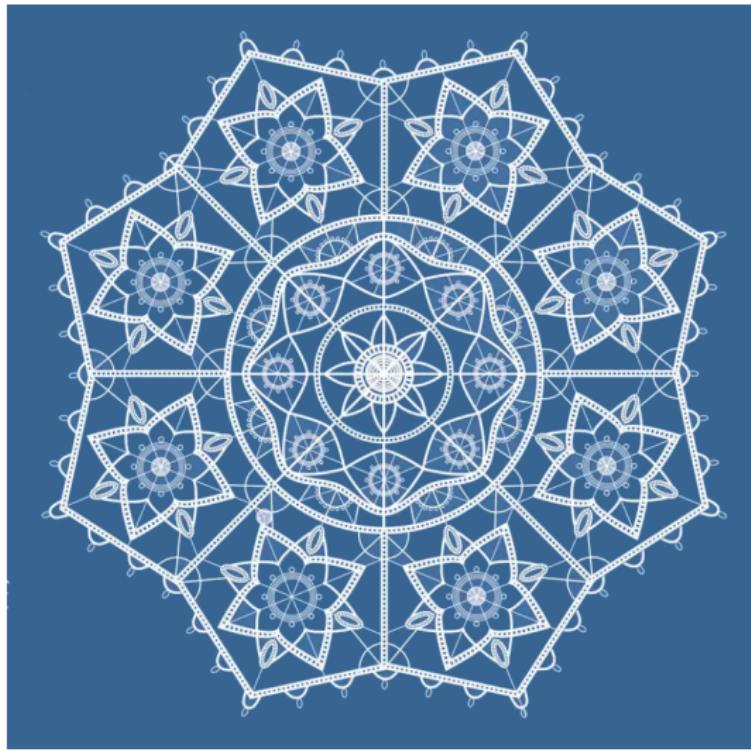
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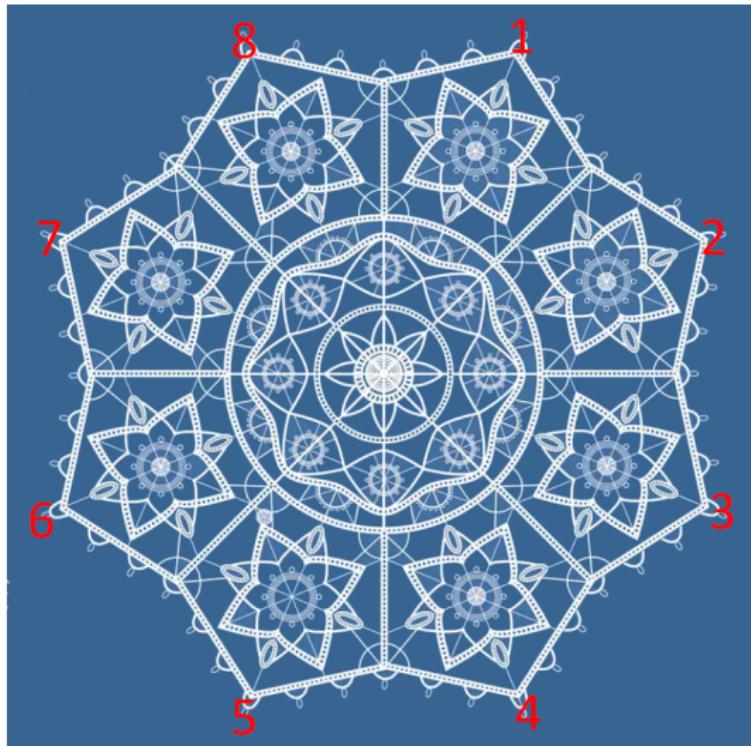
How to choose G ?

PAG in action: Constructing block designs



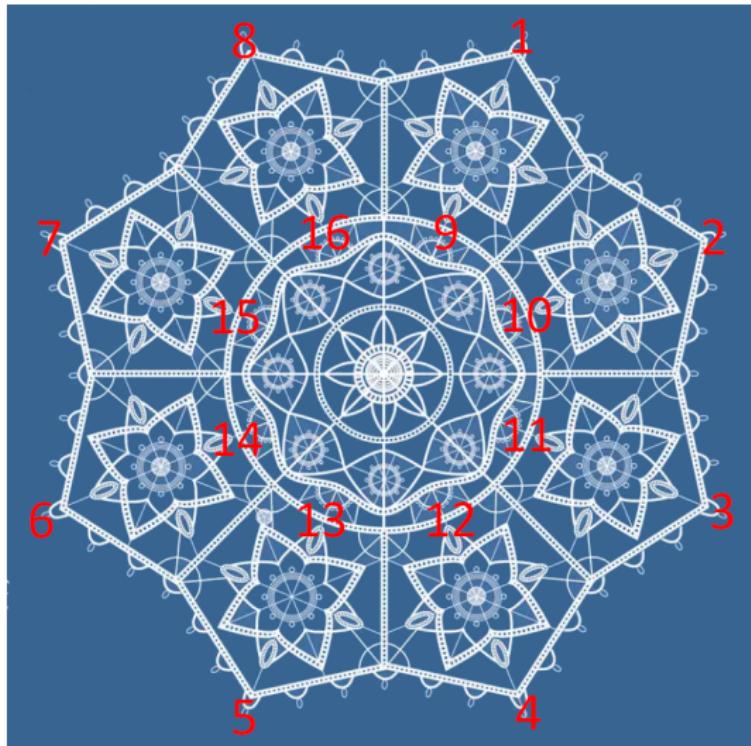
Source: <https://www.plakati.com.hr>

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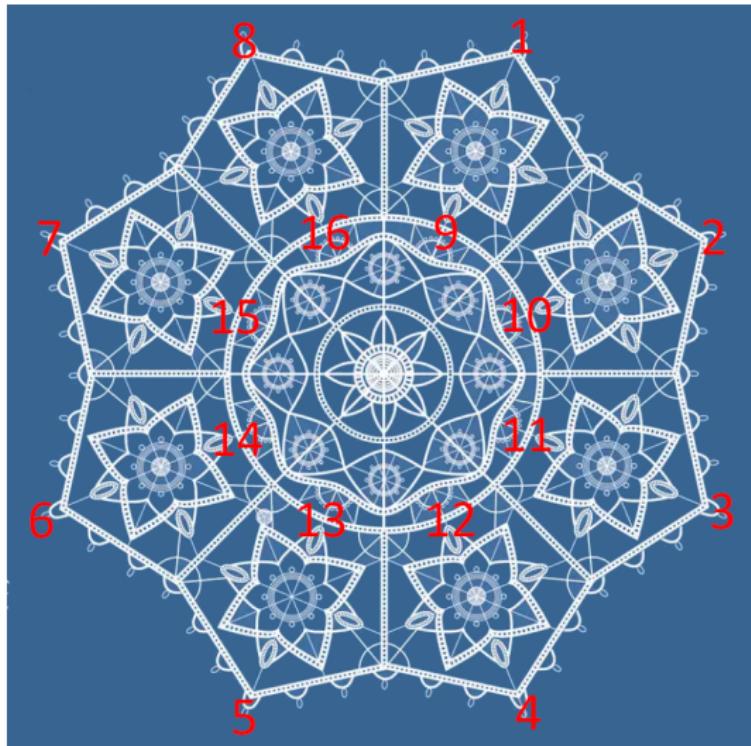
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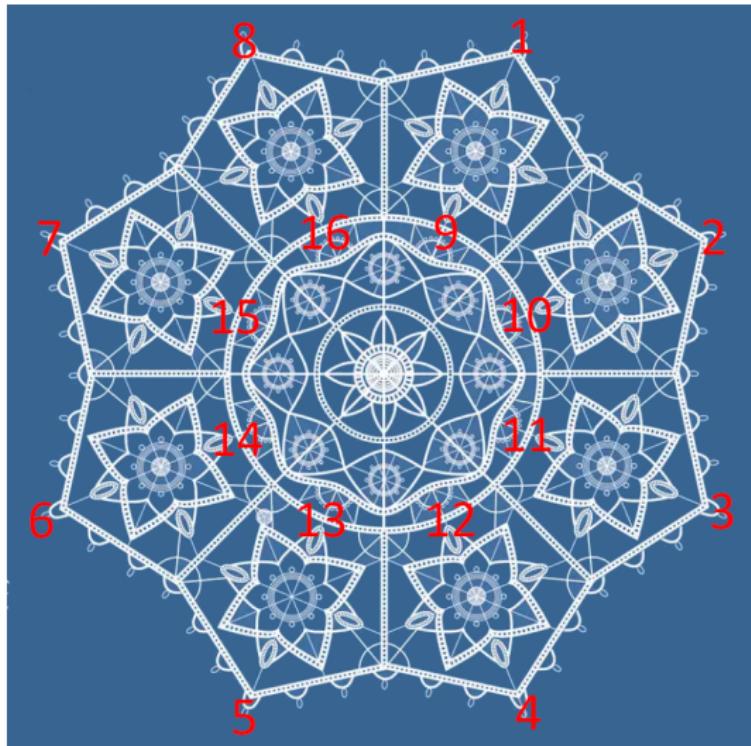
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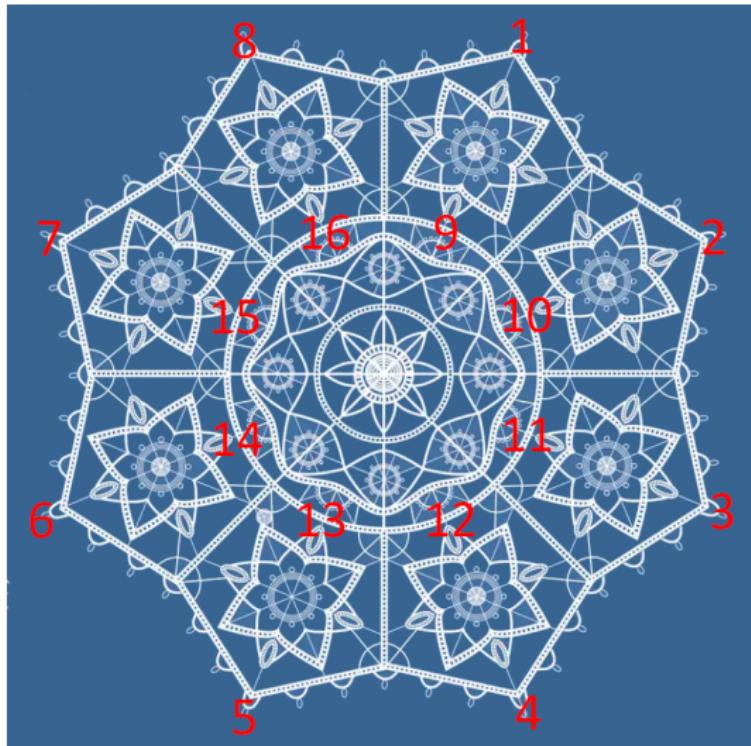
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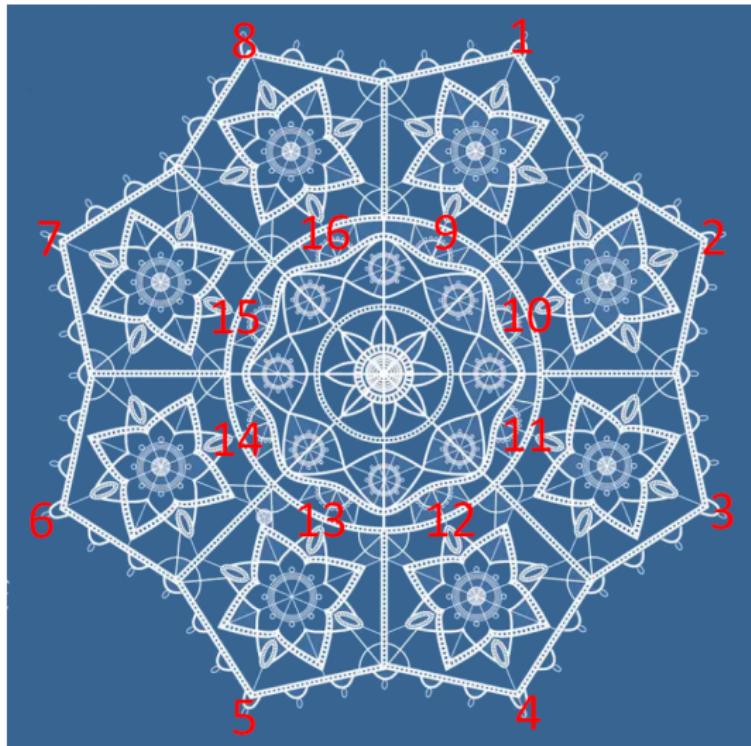
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g:=Group(a,b);  
  
StructureDescription(g);  
"D16"
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Suitable design parameters?

- 2-(16, 6, 2) \rightsquigarrow second smallest biplanes
- 2-(16, 4, 1) \rightsquigarrow affine plane of order 4
- 3-(16, 4, 1) \rightsquigarrow Steiner quadruple systems
- 5-(16, 6, 3) \rightsquigarrow 5-designs

A GAP session

```
gap> GAP 4.11.1 of 2021-03-02
gap> Architecture: x86_64-pc-linux-gnu-default64-kv7
gap> Configuration: gmp 6.2.0, GASMAN, readline
gap> Loading the library and packages ...
gap> Packages: AClib 1.3.2, Alnuth 3.1.2, AtlasRep 2.1.0, AutoDoc 2020.08.11, AutPGrp
CTblLib 1.3.1, FactInt 1.6.3, FGA 1.4.0, Forms 1.2.5, GAPDoc 1.6.4, gen
PrimGrp 3.4.1, RadiRoot 2.8, recog 1.3.2, ResClasses 4.7.2, SmallGrp 1.
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gap> g:=Group(a,b);
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gap> g:=Group(a,b);
Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)(10,15)])
gap>
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```

Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)(10,15)])

gap> StructureDescription(g):

"D16"

gap>

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gap> a:=(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16);
```

(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16)

```
gap> b:=(1,8)(2,7)(3,6)(4,5)(9,16)(10,15)(11,14)(12,13);
```

(1,8)(2,7)(3,6)(4,5)(9,16)(10,15)(11,14)(12,13)

```
gap> g:=Group(a,b);
```

Group(Γ , (1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)(10,15))

```
gap> StructureDescription(g);
```

"D16"

```
gap> KramerMesnerSearch(2,16,6,2,g);
```

A GAP session

```
CTblLib 1.3.1, FactInt 1.6.3, FGA 1.4.0, Forms 1.2.5, GAPDoc 1.6.4, gen  
PrimGrp 3.4.1, RadiRoot 2.8, recog 1.3.2, ResClasses 4.7.2, SmallGrp 1.  
Try '??help' for help. See also '?copyright', '?cite' and '?authors'  
gap> LoadPackage("PAG");  
Loading PAG 0.1.4 (Prescribed Automorphism Groups)  
by Vedran Krcadinac (https://web.math.pmf.unizg.hr/~krcko/homepage.html).  
Homepage: https://gap-packages.github.io/pag  
Report issues at https://github.com/gap-packages/pag/issues  
true  
gap> a:=(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16);  
(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16)  
gap> b:=(1,8)(2,7)(3,6)(4,5)(9,16)(10,15)(11,14)(12,13);  
(1,8)(2,7)(3,6)(4,5)(9,16)(10,15)(11,14)(12,13)  
gap> g:=Group(a,b);  
Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)(10,15)  
gap> StructureDescription(g);  
"D16"  
gap> KramerMesnerSearch(2,16,6,2,g);  
Computing t-subset orbit representatives...  
13  
Computing k-subset orbit representatives...
```

A GAP session

```
0001000000000000000000000000000010000000000  
00000000000010000000000010000000000  
0001000000000000000000000000000010000000000  
00000000000010000000000010000000000  
100000000000000000000000000000001000000000  
0000000000100000000000000010000000000  
100000000000000000000000000000001000000000  
0000000000100000000000000010000000000
```

Prune_cs: 43

Prune_only_zeros: 0 of 42

Prune_hoelder: 0 of 42

Prune_N: 0

Fincke-Pohst: 0

Loops: 93

Total number of solutions: 8

total enumeration time: 0:00:00

```
[ [ [ 1, 2, 3, 6, 13, 15 ], [ 1, 3, 9, 10, 11, 14 ] ], [ [ 1, 2, 3, 6, 13, 15 ], [
[ [ 1, 2, 3, 6, 13, 15 ], [ 1, 3, 9, 10, 11, 14 ] ], [ [ 1, 2, 4, 5, 14, 16 ], [
[ [ 1, 2, 4, 5, 14, 16 ], [ 1, 3, 9, 11, 12, 16 ] ], [ [ 1, 2, 3, 6, 9, 11 ], [ 1,
[ [ 1, 2, 3, 6, 9, 11 ], [ 1, 3, 12, 13, 15, 16 ] ], [ [ 1, 2, 4, 5, 10, 12 ], [ 1,
```

gap>

A GAP session

```
0001000000000000000000000000000010000000000  
00000000000010000000000010000000000  
0001000000000000000000000000000010000000000  
00000000000010000000000010000000000  
100000000000000000000000000000001000000000  
0000000000100000000000000010000000000  
100000000000000000000000000000001000000000  
0000000000100000000000000010000000000
```

Prune_cs: 43

Prune_only_zeros: 0 of 42

Prune_hoelder: 0 of 42

Prune_N: 0

Fincke-Pohst: 0

Loops: 93

Total number of solutions: 8

total enumeration time: 0:00:00

```
[ [ [ 1, 2, 3, 6, 13, 15 ], [ 1, 3, 9, 10, 11, 14 ] ], [ [ 1, 2, 3, 6, 13, 15 ], [
[ [ 1, 2, 3, 6, 13, 15 ], [ 1, 3, 9, 10, 11, 14 ] ], [ [ 1, 2, 4, 5, 14, 16 ], [
[ [ 1, 2, 4, 5, 14, 16 ], [ 1, 3, 9, 11, 12, 16 ] ], [ [ 1, 2, 3, 6, 9, 11 ], [ 1,
[ [ 1, 2, 3, 6, 9, 11 ], [ 1, 3, 12, 13, 15, 16 ] ], [ [ 1, 2, 4, 5, 10, 12 ], [
gap> d:=KramerMesnerSearch(2,16,6,2,g,rec(NonIsomorphic:=true));
```

A GAP session

```
000000000100000000000000010000000  
100000000000000000000000000000001000000  
000000000100000000000000000000000000000001000000  
Prune_cs: 43  
Prune_only_zeros: 0 of 42  
Prune_hoelder: 0 of 42  
Prune_N: 0  
Fincke-Pohst: 0  
Loops: 93  
Total number of solutions: 8
```

```
total enumeration time: 0:00:00
```

```
Performing isomorph rejection...
```

```
2
```

```
[ rec( autGroup := Group([ (1,5)(3,7)(9,13)(11,15), (1,2)(3,12,9,8,15,10)(4,11,14,7),  
    autSubgroup := D16, blocks := [ [ 1, 2, 4, 5, 14, 16 ], [ 1, 2, 6, 7, 11, 13 ],  
        [ 1, 7, 9, 12, 15, 16 ], [ 2, 3, 5, 6, 9, 15 ], [ 2, 3, 7, 8, 12, 14 ], [ 3, 5, 11, 12, 13, 16 ], [ 4, 5, 7, 8, 9, 11 ], [ 4, 6, 9, 12, 13, 14 ],  
        v := 16 ), rec( autGroup := <permutation group with 6 generators>, autSubgroup :=  
            [ 1, 4, 5, 6, 10, 16 ], [ 1, 4, 7, 8, 11, 13 ], [ 1, 7, 9, 12, 15, 16 ],  
            [ 2, 5, 6, 7, 9, 11 ], [ 2, 8, 9, 10, 13, 16 ], [ 3, 4, 5, 8, 9, 15 ], [ 6, 8, 11, 14, 15, 16 ] ], isBinary := true, isBlockDesign := true, v :=
```

```
gap>
```

A GAP session

```
0000000001000000000000000010000000  
100000000000000000000000000000001000000  
0000000001000000000000000000001000000  
Prune_cs: 43  
Prune_only_zeros: 0 of 42  
Prune_hoelder: 0 of 42  
Prune_N: 0  
Fincke-Pohst: 0  
Loops: 93  
Total number of solutions: 8
```

```
total enumeration time: 0:00:00  
Performing isomorph rejection...
```

```
2  
[ rec( autGroup := Group([ (1,5)(3,7)(9,13)(11,15), (1,2)(3,12,9,8,15,10)(4,11,14,7)  
autSubgroup := D16, blocks := [ [ 1, 2, 4, 5, 14, 16 ], [ 1, 2, 6, 7, 11, 13 ]  
[ 1, 7, 9, 12, 15, 16 ], [ 2, 3, 5, 6, 9, 15 ], [ 2, 3, 7, 8, 12, 14 ], [ 3, 5, 11, 12, 13, 16 ], [ 4, 5, 7, 8, 9, 11 ], [ 4, 6, 9, 12, 13, 14 ],  
v := 16 ), rec( autGroup := <permutation group with 6 generators>, autSubgroup  
[ 1, 4, 5, 6, 10, 16 ], [ 1, 4, 7, 8, 11, 13 ], [ 1, 7, 9, 12, 15, 16 ],  
[ 2, 5, 6, 7, 9, 11 ], [ 2, 8, 9, 10, 13, 16 ], [ 3, 4, 5, 8, 9, 15 ], [ 6, 8, 11, 14, 15, 16 ] ], isBinary := true, isBlockDesign := true, v :=
```

```
gap> List(d,AllTDesignLambdas);
```

A GAP session

Prune.cs: 43

Prune only zeros: 0 of 42

Prune hoelder: 0 of 42

Prune N: 0

Fincke-Pohst: 0

Loops: 93

Total number of solutions: 8

total enumeration time: 0:00:00

Performing isomorph rejection...

2

```
[ rec( autGroup := Group([ (1,5)(3,7)(9,13)(11,15), (1,2)(3,12,9,8,15,10)(4,11,14,7),
    autSubgroup := D16, blocks := [ [ 1, 2, 4, 5, 14, 16 ], [ 1, 2, 6, 7, 11, 13 ],
        [ 1, 7, 9, 12, 15, 16 ], [ 2, 3, 5, 6, 9, 15 ], [ 2, 3, 7, 8, 12, 14 ], [
        [ 3, 5, 11, 12, 13, 16 ], [ 4, 5, 7, 8, 9, 11 ], [ 4, 6, 9, 12, 13, 14 ],
        v := 16 ), rec( autGroup := <permutation group with 6 generators>, autSubgroup =
            [ 1, 4, 5, 6, 10, 16 ], [ 1, 4, 7, 8, 11, 13 ], [ 1, 7, 9, 12, 15, 16 ],
            [ 2, 5, 6, 7, 9, 11 ], [ 2, 8, 9, 10, 13, 16 ], [ 3, 4, 5, 8, 9, 15 ], [
            [ 6, 8, 11, 14, 15, 16 ] ], isBinary := true, isBlockDesign := true, v :=
```

```
gap> List(d,AllTDesignLambdas);
```

```
[ [ 16, 6, 2 ], [ 16, 6, 2 ] ]
```

gap>

A GAP session

00000000010000000000000001000000

Prune.cs: 43

Prune only zeros: 0 of 42

Prune hoelder: 0 of 42

Prune N: 0

Fincke-Pohst: 0

Loops: 93

Total number of solutions: 8

total enumeration time: 0:00:00

Performing isomorph rejection...

2

```
[ rec( autGroup := Group([ (1,5)(3,7)(9,13)(11,15), (1,2)(3,12,9,8,15,10)(4,11,14,7),
    autSubgroup := D16, blocks := [ [ 1, 2, 4, 5, 14, 16 ], [ 1, 2, 6, 7, 11, 13 ],
        [ 1, 7, 9, 12, 15, 16 ], [ 2, 3, 5, 6, 9, 15 ], [ 2, 3, 7, 8, 12, 14 ], [
        [ 3, 5, 11, 12, 13, 16 ], [ 4, 5, 7, 8, 9, 11 ], [ 4, 6, 9, 12, 13, 14 ],
        v := 16 ), rec( autGroup := <permutation group with 6 generators>, autSubgroup =
            [ 1, 4, 5, 6, 10, 16 ], [ 1, 4, 7, 8, 11, 13 ], [ 1, 7, 9, 12, 15, 16 ],
            [ 2, 5, 6, 7, 9, 11 ], [ 2, 8, 9, 10, 13, 16 ], [ 3, 4, 5, 8, 9, 15 ], [
            [ 6, 8, 11, 14, 15, 16 ] ], isBinary := true, isBlockDesign := true, v :=
```

```
gap> List(d,AllTDesignLambdas);
```

```
[ [ 16, 6, 2 ], [ 16, 6, 2 ] ]
```

```
gap> d:=KramerMesnerSearch(2,16,4,1,g,rec(NonIsomorphic:=true));
```

A GAP session

Dimension of solution space (k): 2 compared to s-z+2: -5

Number of nonzero entries in the last row: 1

Fq: 1.000000

Fd: 9.009000

16.000 13.000

0 1

0 1

Prune_cs: 1

Prune_only_zeros: 0 of 0

Prune_hoelder: 0 of 0

Prune_N: 0

Fincke-Pohst: 0

Loops: 1

Total number of solutions: 0

total enumeration time: 0:00:00

Performing isomorph rejection...

0

gap>

A GAP session

Dimension of solution space (k): 2 compared to s-z+2: -5

Number of nonzero entries in the last row: 1

Fq: 1.000000

Fd: 9.009000

16.000 13.000

0 1

0 1

Prune_cs: 1

Prune_only_zeros: 0 of 0

Prune_hoelder: 0 of 0

Prune_N: 0

Fincke-Pohst: 0

Loops: 1

Total number of solutions: 0

total enumeration time: 0:00:00

Performing isomorph rejection...

0

gap> g2:=Group(a);

A GAP session

Fq: 1.000000

Fd: 9.009000

16.000 13.000

0 1

0 1

Prune_cs: 1

Prune_only_zeros: 0 of 0

Prune_hoelder: 0 of 0

Prune_N: 0

Fincke-Pohst: 0

Loops: 1

Total number of solutions: 0

total enumeration time: 0:00:00

Performing isomorph rejection...

0

gap> g2:=Group(a);

Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16)])

gap>

A GAP session

```
Fq: 1.000000
```

```
Fd: 9.009000
```

```
16.000 13.000
```

```
0 1
```

```
0 1
```

```
Prune_cs: 1
```

```
Prune_only_zeros: 0 of 0
```

```
Prune_hoelder: 0 of 0
```

```
Prune_N: 0
```

```
Fincke-Pohst: 0
```

```
Loops: 1
```

```
Total number of solutions: 0
```

```
total enumeration time: 0:00:00
```

```
Performing isomorph rejection...
```

```
0
```

```
gap> g2:=Group(a);
```

```
Group([ (1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16) ])
```

```
gap> StructureDescription(g2);
```

A GAP session

16.000 13.000

0 1
0 1

Prune_cs: 1
Prune_only_zeros: 0 of 0
Prune_hoelder: 0 of 0
Prune_N: 0
Fincke-Pohst: 0
Loops: 1
Total number of solutions: 0

total enumeration time: 0:00:00
Performing isomorph rejection...
0
gap> g2:=Group(a);
Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16)])
gap> StructureDescription(g2);
"C8"
gap>

A GAP session

16.000 13.000

0 1

0 1

Prune_cs: 1

Prune_only_zeros: 0 of 0

Prune_hoelder: 0 of 0

Prune_N: 0

Fincke-Pohst: 0

Loops: 1

Total number of solutions: 0

total enumeration time: 0:00:00

Performing isomorph rejection...

0

gap> g2:=Group(a);

Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16)])

gap> StructureDescription(g2);

"C8"

gap> d:=KramerMesnerSearch(2,16,4,1,g2,rec(NonIsomorphic:=true));

A GAP session

total enumeration time: 0:00:00

Performing isomorph rejection...

1

```
[ rec( autGroup := <permutation group with 6 generators>, autSubgroup := C8,
      blocks := [ [ 1, 2, 7, 14 ], [ 1, 3, 4, 16 ], [ 1, 5, 11, 15 ],
                   [ 1, 6, 8, 13 ], [ 1, 9, 10, 12 ], [ 2, 3, 8, 15 ], [ 2, 4, 5, 9 ],
                   [ 2, 6, 12, 16 ], [ 2, 10, 11, 13 ], [ 3, 5, 6, 10 ],
                   [ 3, 7, 9, 13 ], [ 3, 11, 12, 14 ], [ 4, 6, 7, 11 ],
                   [ 4, 8, 10, 14 ], [ 4, 12, 13, 15 ], [ 5, 7, 8, 12 ],
                   [ 5, 13, 14, 16 ], [ 6, 9, 14, 15 ], [ 7, 10, 15, 16 ],
                   [ 8, 9, 11, 16 ] ], isBinary := true, isBlockDesign := true,
      v := 16 ) ]
```

gap>

A GAP session

A GAP session

```
Prune_cs: 792
Prune_only_zeros: 70 of 948
Prune_hoelder: 86 of 878
Prune_N: 0
Fincke-Pohst: 0
Loops: 1756
Total number of solutions: 16
```

```
total enumeration time: 0:00:00
```

```
Performing isomorph rejection...
```

```
1
```

```
[ rec( autGroup := <permutation group with 6 generators>, autSubgroup := C8,
      blocks := [ [ 1, 2, 7, 14 ], [ 1, 3, 4, 16 ], [ 1, 5, 11, 15 ],
                  [ 1, 6, 8, 13 ], [ 1, 9, 10, 12 ], [ 2, 3, 8, 15 ], [ 2, 4, 5, 9 ],
                  [ 2, 6, 12, 16 ], [ 2, 10, 11, 13 ], [ 3, 5, 6, 10 ],
                  [ 3, 7, 9, 13 ], [ 3, 11, 12, 14 ], [ 4, 6, 7, 11 ],
                  [ 4, 8, 10, 14 ], [ 4, 12, 13, 15 ], [ 5, 7, 8, 12 ],
                  [ 5, 13, 14, 16 ], [ 6, 9, 14, 15 ], [ 7, 10, 15, 16 ],
                  [ 8, 9, 11, 16 ] ], isBinary := true, isBlockDesign := true,
      v := 16 ) ]
```

```
gap> List(d,AllTDesignLambdas);
```

```
[ [ 20, 5, 1 ] ]
```

```
gap>
```

A GAP session

```
Prune_cs: 792
Prune_only_zeros: 70 of 948
Prune_hoelder: 86 of 878
Prune_N: 0
Fincke-Pohst: 0
Loops: 1756
Total number of solutions: 16
```

```
total enumeration time: 0:00:00
```

```
Performing isomorph rejection...
```

```
1
```

```
[ rec( autGroup := <permutation group with 6 generators>, autSubgroup := C8,
      blocks := [ [ 1, 2, 7, 14 ], [ 1, 3, 4, 16 ], [ 1, 5, 11, 15 ],
                  [ 1, 6, 8, 13 ], [ 1, 9, 10, 12 ], [ 2, 3, 8, 15 ], [ 2, 4, 5, 9 ],
                  [ 2, 6, 12, 16 ], [ 2, 10, 11, 13 ], [ 3, 5, 6, 10 ],
                  [ 3, 7, 9, 13 ], [ 3, 11, 12, 14 ], [ 4, 6, 7, 11 ],
                  [ 4, 8, 10, 14 ], [ 4, 12, 13, 15 ], [ 5, 7, 8, 12 ],
                  [ 5, 13, 14, 16 ], [ 6, 9, 14, 15 ], [ 7, 10, 15, 16 ],
                  [ 8, 9, 11, 16 ] ], isBinary := true, isBlockDesign := true,
      v := 16 ) ]
```

```
gap> List(d,AllTDesignLambdas);
```

```
[ [ 20, 5, 1 ] ]
```

```
gap> d:=KramerMesnerSearch(3,16,4,1,g,rec(NonIsomorphic:=true));;
```

A GAP session

```
total enumeration time: 0:00:00
Performing isomorph rejection...
30
gap>
```

A GAP session

```
total enumeration time: 0:00:00
Performing isomorph rejection...
30
gap> List(d,AllTDDesignLambdas);
```

A GAP session

```
100000001000000100001000000100000100100000100000000101001100000000000000001001001  
1000000010000000000100001000100000001100000010010000101001100000000000000001001001  
Prune_cs: 2035  
Prune_only_zeros: 397 of 3136  
Prune_hoelder: 593 of 2739  
Prune_N: 121  
Fincke-Pohst: 0  
Loops: 5323  
Total number of solutions: 152
```

total enumeration time: 0:00:00

Performing isomorph rejection...

30

```
gap> List(d,AllTDesignLambdas);
```

```
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

```
gap>
```

A GAP session

```
100000001000000100001000000100000100100000100000000101001100000000000000001001001  
1000000010000000000100001000100000001100000010010000101001100000000000000001001001  
Prune_cs: 2035  
Prune_only_zeros: 397 of 3136  
Prune_hoelder: 593 of 2739  
Prune_N: 121  
Fincke-Pohst: 0  
Loops: 5323  
Total number of solutions: 152
```

total enumeration time: 0:00:00

Performing isomorph rejection...

30

```
gap> List(d,AllTDesignLambdas);
```

```
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],  
[ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

```
gap> c:=(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16);
```

A GAP session

```
Prune_cs: 2035
Prune_only_zeros: 397 of 3136
Prune_hoelder: 593 of 2739
Prune_N: 121
Fincke-Pohst: 0
Loops: 5323
Total number of solutions: 152
```

```
total enumeration time: 0:00:00
Performing isomorph rejection...
30
```

```
gap> List(d,AllTDesignLambdas);
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
gap> c:=(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16);
(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)
gap>
```

A GAP session

```
Prune_cs: 2035
Prune_only_zeros: 397 of 3136
Prune_hoelder: 593 of 2739
Prune_N: 121
Fincke-Pohst: 0
Loops: 5323
Total number of solutions: 152
```

total enumeration time: 0:00:00

Performing isomorph rejection...

30

```
gap> List(d,AllTDesignLambdas);
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

```
gap> c:=(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16);
```

```
(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)
```

```
gap> g3:=Group(a,b,c);
```

A GAP session

```
Prune_N: 121
Fincke-Pohst: 0
Loops: 5323
Total number of solutions: 152
```

```
total enumeration time: 0:00:00
```

```
Performing isomorph rejection...
```

```
30
```

```
gap> List(d,AllTDesignLambdas);
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

```
gap> c:=(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16);
```

```
(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)
```

```
gap> g3:=Group(a,b,c);
```

```
Group([ (1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)
       (10,15)(11,14)(12,13), (1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16) ])
```

```
gap>
```

A GAP session

```
Prune_N: 121
Fincke-Pohst: 0
Loops: 5323
Total number of solutions: 152
```

```
total enumeration time: 0:00:00
```

```
Performing isomorph rejection...
```

```
30
```

```
gap> List(d,AllTDesignLambdas);
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

```
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```

```
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```

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Group([ (1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)
       (10,15)(11,14)(12,13), (1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16) ])
```

```
gap> StructureDescription(g3);
```

A GAP session

Loops: 5323

Total number of solutions: 152

total enumeration time: 0:00:00

Performing isomorph rejection...

30

gap> List(d,AllTDesignLambdas);

```
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

gap> c:=(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16);

(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)

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Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)
 (10,15)(11,14)(12,13), (1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)])

gap> StructureDescription(g3);

"C2 x D16"

gap>

A GAP session

Loops: 5323

Total number of solutions: 152

total enumeration time: 0:00:00

Performing isomorph rejection...

30

gap> List(d,AllTDesignLambdas);

```
[ [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ],
  [ 140, 35, 7, 1 ], [ 140, 35, 7, 1 ] ]
```

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(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)

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Group([(1,2,3,4,5,6,7,8)(9,10,11,12,13,14,15,16), (1,8)(2,7)(3,6)(4,5)(9,16)
 (10,15)(11,14)(12,13), (1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15)(8,16)])

gap> StructureDescription(g3);

"C2 x D16"

gap> d:=KramerMesnerSearch(5,16,6,3,g3,rec(NonIsomorphic:=true));;

A GAP session

```
1000000 loops, solutions: 4, fipo: 0
2000000 loops, solutions: 4, fipo: 0
3000000 loops, solutions: 4, fipo: 0
4000000 loops, solutions: 4, fipo: 0
5000000 loops, solutions: 4, fipo: 0
6000000 loops, solutions: 4, fipo: 0
7000000 loops, solutions: 4, fipo: 0
8000000 loops, solutions: 4, fipo: 0
9000000 loops, solutions: 4, fipo: 0
10000000 loops, solutions: 4, fipo: 0
11000000 loops, solutions: 4, fipo: 0
1000100000101000001101000010010000000011001010011010010000000001000001011001001010
10001000001010001001010000000101000000101010100110100001000001000010001001001001001
001010011000001001000000001010100010000101100000100000010000010000110011000000001100
0000110110000010010100000100001000010100001100000100100000100010010000110000000001100
12000000 loops, solutions: 8, fipo: 0
13000000 loops, solutions: 8, fipo: 0
14000000 loops, solutions: 8, fipo: 0
15000000 loops, solutions: 8, fipo: 0
16000000 loops, solutions: 8, fipo: 0
17000000 loops, solutions: 8, fipo: 0
18000000 loops, solutions: 8, fipo: 0
^CPerforming isomorph rejection...
```

A GAP session

```
3000000 loops, solutions: 4, fipo: 0
4000000 loops, solutions: 4, fipo: 0
5000000 loops, solutions: 4, fipo: 0
6000000 loops, solutions: 4, fipo: 0
7000000 loops, solutions: 4, fipo: 0
8000000 loops, solutions: 4, fipo: 0
9000000 loops, solutions: 4, fipo: 0
10000000 loops, solutions: 4, fipo: 0
11000000 loops, solutions: 4, fipo: 0
10001000001010000011010000100100000000110010100110100100000000001000001011001001010
10001000001010001001010000000101000000101010100110100001000001000010001001001001001
001010011000001001000000001010100010000101100000100000010000010000110011000000001100
00001101100000100101000001000010000101000011000000100100000100010000110000000001100
12000000 loops, solutions: 8, fipo: 0
13000000 loops, solutions: 8, fipo: 0
14000000 loops, solutions: 8, fipo: 0
15000000 loops, solutions: 8, fipo: 0
16000000 loops, solutions: 8, fipo: 0
17000000 loops, solutions: 8, fipo: 0
18000000 loops, solutions: 8, fipo: 0
^CPerforming isomorph rejection...
```

4

gap>

A GAP session

```
3000000 loops, solutions: 4, fipo: 0
4000000 loops, solutions: 4, fipo: 0
5000000 loops, solutions: 4, fipo: 0
6000000 loops, solutions: 4, fipo: 0
7000000 loops, solutions: 4, fipo: 0
8000000 loops, solutions: 4, fipo: 0
9000000 loops, solutions: 4, fipo: 0
10000000 loops, solutions: 4, fipo: 0
11000000 loops, solutions: 4, fipo: 0
10001000001010000011010000100100000000110010100110100100000000001000001011001001010
10001000001010001001010000000101000000101010100110100001000001000010001001001001001
001010011000001001000000001010100010000101100000100000010000010000110011000000001100
00001101100000100101000001000010000101000011000000100100000100010000110000000001100
12000000 loops, solutions: 8, fipo: 0
13000000 loops, solutions: 8, fipo: 0
14000000 loops, solutions: 8, fipo: 0
15000000 loops, solutions: 8, fipo: 0
16000000 loops, solutions: 8, fipo: 0
17000000 loops, solutions: 8, fipo: 0
18000000 loops, solutions: 8, fipo: 0
^CPerforming isomorph rejection...
4
gap> List(d,AllTDesignLambdas);
```

A GAP session

```
6000000 loops, solutions: 4, fipo: 0
7000000 loops, solutions: 4, fipo: 0
8000000 loops, solutions: 4, fipo: 0
9000000 loops, solutions: 4, fipo: 0
10000000 loops, solutions: 4, fipo: 0
11000000 loops, solutions: 4, fipo: 0
100010000010100001101000010010000000110010100110100100000000001000001011001001010
10001000001010001001010000000101000000101010100110100001000001000010001001001001001
001010011000001001000000001010100010000101100000100000010000010000110011000000001100
0000110110000010010100000100001000010100001100000100100000100010000110000000001100
12000000 loops, solutions: 8, fipo: 0
13000000 loops, solutions: 8, fipo: 0
14000000 loops, solutions: 8, fipo: 0
15000000 loops, solutions: 8, fipo: 0
16000000 loops, solutions: 8, fipo: 0
17000000 loops, solutions: 8, fipo: 0
18000000 loops, solutions: 8, fipo: 0
^CPerforming isomorph rejection...
4
gap> List(d,AllTDesignLambdas);
[ [ 2184, 819, 273, 78, 18, 3 ], [ 2184, 819, 273, 78, 18, 3 ],
  [ 2184, 819, 273, 78, 18, 3 ], [ 2184, 819, 273, 78, 18, 3 ] ]
gap>
```

KramerMesnerSearch: behind the scenes

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- Generate G -orbits of t -subsets and k -subsets of V [GAP code]

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- Isomorph rejection using B. D. McKay's program **nauty** [interface to C program]

PAG – Implemented so far

Version 0.1.2:

- Auxiliary functions for permutation groups: MoveGroup, MultiGroup, RestrictedGroup, AllSubgroupsConjugation...

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Version 0.1.4:

- Latin squares and MOLS sets with prescribed autotopism groups: KramerMesnerMOLS, MOLSAut, MOLSFILTER...

PAG – Future developments

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The End

Thanks for your attention!