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The Loci of Vertices of Nedian Triangles

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ABSTRACT

In this article we observe nedians and nedian triangles of ratio η for a given triangle. The locus of vertices of the nedian triangles for $\eta \in \mathbb{R}$ is found and its correlation with isotomic conjugates of the given triangle is shown. Furthermore, the curve on which lie vertices of a nedian triangle for fixed η, when we iterate nedian triangles, is found.

Key words: triangle, cevian, nedian, nedian triangle, isotomic conjugate

MSC2010: 51M04, 51N20

Geometrijsko mjesto točaka vrhova nedijalnog trokuta

SAŽETAK

U ovom članku proučavaju se nedijane i nedijalni trokuti omjera η za dani trokut. Određuju se geometrijska mjesta vrhova nedijalnih trokuta za $\eta \in \mathbb{R}$ te ih se povezuje s pojmom izotomično konjugiranih točaka danog trokuta. Nadalje, određuje se krivulja na kojoj leže vrhovi nedijalnih trokuta za čvrsti η kada se iteriraju nedijalni trokuti.

Ključne riječi: trokut, cevain pravac, nedijalni pravac, nedijalni trokut, izotomično konjugirana točka

1 Introduction

The triangle is one of the first simple figures that mathematicians have studied but, as we can see in [9], it is still an interesting topic. There are numerous triangle points, lines, circles, conics, cubics, transformations, other specific triangles etc. that can be attached in a certain way to a single triangle. These elements can be repeated on the same triangle whereby then they can create a structure without an end. In this article we will add some interesting facts about the nedian triangles to the triangle geometry.

Definition 1 *Let a triangle $\triangle ABC$ be given and $\eta \in \mathbb{R}$. Let A_η, B_η and C_η be the points on the lines BC, CA and AB respectively, such that*

$$\begin{aligned} |AC_\eta| : |AB| &= \eta, \\ |BA_\eta| : |BC| &= \eta, \\ |CB_\eta| : |CA| &= \eta. \end{aligned} \tag{1}$$

is valid, assuming the sides have counterclockwise orientation. Then the cevians AA_η, BB_η and CC_η are called ***nedians of ratio*** η.

The segments are observed as oriented segments such that $|AB| = -|BA|$.

Definition 2 *Let a triangle $\triangle ABC$ be given and $\eta \in \mathbb{R}$ then the triangle $\triangle A_1B_1C_1$ is called the* **nedian triangle of ratio** η *of the given triangle $\triangle ABC$, whereby*

$$\begin{aligned} A_1 &= AA_\eta \cap CC_\eta, \\ B_1 &= BB_\eta \cap AA_\eta, \\ C_1 &= CC_\eta \cap BB_\eta. \end{aligned} \tag{2}$$

The nedian triangle of ratio 0 is the given triangle $\triangle ABC$ and the nedian triangle of ratio 1 is the triangle $\triangle BCA$. For $\eta = 1/2$, the points $A_{1/2}$, $B_{1/2}$ and $C_{1/2}$ are midpoints of sides of the triangle $\triangle ABC$, the nedians are the medians of the triangle $\triangle ABC$ which are concurrent at the centroid G

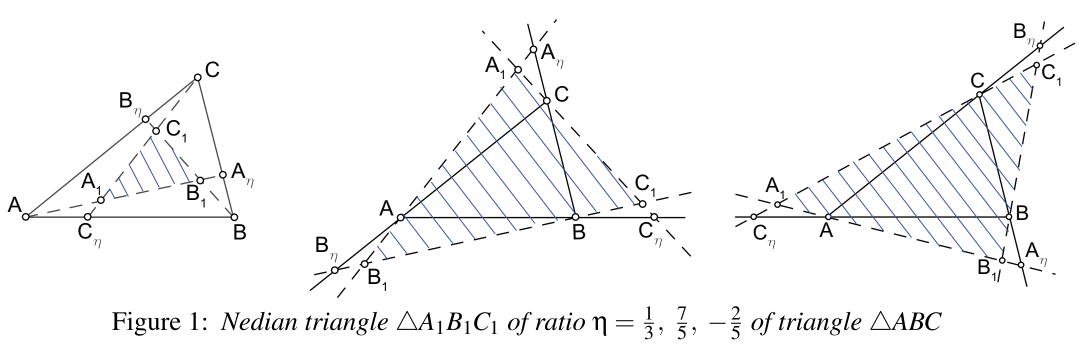

Figure 1: *Nedian triangle $\triangle A_1B_1C_1$ of ratio $\eta = \frac{1}{3},\ \frac{7}{5},\ -\frac{2}{5}$ of triangle $\triangle ABC$*

of the triangle $\triangle ABC$. Thus, the nedian triangle of ratio $1/2$ is deformed to a point (special case).

We can also define the nedian triangle of ratio $\eta = \infty$ for a given triangle $\triangle ABC$. The points A_∞, B_∞ and C_∞ are the points at infinity of the lines BC, CA and AB respectively. Accordingly, the nedians are the lines through the vertices of the triangle $\triangle ABC$ parallel to the opposite triangle side, i.e. the nedians are the exmedians of the given triangle, ([4], pp. 175-176). Let the intersections of nedians be denoted with G_A, G_B, G_C (see Fig. 2), thus the triangle $\triangle G_AG_BG_C$ is the nedian triangle of ratio ∞. Furthermore, in terms of triangle geometry, the exmedians form the anticomplementary triangle of the given triangle, whose vertices are called exmedian points of the given triangle, ([4], p. 176, [9]).

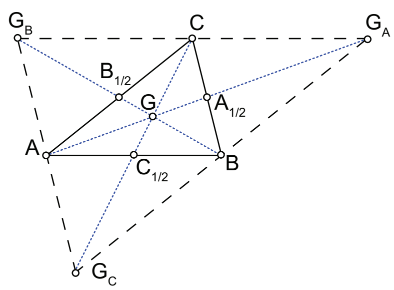

Figure 2: *Nedian triangle $\triangle G_AG_BG_C$ of ratio ∞*

The terminology for the nedian triangles comes from [6, 7], where J. Satterly observes the triangle we now call nedian triangle of ratio $1/n$, obtained by dividing the side of a given triangle in n parts. As he states, 'the name recalls the n', and the similarity to the medians. In [2], H.R. Chillingworth discusses that the ratio can be any real number.

Remark 1 *We will abide by the term nedian triangle from [6, 7], even though we could call it nedial triangle, and the nedian triangle the triangle whose sides have the same lengths as the nedians, i.e. $|AA_\eta|$, $|BB_\eta|$, $|CC_\eta|$. This terminology would follow the notion of the medial ([3], p. 18) and the median ([4], p. 282) triangle of a given triangle.*

Let $A(x_A,y_A)$, $B(x_B,y_B)$ and $C(x_C,y_C)$ be the vertices of a given triangle $\triangle ABC$, then the points A_η, B_η and C_η from Definition 1 have the following coordinates

$$\begin{aligned}
A_\eta &= ((x_C-x_B)\eta+x_B,(y_C-y_B)\eta+y_B),\\
B_\eta &= ((x_A-x_C)\eta+x_C,(y_A-y_C)\eta+y_C),\\
C_\eta &= ((x_B-x_A)\eta+x_A,(y_B-y_A)\eta+y_A),
\end{aligned} \tag{3}$$

and the vertices of the nedian triangle $\triangle A_1B_1C_1$ of ratio η have the coordinates

$$\begin{aligned}
A_1 &= \Big(\frac{\eta^2x_C+(\eta-1)^2x_A-\eta(\eta-1)x_B}{\eta^2-\eta+1},\frac{\eta^2y_C+(\eta-1)^2y_A-\eta(\eta-1)y_B}{\eta^2-\eta+1}\Big),\\
B_1 &= \Big(\frac{\eta^2x_A+(\eta-1)^2x_B-\eta(\eta-1)x_C}{\eta^2-\eta+1},\frac{\eta^2y_A+(\eta-1)^2y_B-\eta(\eta-1)y_C}{\eta^2-\eta+1}\Big),\\
C_1 &= \Big(\frac{\eta^2x_B+(\eta-1)^2x_C-\eta(\eta-1)x_A}{\eta^2-\eta+1},\frac{\eta^2y_B+(\eta-1)^2y_C-\eta(\eta-1)y_A}{\eta^2-\eta+1}\Big).
\end{aligned} \tag{4}$$

2 The affine transformation

For every two triangles in the same plane we can find an affine transformation that maps one triangle into the other triangle. Affine transformations have 6 degrees of freedom. They preserve affine properties such as parallelism and ratios of lengths as well as projective properties such as cross-ratios and concurrence, ([1] in 2.3.3, 3.3.2, 3.5.1).

Definition 3 **The standard triangle** *is the triangle with the vertices $A(0,0)$, $B(1,0)$ and $C(0,1)$.*

The affine transformation that maps the standard triangle into its nedian triangle of ratio η has the matrix

$$SN_\eta = \begin{pmatrix} \frac{(2\eta-1)(\eta-1)}{\eta^2-\eta+1} & \frac{(2\eta-1)\eta}{\eta^2-\eta+1} & -\frac{(\eta-1)\eta}{\eta^2-\eta+1}\\ -\frac{(2\eta-1)\eta}{\eta^2-\eta+1} & -\frac{2\eta-1}{\eta^2-\eta+1} & \frac{\eta^2}{\eta^2-\eta+1}\\ 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

The affine transformation for a general triangle can be found by composing transformation (5) and the transformation that maps the standard triangle into the general one, which has the following matrix

$$SG = \begin{pmatrix} -x_A+x_B & -x_A+x_C & x_A\\ -y_A+y_B & -y_A+y_C & y_A\\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

Since the centroid is an affine invariant, we can state the following

Theorem 1 *The centroid of all nedian triangles of a given triangle is the centroid of the given triangle.*

Another property of affine transformations is that they multiply the area of a given figure by $|\det(T_{aff})|$, where T_{aff} is the transformation matrix. Thus, in our example, the ratio of the area of the nedian triangle of ratio η and the area of the standard triangle equals

$$|\det(SN_\eta)| = \frac{(2\eta-1)^2}{\eta^2-\eta+1}. \tag{7}$$

3 Locus of vertices of the nedian triangles

Theorem 2 *Vertices of all nedian triangles of ratio η of a given triangle $\triangle ABC$ lie on three ellipses such that they intersect at the centroid of the triangle $\triangle ABC$, each of the ellipse passes through an exmedian point and two vertices of the triangle $\triangle ABC$ and is tangent to two sides of the triangle $\triangle ABC$.*

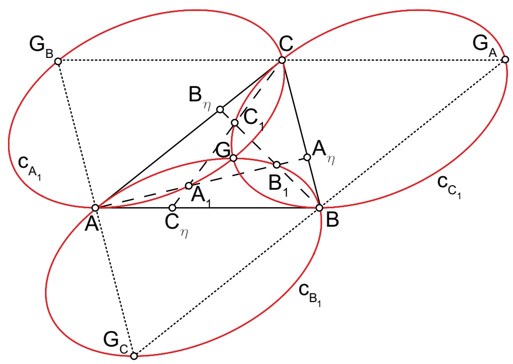

Figure 3: *Locus of vertices of nedian triangles*

Proof. For a given triangle $\triangle ABC$ the ratios (1) define 3 pairs of projective pencils $(A)\barwedge(B)$, $(B)\barwedge(C)$, $(C)\barwedge(A)$. The result of a pair of projective pencils is a conic which passes through the vertices of the pencils. Furthermore, for

i) $\eta = 0$,
$AB \in (A) \to BC \in (B)$,
$BC \in (B) \to CA \in (C)$,
$CA \in (C) \to AB \in (A)$,

ii) $\eta = 1$,
$AC \in (A) \to BA \in (B)$,
$BA \in (B) \to CB \in (C)$,
$CB \in (C) \to AC \in (A)$,

iii) $\eta = 1/2$,
$AA_{1/2} \in (A) \to BB_{1/2} \in (B)$,
$BB_{1/2} \in (B) \to CC_{1/2} \in (C)$,
$CC_{1/2} \in (C) \to AA_{1/2} \in (A)$,

iv) $\eta = \infty$,
$AA_\infty \in (A) \to BB_\infty \in (B)$,
$BB_\infty \in (B) \to CC_\infty \in (C)$,
$CC_\infty \in (C) \to AA_\infty \in (A)$,

whereby from (i) and (ii) follows that each conic is tangent to one pair of lines on which lie the triangle sides, from (iii) that each conic passes through the centroid G of the triangle $\triangle ABC$. Furthermore from (iv) follows that each conic passes through a vertex of the nedian triangle $\triangle G_A G_B G_C$ of ratio ∞, i.e. through an exmedian vertex of the given triangle $\triangle ABC$.

Each pair of projective pencils determines two ranges of points on the line at infinity with no real double points, therefore the conics are ellipses. □

For the standard triangle from (3) and (4) follows

$$A_\eta = (1-\eta, \eta),\ B_\eta = (0, 1-\eta),\ C_\eta = (\eta, 0), \tag{8}$$

$$\begin{aligned} A_1 &= \left(\frac{-\eta^2+\eta}{\eta^2-\eta+1}, \frac{\eta^2}{\eta^2-\eta+1}\right), \\ B_1 &= \left(\frac{\eta^2-2\eta+1}{\eta^2-\eta+1}, \frac{-\eta^2+\eta}{\eta^2-\eta+1}\right), \\ C_1 &= \left(\frac{\eta^2}{\eta^2-\eta+1}, \frac{\eta^2-2\eta+1}{\eta^2-\eta+1}\right). \end{aligned} \tag{9}$$

Points A_1, B_1, and C_1 are the parametrization of the ellipses in Theorem 2, denoted with $c_{A_1}, c_{B_1}, c_{C_1}$ (see Fig. 3), whose equations in terms of affine point coordinates are obtained by eliminating the parameter η. This yields:

$$\begin{aligned} c_{A_1} &: x^2+y^2+xy-y=0, \\ c_{B_1} &: x^2+y^2+xy-x=0, \\ c_{C_1} &: x^2+y^2+xy-2x-2y+1=0. \end{aligned} \tag{10}$$

The equations of those ellipses can be found for a general triangle with the inverse of the transformation (6).

4 Isotomic conjugates of a triangle

Definition 4 *If two cevians from a triangle vertex are intersecting the opposite side in points that are equidistant from its midpoint then they are called **isotomic lines** of the triangle and the points are called **isotomic points** of the triangle, ([5], p. 126).*

Definition 5 *Let a point P and a triangle $\triangle ABC$ be given. Then the isotomic lines of the cevians for the point P are concurrent at a point $\overline{P}$. The points P and $\overline{P}$ are called **isotomic conjugates** of the triangle $\triangle ABC$, ([4], pp. 157-158, [5], p. 127).*

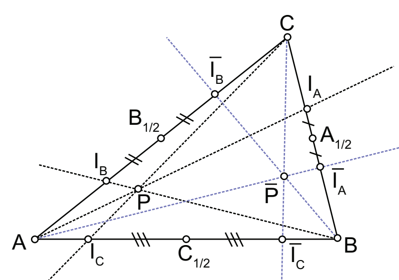

Figure 4: *Isotomic conjugates P and $\overline{P}$ of triangle $\triangle ABC$*

In ([4], pp. 157-158), ([5], p. 127) and [8] can be found various properties of the isotomic conjugates of a triangle, but here we will point out only the following:

- self-isotomic points of a triangle $\triangle ABC$ are the centroid G and the exmedian points G_A, G_B, G_C of the triangle $\triangle ABC$
- self-isotomic lines are the medians m_a, m_b, m_c and exmedians e_a, e_b, e_c of the triangle $\triangle ABC$
- the isotomic conjugate of the line at infinity for the triangle $\triangle ABC$ is the Steiner ellipse s
- every conic through points $AGBG_C$, $BGCG_A$ or $CGAG_B$ is self-isotomic. Each of these family of conics contains an special ellipse, denoted with s_c, s_a and s_b, which is a translation of the Steiner ellipse by the vector $\overrightarrow{CG}$, $\overrightarrow{AG}$ or $\overrightarrow{BG}$, respectively. Furthermore ellipses s_a, s_b and s_c are tangent to the triangle sides which are opposite to the vertices that the corresponding ellipse passes through.

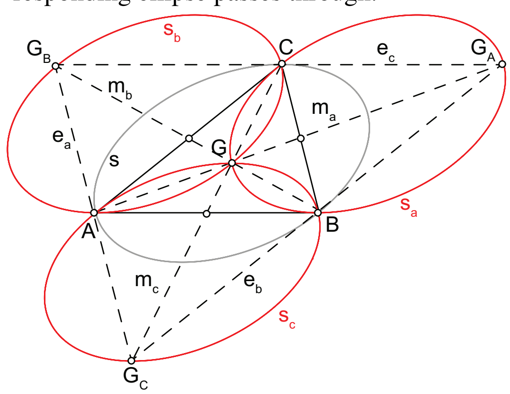

Figure 5: *Steiner ellipse s and self-isotomic elements of triangle $\triangle ABC$*

Theorem 3 *The vertices of all nedian triangles of a given triangle lie on the self-isotomic ellipses of the given triangle.*

Proof. Let the isotomic conjugate of the vertex B_1 of a nedian triangle be denoted with $\overline{B_1}$ and the intersections of cevians through $\overline{B_1}$ with lines AB, BC and CA be denoted with $\overline{C_\eta}$, $\overline{A_\eta}$ and $\overline{B_\eta}$ respectively. Then the points A_η, $\overline{A_\eta}$ and B_η, $\overline{B_\eta}$ are isotomic points of the triangle $\triangle ABC$, thus

$$|A_\eta A_{1/2}| = |A_{1/2}\overline{A_\eta}| \Rightarrow |BA_\eta| = |\overline{A_\eta}C|, \quad (11)$$
$$|B_\eta B_{1/2}| = |B_{1/2}\overline{B_\eta}| \Rightarrow |CB_\eta| = |\overline{B_\eta}A|.$$

On the other hand the point B_1 is a vertex of a nedian triangle and therefore from Definition 1 and a simple calculation follows

$$|B\overline{A_\eta}| : |BC| = 1-\eta, \quad (12)$$
$$|C\overline{B_\eta}| : |CA| = 1-\eta,$$

hence, $\overline{B_1}$ is a vertex of a nedian triangle when ratio is $1-\eta$ for every $\eta \in \mathbb{R} \cup \{\infty\}$. Analogously we can conclude for vertices A_1, C_1 and its isotomic conjugates, therefore the ellipses on which lie vertices of the nedian triangles are self-isotomic. □

From Theorem 2., Theorem 3. and properties of the special self-isotomic ellipses of a triangle we can conclude the following statement

Corollary 1 *The three ellipses on which lie the vertices of all nedian triangles of a given triangle are congruent.*

4.1 Equilateral triangle and its nedian triangle

Theorem 4 *The nedian triangle of ratio η of an equilateral triangle is equilateral.*

Proof. From $\triangle AA_\eta B \cong \triangle BB_\eta C \cong \triangle CC_\eta A$ follows $|AA_\eta| = |BB_\eta| = |CC_\eta|$. The ratios of segments on nedians can be computed and we obtain ratios

$$\frac{|A_1B_1|}{|AA_\eta|} = \frac{|B_1C_1|}{|BB_\eta|} = \frac{|C_1A_1|}{|CC_\eta|} = \frac{2\eta-1}{\eta^2-\eta+1}, \quad (13)$$

hence $|A_1B_1| = |B_1C_1| = |C_1A_1|$. □

Proposition 1 *Vertices of all nedian triangles of an given equilateral triangle lie on three circles such that each passes through the centroid, two vertices and an exmedian point of the given triangle.*

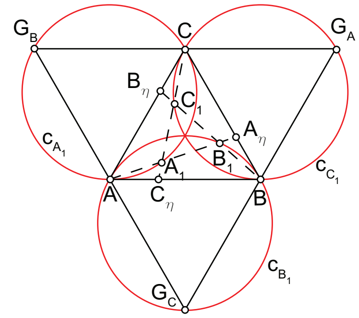

Figure 6: *Locus of vertices of nedian triangles of equilateral triangle*

Proof. W.l.o.g., due to the affine transformations, we will prove this statement for the equilateral triangle $A(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$, $B(\frac{\sqrt{3}}{2}, -\frac{1}{2})$, $C(0,1)$. From Theorem 2 the locus of vertices of the nedian triangles are passing through through the centroid, two vertices and an exmedian point of the given triangle. The Steiner ellipse for the triangle $\triangle ABC$ is the circle with equation $x^2+y^2=1$. Then by Theorem 3 the conics on which the vertices of the nedian triangle lie are circles with following equations

$$c_{A_1} : \left(x+\frac{\sqrt{3}}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 = 1,$$
$$c_{B_1} : x^2 + (y+1)^2 = 1, \quad (14)$$
$$c_{C_1} : \left(x-\frac{\sqrt{3}}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 = 1.$$

The same conclusion can be made if we apply the affine transformation on (10) which maps the standard triangle to the chosen equilateral triangle. □

5 Iterations in the standard triangle

We will explore what happens when we repeat the process of finding a nedian triangle. For a fixed η, the triangle $\triangle A_nB_nC_n$, $n \geq 0$ is defined as the nedian triangle of the triangle $\triangle A_{n-1}B_{n-1}C_{n-1}$ such that the triangle $\triangle A_0B_0C_0$ is the given triangle.

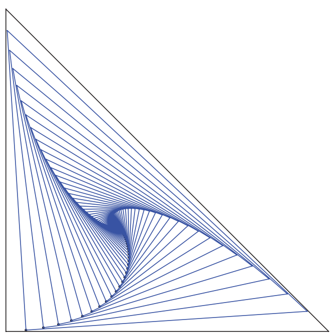

Figure 7: *First 50 iterations with* $\eta = 1/16$

To simplify the computations, we will work with the standard triangle. We will show that points A_n, $n \geq 0$ (see Fig. 8) lie on a curve and give its parametric equation.

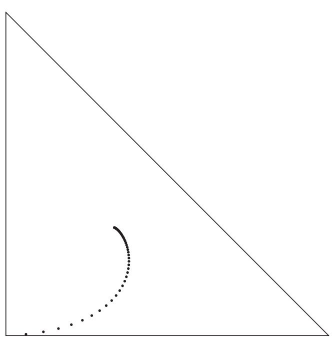

Figure 8: *Points* A_n *with* $\eta = 1/16$

We will observe the affine transformation (5) using barycentric coordinates. In this case, the transformation matrix for the nedian triangle of ratio η, as we can see from (4) is given by:

$$Q = \frac{(\eta-1)^2}{\eta^2-\eta+1} Q_1, \tag{15}$$

where

$$Q_1 = \begin{pmatrix} 1 & t & t^2 \\ t^2 & 1 & t \\ t & t^2 & 1 \end{pmatrix}, \quad t = \frac{1}{1-\eta}. \tag{16}$$

The eigenvalues of the matrix Q_1 are

$$\begin{aligned} \lambda_1 &= -\frac{1}{2}(t^2-t-2) + \frac{\sqrt{3}}{2}(t^2-t)i, \\ \lambda_2 &= -\frac{1}{2}(t^2-t-2) - \frac{\sqrt{3}}{2}(t^2-t)i, \\ \lambda_3 &= t^2+t+1. \end{aligned} \tag{17}$$

If we denote by $\sigma = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, which satisfies $\sigma^6 = 1$, the eigenvectors for Q_1 are

$$(1,\bar{\sigma},\sigma), (1,\sigma,\bar{\sigma}), (1,1,1). \tag{18}$$

If M is the matrix whose columns are the eigenvectors (18), then we can write

$$Q_1 = MDM^{-1}, \tag{19}$$

where D is the diagonal matrix

$$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}. \tag{20}$$

The triangle $A_nB_nC_n$, i. e. the n-th iteration of the nedian triangle, will be obtained by the transformation

$$Q_1^n = MD^nM^{-1}. \tag{21}$$

To find the equations of the curve which passes through the points A_n, $n \geq 0$, we have to make (21) continuous in the variable n. This can be done in the following way. Let us write the matrix D as $\exp D_1$, where D_1 is a diagonal matrix whose entries are $\ln \lambda_i$. From (17) we can compute that λ_1, λ_2 and λ_3 are never negative real numbers, so we can choose the principal logarithm branch.
This way, we have

$$\begin{aligned} Q^r &= \left(\frac{(\eta-1)^2}{\eta^2-\eta+1}\right)^r Q_1^r \\ Q^r &= \left(\frac{(\eta-1)^2}{\eta^2-\eta+1}\right)^r M \exp rD_1 M^{-1}, \end{aligned} \tag{22}$$

where r is any real number.
The matrix $\exp rD_1$ is the matrix

$$\begin{pmatrix} \lambda_1^r & 0 & 0 \\ 0 & \lambda_2^r & 0 \\ 0 & 0 & \lambda_3^r \end{pmatrix}. \tag{23}$$

The trajectory of the point A, for $r \in \mathbb{R}$ will be a curve, and all the points A_n, $n \geq 1$, lie on this curve.
The affine coordinates (x,y) of the point on the curve can be obtained from barycentric coordinates as the elements of the matrix $Q^r = (q_{ij}^r)$

$$x = q_{12}^r, \quad y = q_{13}^r, \tag{24}$$

and from this we get parametric equations of the curve. To find these equations and to prove that x and y are real numbers, we write λ_1, λ_2 in the trigonometric form as

$$\begin{aligned}\lambda_1 =& r_\lambda(\cos\varphi_\lambda + i\sin\varphi_\lambda), \\ \lambda_2 =& r_\lambda(\cos\varphi_\lambda - i\sin\varphi_\lambda),\end{aligned} \tag{25}$$

such that

$$\begin{aligned} r_\lambda =& \frac{1}{4}\sqrt{3(t^2-t)^2+(-t^2-t+2)^2} \\ \varphi_\lambda =& \arctan\left(\frac{\sqrt{3}(t^2-t)}{-t^2-t+2}\right)\end{aligned} \tag{26}$$

Then follows

$$\lambda_1^r = r_\lambda^r(\cos r\varphi_\lambda + i\sin r\varphi_\lambda). \tag{27}$$

From (24) we have

$$\begin{aligned} x =& \frac{1}{3}\left(\frac{(\eta-1)^2}{\eta^2-\eta+1}\right)^r(\bar{\sigma}\lambda_1^r+\sigma\lambda_2^r+\lambda_3^r), \\ y =& \frac{1}{3}\left(\frac{(\eta-1)^2}{\eta^2-\eta+1}\right)^r(\sigma\lambda_1^r+\bar{\sigma}\lambda_2^r+\lambda_3^r).\end{aligned} \tag{28}$$

These expressions are real, and substituting the trigonometric form of λ_1, λ_2 we get the following parametric equation for the curve which passes through A_n, $n \geqslant 0$

$$\begin{aligned} x =& \frac{1}{3}\left(\frac{(\eta-1)^2}{\eta^2-\eta+1}\right)^r\left(r_\lambda^r(\sqrt{3}\sin r\varphi_\lambda-\cos r\varphi_\lambda)+\lambda_3^r\right), \\ y =& \frac{1}{3}\left(\frac{(\eta-1)^2}{\eta^2-\eta+1}\right)^r\left(r_\lambda^r(\sqrt{3}\sin r\varphi_\lambda+\cos r\varphi_\lambda)+\lambda_3^r\right).\end{aligned} \tag{29}$$

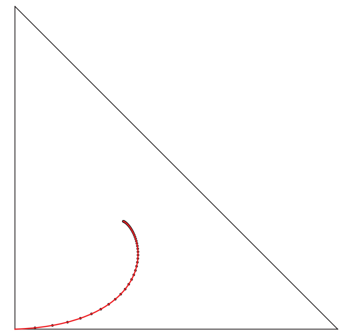

Figure 9: *First 50 iterations with* $\eta = 1/16$

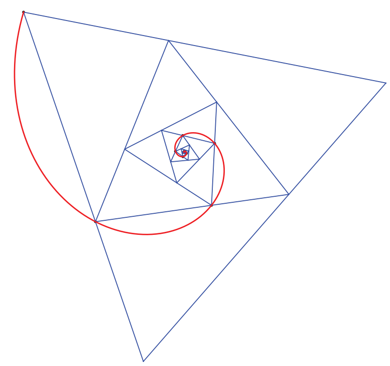

Figure 10: *First 10 iterations with* $\eta = 3$ *for equilateral triangle*

References

[1] D.A. Brannan, M.F. Esplen, J.J. Gray, *Geometry*, Cambridge University Press, New York, 2012.

[2] H.R. Chillingworth, A further note on the nedians of a plane triangle, *Gaz. Math.*, **40**/331 (1956), 52–53.

[3] H.S. Coxeter, S.L. Greitzer, *Geometry revisited*, The Mathematical Association of America, Washingthon D.C., 1967.

[4] R.A. Johnson, *Advanced Euclidean geometry: An elementary treatise on the geometry of the triangle and the circle*, Dover Publications Inc., New York, 1960.

[5] D. Palman, *Trokut i kružnica*, Element, Zagreb, 1994.

[6] J. Satterly, The nedians of a plane triangle, *Gaz. Math.*, **38**/324 (1954), 111–113.

[7] J. Satterly, The nedians, the nedian triangle and the aliquot triangle of a plane triangle, *Gaz. Math.*, **40**/332 (1956), 109–113.

[8] S. Sigur, Where are the conjugates?, *Forum Geom.*, **5** (2005), 1–15.

[9] C. Kimberling, Encyclopedia of Triangle Centers, `http://faculty.evansville.edu/ck6/encyclopedia/ETC.html`

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