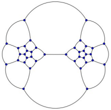

Simplicial complex of polyomino type tilings $K_P(\mathbb{T}_{2\times n})$

(This is joint work with Đorđe Baralić)

4th Croatian Combinatorial Days

University of Bihać
Edin Liđan
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lidan.edin@gmail.com

Zagreb, September 22-23, 2022

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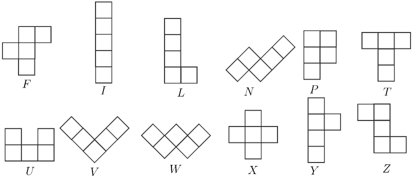

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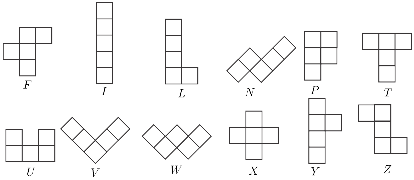

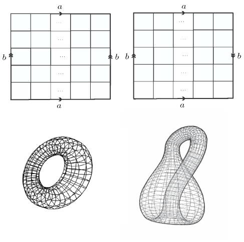

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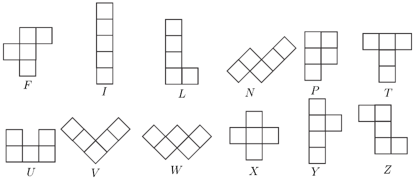

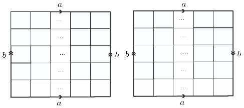

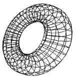

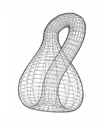

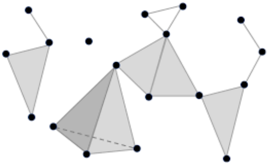

Polyomino

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Figure: Polyomino

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Figure: Not a polyomino

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- Solomon W. Golomb (1965.)

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- Martin Gardner Scientific American, "Mathematical Games"

Polyomino type tilings

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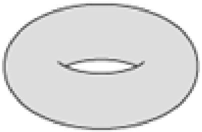

Figure: Torus $g = 1$

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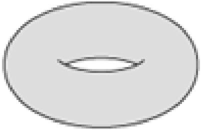

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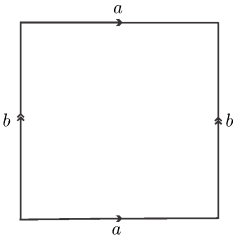

Figure: Plane model

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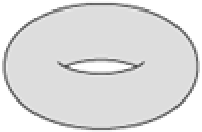

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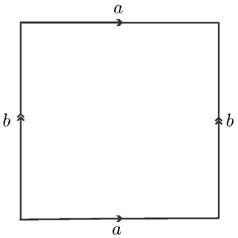

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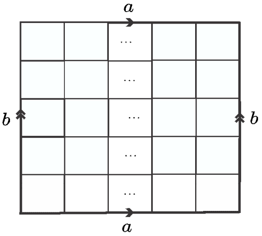

Figure: Square torus grid

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Is possible to tile square torus grid 5×5 with L–pentominoes?

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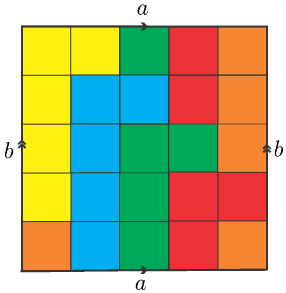

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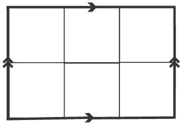

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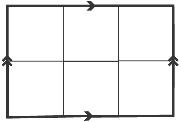

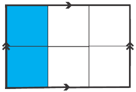

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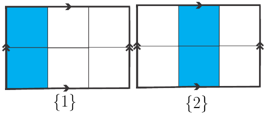
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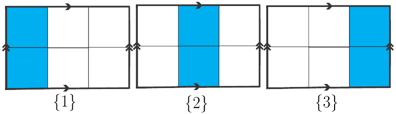
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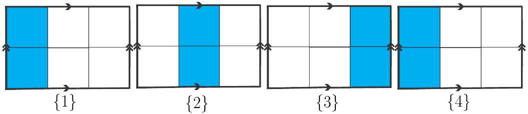
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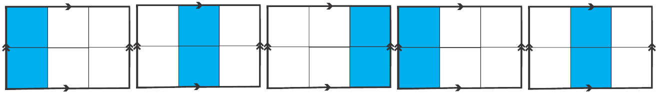

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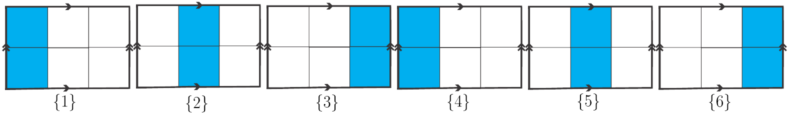
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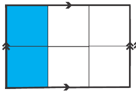

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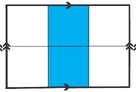

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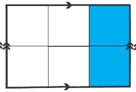

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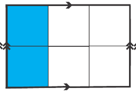

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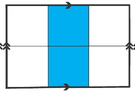

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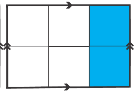

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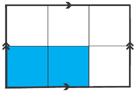

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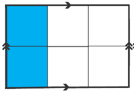

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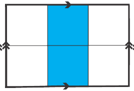

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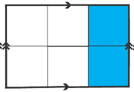

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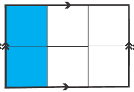

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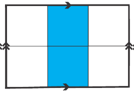

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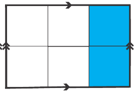

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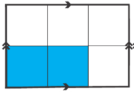

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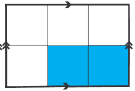

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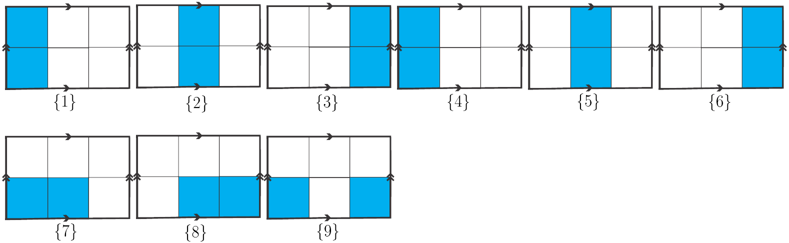
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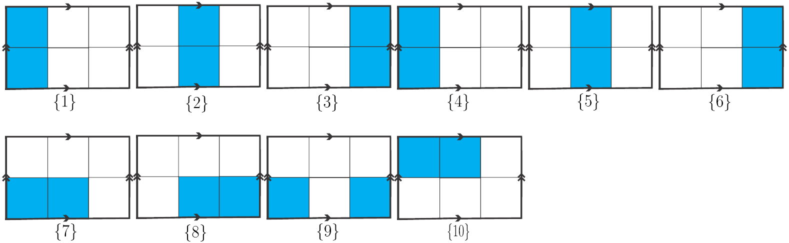
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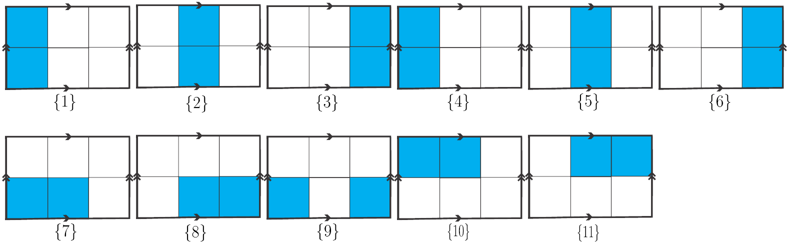
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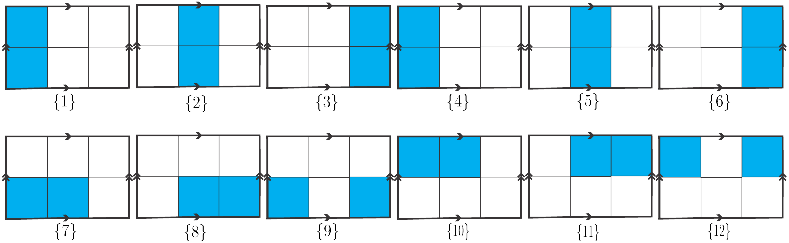
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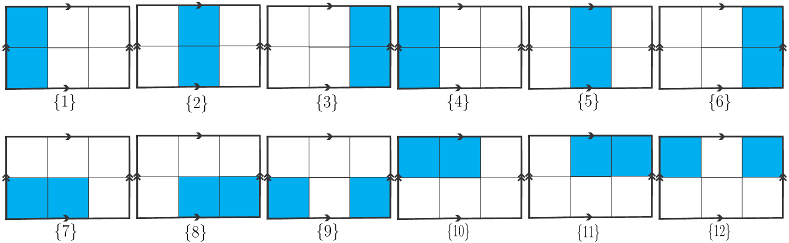

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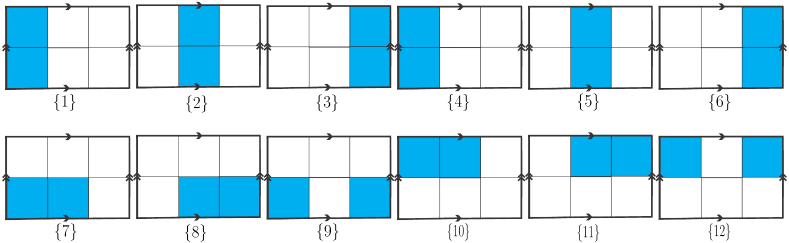

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- Faces of dimension 0 are called *vertices* (0-simplex): ({1},{2},{3},{4},{5},{6},{7},{8},{9},{10},{11},{12})

- edges (1-simplex): ({1,2}, {1,3}, {1,5}, {1,6}, {1,8}, {1,11}, {2,3}, {2,5}, {2,6}, {2,9}, {2,12}, {3,4}, {3,6}, {3,7}, {3,10}, {4,5}, {4,6}, {4,8}, {4,11}, {5,6}, {5,9}, {5,12}, {6,7}, {6,10}, {7,10}, {7,11}, {7,12}, {8,10}, {8,11}, {8,12}, {9,10}, {9,11}, {9,12}

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- *The dimension of* K, $\dim K$, is defined as the maximum dimension of the faces of K.

- Simplicial complex $K_{I_2}(\mathbb{T}_{2\times 3})$

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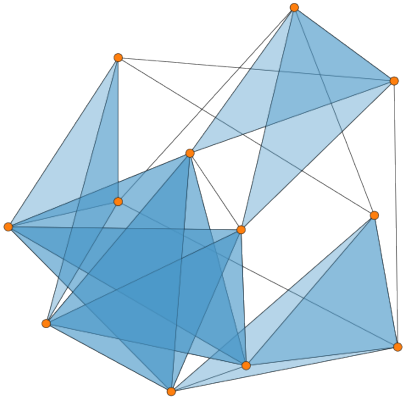

Figure: Simplices complex $K_{I_2}(\mathbb{T}_{2\times 3})$ presented in Sage 9.0

Simplicial complex of a polyomino tiling problem

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Proposition

Maximal number of polyomino shapes from $\mathcal{T}$ that may be placed on M without overlapping is $\dim(K(M;\mathcal{T}))+1$.

f-vector of simplicial complex

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- *The* **f**-*vector* of an $(n-1)$-dimensional simplicial complex K^{n-1} is the integer vector

$$\mathbf{f}(K^{n-1}) = (f_{-1}, f_0, f_1, \ldots, f_{n-1}),$$

where $f_{-1} = 1$ and $f_i = f_i(K^{n-1})$ denotes the number of i-faces of K^{n-1} for all $i = 1, \ldots, n-1$.

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- The $\mathbf{f}$*-polynomial* of an $(n-1)$-dimensional simplicial complex K is

$$\mathbf{f}(t) = t^n + f_0 t^{n-1} + \cdots + f_{n-1}.$$

Theorem

$\mathbf{f}$*–vector of simplicial complex* $K_{I_m}(\mathbb{T}_{1\times n})$ *is given by*

$$\mathbf{f}_k(K_{I_m}(\mathbb{T}_{1\times n})) = (m-1)\binom{n+(1-m)k-m}{k} + \binom{n-(m-1)(k+1)}{k+1}.$$

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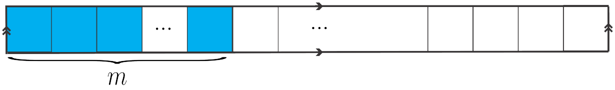

- Every placement of $k+1$ $1 \times m$ polyominoes on the square torus grid yields to a $k+2$-tuple of nonegative integers $(a_1, a_2, \dots, a_{k+2})$ satisfying (1),

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- where a_i is the number of noncovered cells between ith and $(i+1)$th shape as seen from the left to the right.
- Also, any $k+2$-tuple of nonegative integers such that

$$a_1 + a_2 + a_3 + \dots + a_{k+2} = n - mk - m, \qquad (1)$$

defines a placement of $k+1$ $1\times m$ I-minoes on the board without overlapping.

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- Indeed, the number of k-simplices of is equal to the number of nonegative integer solutions of the equation (1) so

$$\binom{n-(m-1)(k+1)}{k+1}.$$

2. Placement of $k+1$ $1 \times m$ I–minoes on the square torus grid with overlapping.

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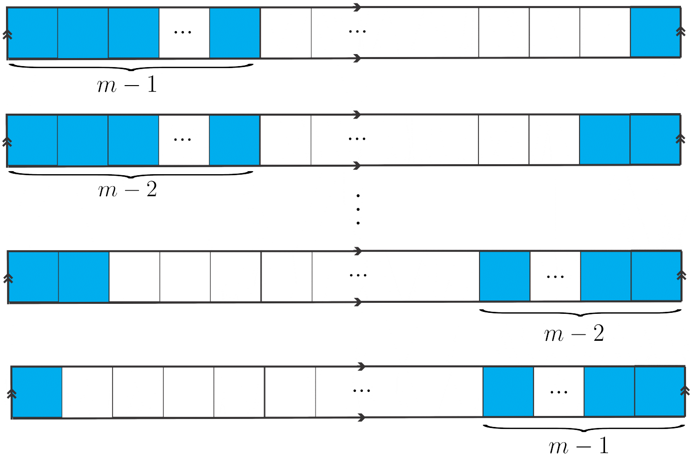

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$$a_1 + a_2 + a_3 + ... + a_{k+2} = n - mk - m, \qquad (2)$$

- Analoguous like in the first case, we obtained the number of k-simplices of is equal to the number of nonegative integer solutions of the equation (2) so

$$\binom{n+(1-m)k-m}{k}.$$

$$\mathbf{f}_k(K_{I_m}(\mathbb{T}_{1\times n})) \quad = \quad (m-1)\binom{n+(1-m)k-m}{k}+\binom{n-(m-1)(k+1)}{k+1}.$$

□

Join of simplicial complex

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Definition

Let K and L be simplicial complex with vertices S and S', where S and S' are mutually disjoint. Simplicial complex

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Proposition

Let K and L be simplicial complex. Then it is valid

$$\mathbf{f}(K * L) = \mathbf{f}(K) * \mathbf{f}(L).$$

Theorem

f–*vector of simplicial complex* $K_{I_3}(\mathbb{T}_{2\times n})$ *is given by*

$$
\begin{aligned}
\mathbf{f}_k(K_{I_3}(\mathbb{T}_{2\times n})) &= 4\sum_{j=0}^{k}\binom{n-j-2}{j}\binom{n-k+j-3}{k-j+1}\\
&+ 2\sum_{j=0}^{k}\binom{n-j-2}{j}\binom{n-k+j-3}{k-j+2}\\
&+ 2\sum_{j=0}^{k}\binom{n-j-2}{j+1}\binom{n-k+j+3}{k-j+1}\\
&+ \sum_{j=0}^{k}\binom{n-j-2}{j+1}\binom{n-k+j+3}{k-j+2}.
\end{aligned}
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- Example: $\mathbf{f}(K_{I_2}(\mathbb{T}_{2,3})) = (12, 33, 14)$.

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Table: Review **f**–vector simplicial complex $K_{I_2}(\mathbb{T}_{2\times n})$ for some concrete value of n

n	$\mathbf{f}_0$	$\mathbf{f}_1$	$\mathbf{f}_2$	$\mathbf{f}_3$	$\mathbf{f}_4$
3	12	33	14		
4	16	76	112	36	
5	20	136	371	376	102

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Table: Review **f**–vector simplicial complex $K_{I_2}(\mathbb{T}_{2\times n})$ for some concrete value of n

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5	20	136	371	376	102

Proposition

$$\begin{aligned} \mathbf{f}_0(\mathbb{T}_{2\times n}) &= 4n, \\ \mathbf{f}_1(\mathbb{T}_{2\times n}) &= 8n^2 - 13n. \end{aligned}$$

Other properties of simplicial complex of polyomino type tilings

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- Connectivity of simplicial complex of polyomino type tilings
- Homotopy of simplicial complex of polyomino type tilings

References

E. Liđan: *Topological characteristics of generalized polyomino tilings*, Doctoral dissertation, Faculty of Natural sciences, Podgorica, 2022.

Thank you for your attention.