

On automorphisms of a Fano plane 2-analog design

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A classical example of a (v, k, λ) -design is a Fano plane, a design with parameters $(7, 3, 1)$.

A 2-analog of a Fano plane is a collection of 3-dimensional blocks from \mathbb{F}_{2^7} such that any 2-dimensional subspace of \mathbb{F}_{2^7} is contained in one block from a collection of blocks



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in general $|E_{2^k}[E_{2^n}]| = \begin{bmatrix} n \\ k \end{bmatrix}_2$, where $\begin{bmatrix} n \\ k \end{bmatrix}_2$ is a gaussian 2-coefficient.



if $T \in E_{2^t}[E_{2^7}]$, then $|E_{2^k}[T]^{-1}| = |E_{2^{k-t}}[E_{2^7}/T]| = |E_{2^{k-t}}[E_{2^{7-t}}]| =$
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If $A \in E_{2^k}[E_{2^7}]$ and $\alpha \in Aut(E_{2^7})$ is of order m , we use a group ring $\mathbb{Z}[E_{2^k}[E_{2^7}]]$ to express α -orbit of A .





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The method of Kramer and Mesner



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V vector space over finite field \mathbb{F}_q , G group act on V , $\begin{bmatrix} V \\ t \end{bmatrix}_q$ collection of t -dimensional subspaces of V



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The estimated time for involution is 8×10^{12} CPU-years



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Using the formula of inclusion and exclusion we get $127 = X_1 - X_2 + X_3 - X_4 + \dots$, where $X_j = \sum_{P \in \binom{[127]}{j}} |\bigcap_{s \in P} (A^*)^{\alpha^s}|$, $j \geq 1$, where

is a collection of j -element subsets of $[127] = \{1, 2, \dots, 127\}$. Thus, we get $127 = 7 \cdot 127 - X_2 + X_3 - X_4 + \dots$.



We get

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Let us assume the opposite. Let $\langle \alpha \rangle \curvearrowright \mathcal{H}$, where α is of order 31. Let $\mathcal{H}_g = \{H \in \mathcal{H} \mid g \in H\}$. One can see that $|\mathcal{H}_g| = 21$. Also, from $|\{\mathcal{H}_g\}_{g \neq 1}| = 127$, we get $|\text{Fix}(\alpha, \{\mathcal{H}_g\}_{g \neq 1})| \equiv 3 \pmod{31}$.



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Automorphism of order 7



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Automorphism of order 7

Difficult case, since order of Singer automorphism of E_8 is SEVEN



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Automorphism of order 7

Difficult case, since order of Singer automorphism of E_8 is SEVEN

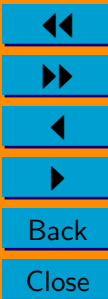
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Lemma: Let $\langle \alpha \rangle \hookrightarrow \mathcal{H}$ be of order 7. Then, $|Fix(\alpha)| = 1$.



Theorem: Let $\alpha \in \text{Aut}(E_{2^6})$ be of order 7 and $\text{Fix}(\alpha) = \phi$. If $\langle g^{\langle \alpha \rangle} \rangle < E_{2^6}$, then, $\langle g^{\langle \alpha \rangle} \rangle \cong E_{2^3}$. Furthermore, $\text{Fix}(\alpha, E_{2^3}[E_{2^6}]) = \{A, B\}$ and $E_{2^6} = A \times B$.



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Sketch:

Let's assume the opposite. Then, $|\text{Fix}(\alpha)| = 7$, $\text{Fix}(\alpha, E_{2^3}[E_{2^7}]) = \text{Fix}(\alpha, \mathcal{H}) = H_0$.



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We can expand the natural epimorphism $E_{2^7} \rightarrow E_{2^7}/H_0$ to a group ring by $\varphi : \mathbb{Z}[E_{2^7}] \rightarrow \mathbb{Z}[E_{2^7}/H_0]$, where $\varphi(H_i^{\alpha^j}) = 2H_i^{\alpha^j}/H_0$, $i \in [4]$, $j \in [5]$.





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$E_{2^2}[E_{2^6}]$, $j \in [7 - m]$, $\beta \in \text{Aut}(E_{2^6})$ of order 3, such that

$$\sum_{i=1}^{3m} A_i + \sum_{j=1}^{7-m} B_j^{\langle \alpha \rangle} = E_{2^6} + 20$$

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Automorphism of order 4



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Automorphism of order 4

Lemma: Let $\beta \in \text{Aut}(E_{2^n})$ be of order 2. Let $F = 1 + \text{Fix}(\beta)$. Then $|F| \geq 2^{n/2}$.



where $A_i^\beta = A_i$, $B_j \cap B_j^\beta = 1$, and $A_i \cong B_j \cong E_{2^2}$.



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where $A_i^\beta = A_i$, $B_j \cap B_j^\beta = 1$, and $A_i \cong B_j \cong E_{2^2}$.

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Lemma: Let $\alpha \in \text{Aut}(\mathcal{H})$ is of order 4. Then there are 28 α -orbits on E_{2^7} of a size 4. Furthermore, $\text{Fix}(\alpha^2) = \text{Fix}(\alpha) + \sum_{i=1}^{a_2} x_i^{\langle \alpha \rangle}$, where a_2 is the number of α -orbits on E_{2^7} of a size 2.



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Theorem: If $\alpha \in \text{Aut}(E_{27})$ and $o(\alpha) = 4$, then $\langle \alpha \rangle \not\hookrightarrow \mathcal{H}$.





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So, finally, we have proved the following:





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Thank You!



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