

Za vlačnu je silu nehomogena diferencijalna jednačba

$$w_{\text{el}}''(x) - \frac{h^2}{\ell^2} w_{\text{el}}(x) = \frac{h^2}{\ell^2} w_{\text{imp}}(x). \quad (214)$$

Uvrštavanjem izrazā za pretpostavljeno rješenje, njegovu drugu derivaciju i odabranu početnu imperfekciju dobivamo

$$2 c_2 - \frac{h^2}{\ell^2} \left(c_0 + c_1 x + c_2 x^2 \right) = w_0 \frac{h^2}{\ell^2} \left(\frac{4}{\ell} x - \frac{4}{\ell^2} x^2 \right)$$

i odatle

$$\begin{aligned} -\frac{h^2}{\ell^2} c_2 + \frac{4 w_0 h^2}{\ell^4} &= 0 \quad \Rightarrow \quad c_2 = \frac{4 w_0}{\ell^2}, \\ -\frac{h^2}{\ell^2} c_1 - \frac{4 w_0 h^2}{\ell^3} &= 0 \quad \Rightarrow \quad c_1 = -\frac{4 w_0}{\ell}, \\ 2 c_2 - \frac{h^2}{\ell^2} c_0 &= 0 \quad \Rightarrow \quad c_0 = \frac{8 w_0}{h^2}, \end{aligned}$$

tako da je partikularno rješenje

$$w_{\text{el,p}}(x) = w_0 \left(\frac{8}{h^2} - \frac{4}{\ell} x + \frac{4}{\ell^2} x^2 \right) = \frac{8 w_0}{h^2} - w_{\text{imp}}(x),$$

dok je opće rješenje nehomogene jednačbe (214)

$$w_{\text{el}}(x) = a_1 \operatorname{sh} \left(\frac{h}{\ell} x \right) + a_2 \operatorname{ch} \left(\frac{h}{\ell} x \right) + \frac{8 w_0}{h^2} - w_{\text{imp}}(x). \quad (215)$$

Rubni uvjeti za slobodno oslonjenu gredu, $w_{\text{el}}(0) = w_{\text{el}}(\ell) = 0$, daju sustav jednačbi

$$\begin{aligned} a_2 + \frac{8 w_0}{h^2} &= 0, \\ a_1 \operatorname{sh} h + a_2 \operatorname{ch} h + \frac{8 w_0}{h^2} &= 0, \end{aligned}$$

kojemu je rješenje

$$a_2 = -\frac{8 w_0}{h^2} \quad \& \quad a_1 = -\frac{8 w_0}{h^2} \frac{1 - \operatorname{ch} h}{\operatorname{sh} h} = \frac{8 w_0}{h^2} \frac{\operatorname{ch} h - 1}{\operatorname{sh} h} = \frac{8 w_0}{h^2} \operatorname{th} \frac{h}{2},$$

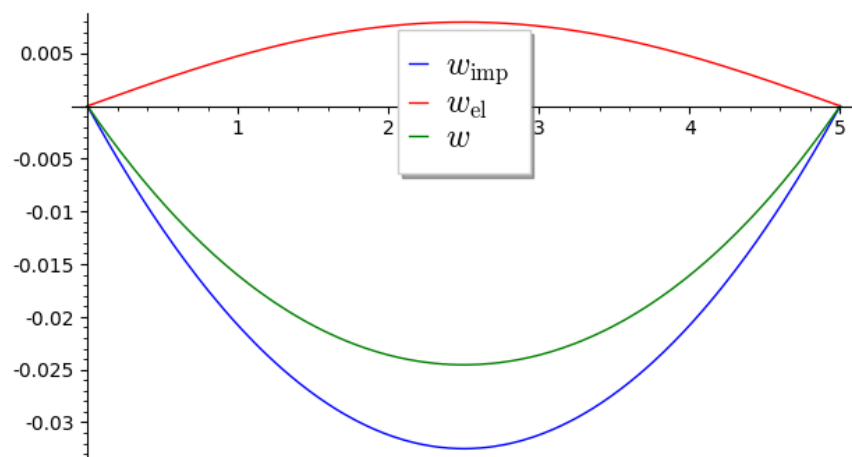
te je rješenje diferencijalne jednačbe (214) za slobodno oslonjenu gredu s imperfekcijom (210)

$$w_{\text{el}}(x) = \frac{8 w_0}{h^2} \operatorname{th} \frac{h}{2} \operatorname{sh} \left(\frac{h}{\ell} x \right) - \frac{8 w_0}{h^2} \operatorname{ch} \left(\frac{h}{\ell} x \right) + \frac{8 w_0}{h^2} - w_{\text{imp}}(x),$$

dok je ravnotežni oblik osi grede u odnosu na os x

$$w = w_{\text{el}}(x) + w_{\text{imp}}(x) = \frac{8 w_0}{h^2} \operatorname{th} \frac{h}{2} \operatorname{sh} \left(\frac{h}{\ell} x \right) - \frac{8 w_0}{h^2} \operatorname{ch} \left(\frac{h}{\ell} x \right) + \frac{8 w_0}{h^2}$$

(slika 44.).



Slika 44.