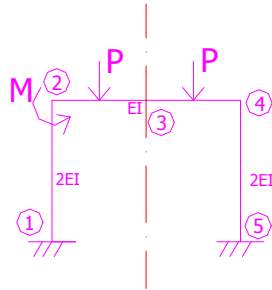


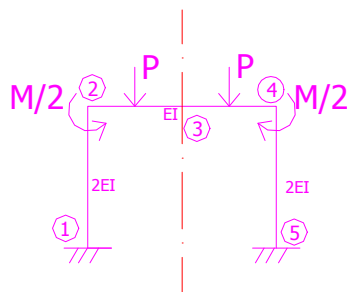
Odredite momentni dijagram metodom pomaka koristeći se simetrijom i antimetrijom!



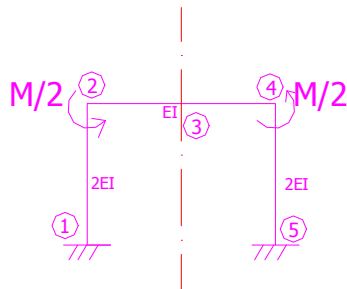
$$P=60\text{kN}$$
$$M=40\text{kNm}$$

-sistem dijelimo na simetrično opterećenu konstrukciju i antimetrično opterećenje
-pošto vrijedi princip superpozicije, konačno rješenje dobivamo zbrajanjem momentnog dijagrama zbog simetričnog i zbog antimetričnog opterećenja

1.) SIMETRIJA

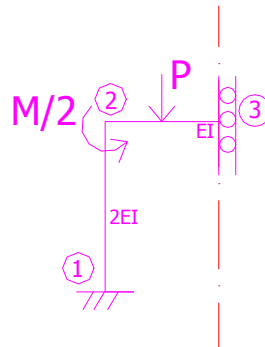


2.) ANTIMETRIJA



1.)

-zamjenjujući sistem:



-nepoznanice: φ_2, v_3

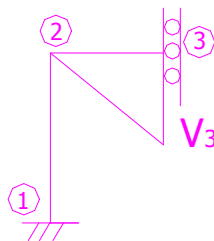
-momenti upetosti:

$$\bar{M}_{12} = \bar{M}_{21} = 0$$

$$\bar{M}_{23} = \frac{P \cdot l}{8} = 15 \text{ kNm}$$

$$\bar{M}_{32} = -15 \text{ kNm}$$

-plan pomaka:



$$\Psi_{23} = -\frac{v_3}{2}$$

-momenti od pomaka/zaokreta:

$$m_{12} = 2 \cdot \frac{2EI}{3} \cdot \varphi_2$$

$$m_{21} = 4 \cdot \frac{2EI}{3} \cdot \varphi_2$$

$$m_{23} = 4 \cdot \frac{EI}{2} \cdot \varphi_2 + 6 \cdot \frac{EI}{2} \cdot \frac{v_3}{2}$$

$$m_{32} = 2 \cdot \frac{EI}{2} \cdot \varphi_2 + 6 \cdot \frac{EI}{2} \cdot \frac{v_3}{2}$$

-ravnoteža čvora 2:

$$\frac{8}{3} \cdot \varphi_2 + 2 \cdot \varphi_2 + \frac{3}{2} \cdot v_3 + 15 - \frac{40}{2} = 0$$

$$\frac{14}{3} \cdot \varphi_2 + \frac{3}{2} \cdot v_3 = 5$$

-jednadžba rada:

$$-\frac{1}{2} \cdot \left(15 - 15 + 2 \cdot \varphi_2 + \frac{3}{2} \cdot v_3 + \varphi_2 + \frac{3}{2} \cdot v_3 \right) + 60 \cdot \frac{1}{2} = 0$$

$$-\frac{3}{2} \cdot \varphi_2 - \frac{3}{2} \cdot v_3 = -30$$

-rješenje sustava:

$$\varphi_2 = -7,89$$

$$v_3 = 27,89$$

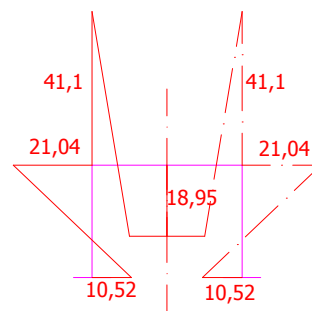
-vrijednosti momenata za simetrično opterećenje (M_s):

$$M_{12} = \frac{4}{3} \cdot (-7,89) = -10,52 \text{ kNm}$$

$$M_{21} = \frac{8}{3} \cdot (-7,89) = -21,04 \text{ kNm}$$

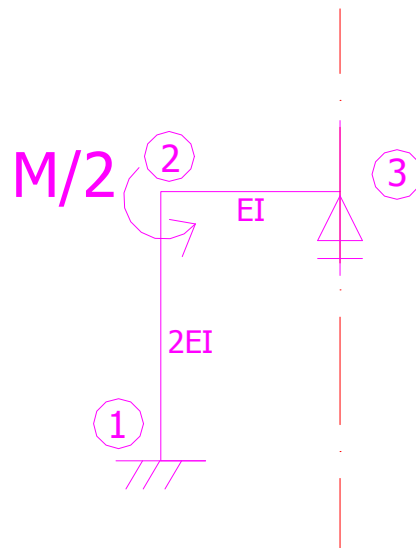
$$M_{23} = 15 + 2 \cdot (-7,89) + \frac{3}{2} \cdot 27,89 = 41,1 \text{ kNm}$$

$$M_{32} = -15 + (-7,89) + \frac{3}{2} \cdot 27,89 = 18,95 \text{ kNm}$$

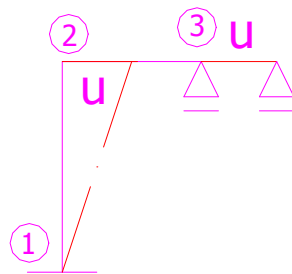


2.)

-zamjenjujući sistem: (vertikalni pomak na osi simetrije je nula, kao i moment)



- nepoznanice: φ_2, u
- momenti upetosti: svi su nula!!
- plan pomaka:



$$\psi_{12} = -\frac{u}{3}$$

-momenti od pomaka/zaokreta:

$$m_{12} = 2 \cdot \frac{2EI}{3} \cdot \varphi_2 + 6 \cdot \frac{2EI}{3} \cdot \frac{u}{3}$$
$$m_{21} = 4 \cdot \frac{2EI}{3} \cdot \varphi_2 + 6 \cdot \frac{2EI}{3} \cdot \frac{u}{3}$$
$$m_{23} = 3 \cdot \frac{EI}{2} \cdot \varphi_2$$

-ravnoteža čvora 2:

$$\frac{8}{3} \cdot \varphi_2 + \frac{4}{3} \cdot u + \frac{3}{2} \cdot \varphi_2 = 20$$
$$\frac{25}{6} \cdot \varphi_2 + \frac{4}{3} \cdot u = 20$$

-jednadžba rada:

$$-\frac{1}{3} \cdot \left(\frac{4}{3} \cdot \varphi_2 + \frac{4}{3} \cdot u + \frac{8}{3} \cdot \varphi_2 + \frac{4}{3} \cdot u \right) - 20 \cdot \frac{1}{3} = 0$$

-rješenje sustava:

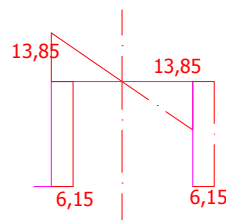
$$\varphi_2 = \frac{120}{13}$$
$$u = -\frac{180}{13}$$

-vrijednosti momenata za antimetrično opterećenje (M_A):

$$M_{12} = \frac{4}{3} \cdot \left(\frac{120}{13} \right) - \frac{4}{3} \cdot \frac{180}{13} = -6,15 \text{ kNm}$$

$$M_{21} = \frac{8}{3} \cdot \frac{120}{13} - \frac{4}{3} \cdot \frac{180}{13} = 6,15 \text{ kNm}$$

$$M_{23} = \frac{3}{2} \cdot \frac{120}{13} = 13,85 \text{ kNm}$$



UKUPNI MOMENTNI DIJAGRAM ($M_{uk} = M_S + M_A$)

$$M_{12} = 10,52 + 6,15 = 16,67 \text{ kNm}$$

$$M_{21} = 21,04 - 6,15 = 14,89 \text{ kNm}$$

$$M_{23} = 41,1 + 13,85 = 54,95 \text{ kNm}$$

$$M_{32} = 18,95 \text{ kNm}$$

$$M_{43} = 41,1 - 13,85 = 27,25 \text{ kNm}$$

$$M_{45} = 21,04 + 6,15 = 27,25 \text{ kNm}$$

$$M_{54} = 10,52 - 6,15 = 4,37 \text{ kNm}$$

