GS 1. — 9. rujna 2025.

Zadatak 2.

Metodom sila izračunajte potrebne vrijednosti i nacrtajte dijagram momenata savijanja! Izračunajte orijentiranu duljinu pomaka desnoga ležaja!

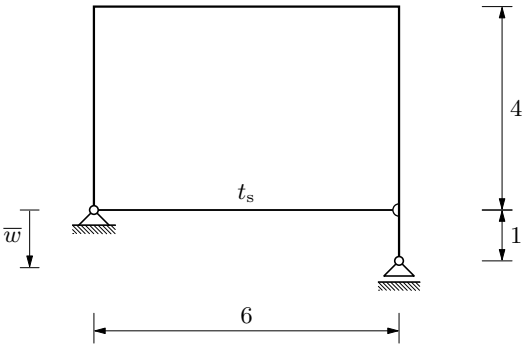
$t_s = -50°\mathrm{C}$

$\overline{w} = 2\,\mathrm{cm}$

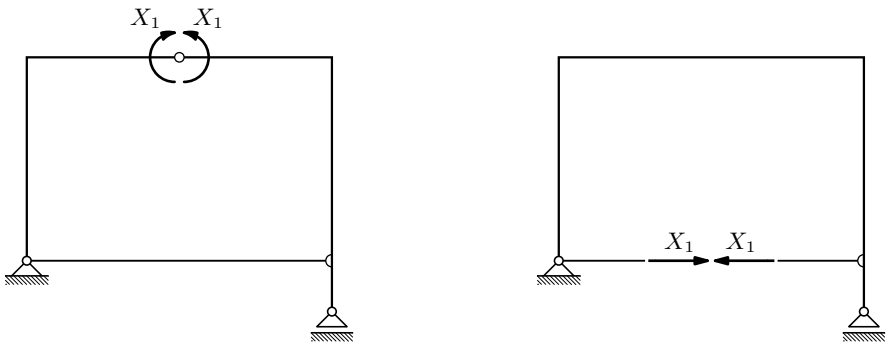
$\alpha_t = 10^{-5}\,\mathrm{K}^{-1}$

okvir: $EI = 162\,000\,\mathrm{kNm}^2$

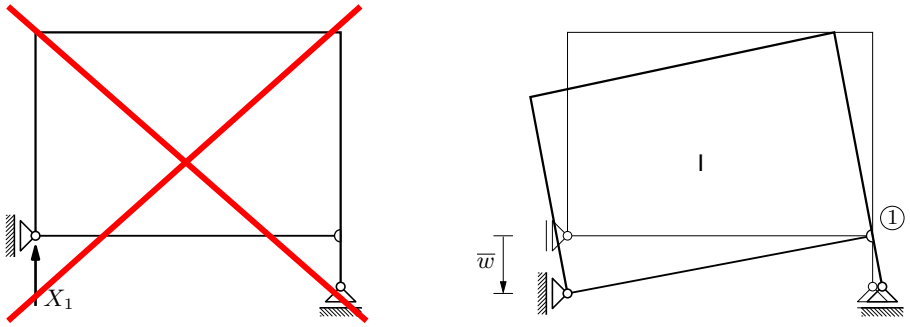
zatega: $EA = 125\,000\,\mathrm{kN}$



dva smislena osnovna sistema:



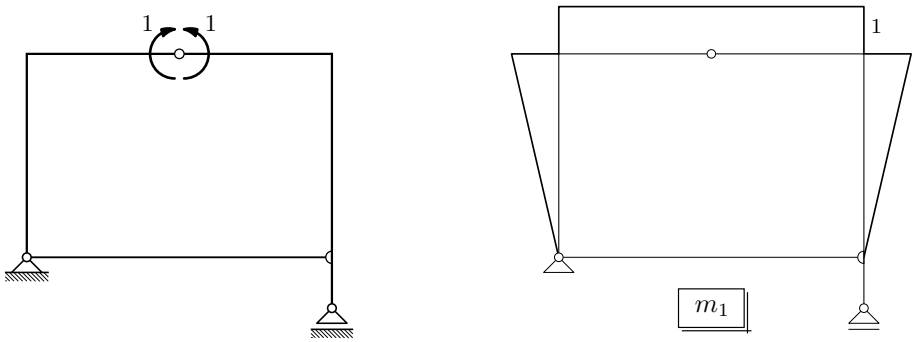
jedan besmisleni osnovni sistem (osnovni sistem koji to nije)**:**



mehanizam

dijagram momenata pomoću lijevoga osnovnog sistema:

(dijagram m_1 — najjednostavnije: superpozicijski postupak)



$n_\mathrm{z} \;=\; {}^1\!/_4$ (trebalo bi biti očito)

$\delta_{1,1}$ i $\delta_{1,0}$ — relativni kutovi zaokreta osi grede neposredno desno u odnosu na os neposredno lijevo od zgloba

$$\delta_{1,1} \;=\; \int \frac{m_1^2}{EI}\,\mathrm{d}x \;+\; \int \frac{n_\mathrm{z}^2}{EA}\,\mathrm{d}x$$

$$=\; \frac{2}{EI}\left(\frac{1}{2}\cdot 1\cdot 4\right)\left(\frac{2}{3}\cdot 1\right) \;+\; \frac{1}{EI}\left(1\cdot 6\right)\cdot 1 \;+\; \frac{1}{EA}\left(\frac{1}{4}\cdot 6\right)\cdot\frac{1}{4} \;=\; 5{,}649\,79\cdot 10^{-5}$$

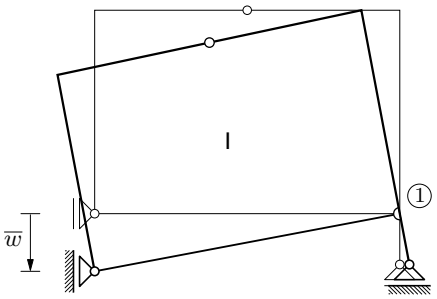
$$\delta_{1,0} \;=\; \delta_{1,0}(\overline{w}) \;+\; \delta_{1,0}(t_\mathrm{s})$$

$\delta_{1,0}(\overline{w})$:

pomoću plana pomakā: $\delta_{1,0}(\overline{w}) \;=\; 0$

ili primjenom virtualnoga rada:

$a_v \;=\; 0 \;\Rightarrow\; \delta_{1,0}(\overline{w}) \;=\; -a_v\,\overline{w} \;=\; 0$



$\delta_{1,0}(t_\mathrm{s})$:

pomoću (virtualne) uzdužne sile n_z:

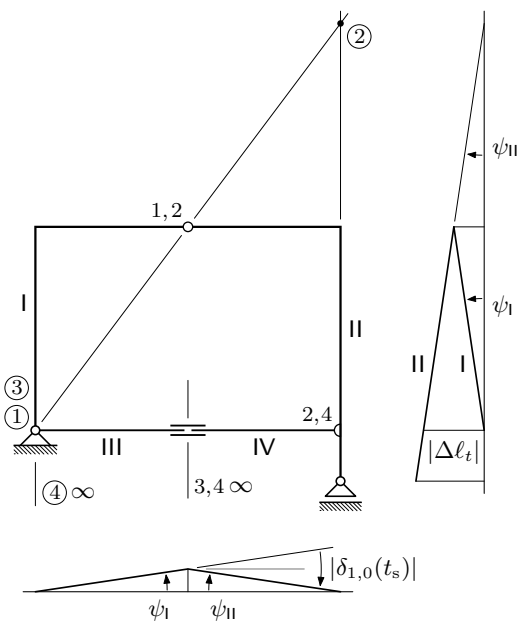
$$\delta_{1,0}(t_\mathrm{s}) \;=\; \int n_\mathrm{z}\,\varepsilon_t\,\mathrm{d}x \;=\; n_\mathrm{z}\left(\alpha_t\,t_\mathrm{s}\right)\ell_\mathrm{z} \;=\; \frac{1}{4}\cdot 10^{-5}\cdot(-50)\cdot 6 \;=\; -75\cdot 10^{-5}$$

ili pomoću dijagramā projekcija pomakā/plana pomakā (crteži na vrhu sljedeće stranice):

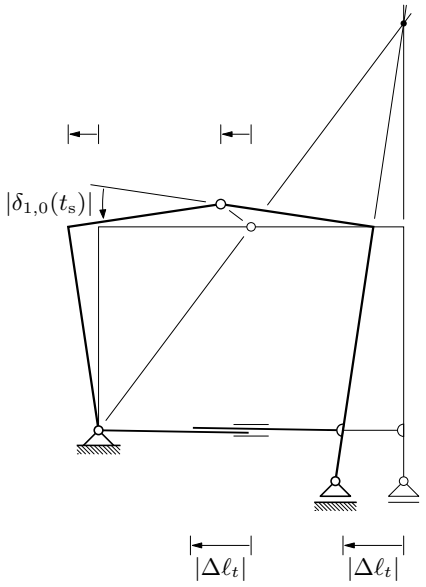
$$\Delta\ell_t \;=\; \varepsilon_t\,\ell_\mathrm{z} \;=\; \left(\alpha_t\,t_\mathrm{s}\right)\ell_\mathrm{z} \;=\; 10^{-5}\cdot(-50)\cdot 6 \;=\; -300\cdot 10^{-5}\ \mathrm{m}$$

$$\psi_2 \;=\; \frac{|\Delta\ell_t|}{8} \;=\; 37{,}5\cdot 10^{-5}, \qquad \psi_1 \;=\; \frac{\psi_2\cdot 4}{4} \;=\; \psi_2$$

$$\delta_{1,0}(t_\mathrm{s}) \;=\; -\left(\psi_1 \;+\; \psi_2\right) \;=\; -75\cdot 10^{-5}$$



ili



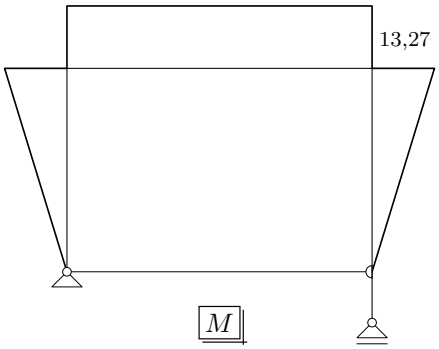
$$\delta_{1,0} \;=\; \delta_{1,0}(\overline{w}) \;+\; \delta_{1,0}(t_{\mathrm{s}}) \;=\; 0 \;+\; \left(-75\cdot 10^{-5}\right) \;=\; -75\cdot 10^{-5}$$

$$\delta_{1,1}\, X_1 \;+\; \delta_{1,0} \;=\; 0$$

$$X_1 \;=\; -\frac{\delta_{1,0}}{\delta_{1,1}} \;=\; -\frac{-75}{5{,}649\,79} \;=\; 13{,}274\,8 \text{ kNm}$$

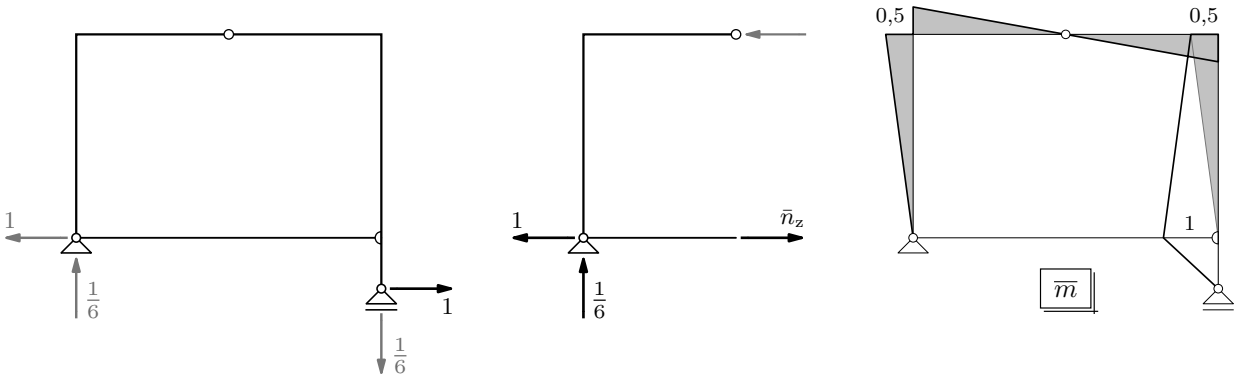
$$M(x) \;=\; X_1\, m_1(x)$$

$$N_{\mathrm{z}} \;=\; X_1\, n_{\mathrm{z}} \;=\; 3{,}318\,70 \text{ kN}$$



orijentirana duljina pomaka desnoga ležaja pomoću lijevoga osnovnog sistema:

$\delta_B \;=\; \delta_B(M) \;+\; \delta_B(t_{\mathrm{s}}) \;+\; \delta_B(\overline{w})$ (redukcijski stavak — na osnovnom sistemu)



$$-4\cdot 1 \;-\; 3\cdot\frac{1}{6} \;+\; 4\cdot \bar{n}_{\mathrm{z}} \;=\; 0 \qquad \Rightarrow \qquad \bar{n}_{\mathrm{z}} \;=\; 1 \;+\; \frac{1}{8} \;=\; 1{,}125$$

$$\delta_B(M) \;=\; \int \frac{\overline{m}\,M}{EI}\,\mathrm{d}x \;+\; \frac{\bar{n}_\mathrm{z}\,N_\mathrm{z}}{EA}\,\ell_\mathrm{z}$$

$$= \underbrace{\text{sim.}\cdot\text{antim.}}_{=\,0} + \frac{1}{EI}\left(\frac{1}{2}\cdot 13{,}27\cdot 4\right)\left(\frac{1}{3}\cdot 1\right)(-1) \;+\; \frac{1}{EA}\cdot 1{,}125\cdot 3{,}318\,70\cdot 6$$

$$= -0{,}000\,054\,609\,1 \;+\; 0{,}000\,179\,210 \;=\; 0{,}000\,124\,601\ \mathrm{m}$$

$$\delta_B(t_\mathrm{s}) \;=\; \bar{n}_\mathrm{z}\left(\alpha_t\,t_\mathrm{s}\right)\ell_\mathrm{z} \;=\; 1{,}125\cdot 10^{-5}\cdot(-50)\cdot 6 \;=\; -0{,}003\,375\,00\ \mathrm{m}$$

ili pomoću dijagrama projekcija pomakā (na vrhu prethodne stranice):

$|\delta_B(t_\mathrm{s})| \;=\; \psi_2\cdot 9 \;=\; 0{,}003\,375\,00\ \mathrm{m}$

smisao suprotan od smisla jedinične sile: $\quad\Rightarrow\quad \delta_B(t_\mathrm{s}) \;=\; -0{,}003\,375\,00\ \mathrm{m}$

$$\delta_B(\overline{w}) \;=\; -\bar{a}_v\,\overline{w} \;=\; -\frac{1}{6}\cdot 0{,}02\cdot(-1) \;=\; 0{,}003\,333\,33\ \mathrm{m}$$

ili pomoću plana pomakā (na sredini stranice 2):

$$\psi_\mathrm{I} \;=\; \frac{\overline{w}}{6} \;=\; \frac{0{,}02}{6} \;=\; 0{,}003\,333\,33$$

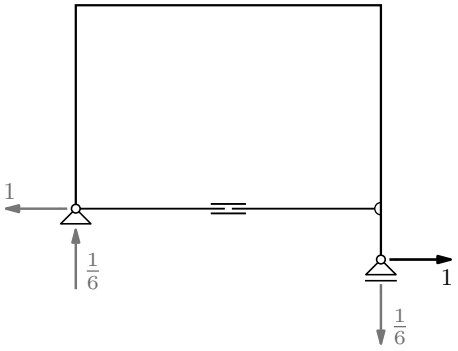
$$\delta_B(\overline{w}) \;=\; \psi_\mathrm{I}\cdot 1 \;=\; 0{,}003\,333\,33\ \mathrm{m}$$

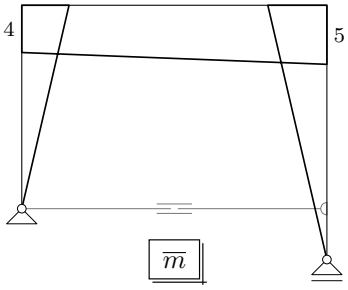
$$\delta_B \;=\; \delta_B(M) \;+\; \delta_B(t_\mathrm{s}) \;+\; \delta_B(\overline{w})$$

$$=\; 0{,}000\,124\,601 \;+\; (-0{,}003\,375\,00) \;+\; 0{,}003\,333\,33 \;=\; 0{,}000\,082\,931\ \mathrm{m}$$

ili **orijentirana duljina pomaka desnoga ležaja pomoću desnoga osnovnog sistema** (redukcijski stavak — *bilo koji* osnovni sistem)**:**

$$\delta_B \;=\; \delta_B(M) \;+\; \delta_B(\overline{w})$$





$n_\mathrm{z} \;=\; 0$

$$\delta_B(M) \;=\; \int \frac{\overline{m}\,M}{EI}\,\mathrm{d}x \;=\; \frac{1}{EI}\left[\left(\frac{1}{2}\cdot 13{,}27\cdot 4\right)\left(\frac{2}{3}\cdot 4\right)(-1) \;+\; (13{,}27\cdot 6)\cdot 4{,}5\cdot(-1)\right.$$

$$\left. +\left(\frac{1}{2}\cdot 13{,}27\cdot 4\right)\left(\frac{2}{3}\cdot 5+\frac{1}{3}\cdot 1\right)(-1)\right]$$

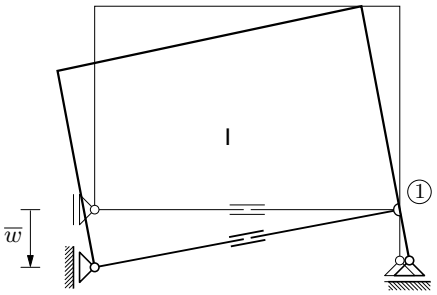
$$=\; -0{,}003\,249\,24\ \mathrm{m}$$

$$\delta_B(\overline{w}) \;=\; -\bar{a}_v\,\overline{w} \;=\; -\frac{1}{6}\cdot 0{,}02\cdot(-1) \;=\; 0{,}003\,333\,33\ \mathrm{m}$$

ili pomoću plana pomakā:

$$\psi_{\mathrm{I}} \;=\; \frac{\overline{w}}{6} \;=\; \frac{0{,}02}{6} \;=\; 0{,}003\,333\,33$$

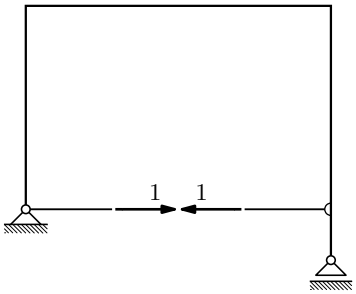
$$\delta_B(\overline{w}) \;=\; \psi_{\mathrm{I}}\cdot 1 \;=\; 0{,}003\,333\,33\ \mathrm{m}$$

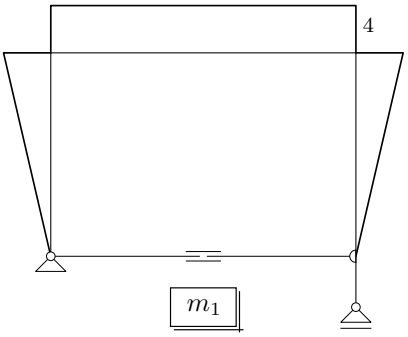


$$\delta_B \;=\; \delta_B(M) \;+\; \delta_B(\overline{w})$$

$$\;=\; -0{,}003\,249\,24 \;+\; 0{,}003\,333\,33 \;=\; 0{,}000\,084\,09\ \mathrm{m}$$

i, potpunosti radi, **dijagram momenata pomoću desnoga osnovnog sistema:**





$$n_{\mathrm{z}} \;=\; 1$$

$\delta_{1,1}$ i $\delta_{1,0}$ — relativni pomaci presjeka neposredno lijevo u odnosu na presjek neposredno desno od reza

$$\delta_{1,1} \;=\; \frac{2}{EI}\left(\frac{1}{2}\cdot 4\cdot 4\right)\left(\frac{2}{3}\cdot 4\right) \;+\; \frac{1}{EI}\left(4\cdot 6\right)\cdot 4 \;+\; \frac{1}{EA}\left(1\cdot 6\right)\cdot 1 \;=\; 9{,}039\,67\cdot 10^{-4}$$

$$\delta_{1,0} \;=\; \delta_{1,0}(t_{\mathrm{s}}) \;+\; \delta_{1,0}(\overline{w})$$

$$\delta_{1,0}(\overline{w}) \;=\; -a_v\,\overline{w} \;=\; 0 \quad\Leftarrow\quad a_v \;=\; 0$$

ili pomoću plana pomakā (na vrhu stranice): $\delta_{1,0}(\overline{w}) \;=\; 0$

$$\delta_{1,1}\,X_1 \;+\; \delta_{1,0} \;=\; 0$$

$$X_1 \;=\; -\frac{\delta_{1,0}}{\delta_{1,1}} \;=\; -\frac{-30}{9{,}039\,67} \;=\; 3{,}318\,71\ \mathrm{kN}$$

$$M(x) \;=\; X_1\,m_1(x) \quad\rightarrow\quad \text{dijagram } M \text{ kao na sredini stranice 3}$$

$$N_{\mathrm{z}} \;=\; X_1\,n_{\mathrm{z}} \;=\; 3{,}318\,71\ \mathrm{kN}$$