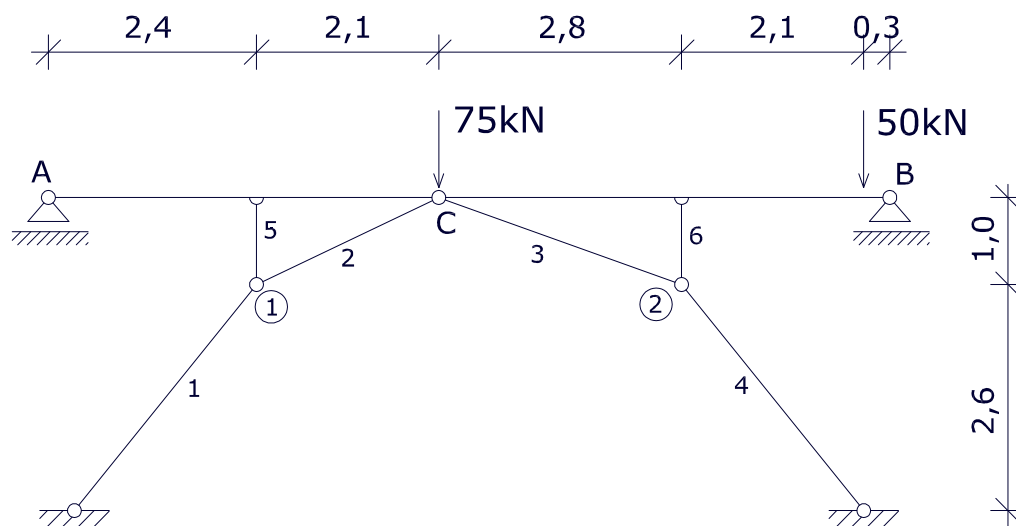


## II) POMACI

Odrediti vertikalni pomak točke C, horizontalni točke B, te relativni kut zaokreta osi u točkama A, B za opterećenje  $P=75\text{kN}$ ,  $F=50\text{kN}$ .



Geometrijske i materijalne karakteristike:

a) štapova potpornog sistema:

$$E_1 = 2 \cdot 10^8 \text{ kN/m}^2$$

$$b_1 / h_1 = 5 / 5 \text{ cm}$$

$$A = 0,05^2 = 2,5 \cdot 10^{-3} \text{ m}^2$$

duljine štapova:

$$l_1 = l_4 = \sqrt{2,1^2 + 2,6^2} = 3,34 \text{ m}$$

$$l_2 = \sqrt{2,1^2 + 1,0^2} = 2,33 \text{ m}$$

$$l_3 = \sqrt{2,8^2 + 1,0^2} = 2,97 \text{ m}$$

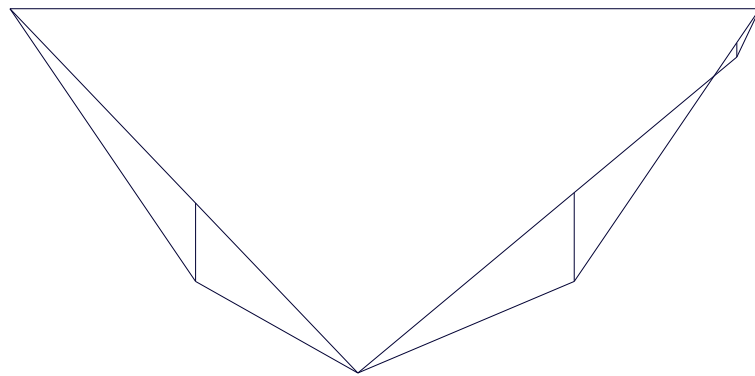
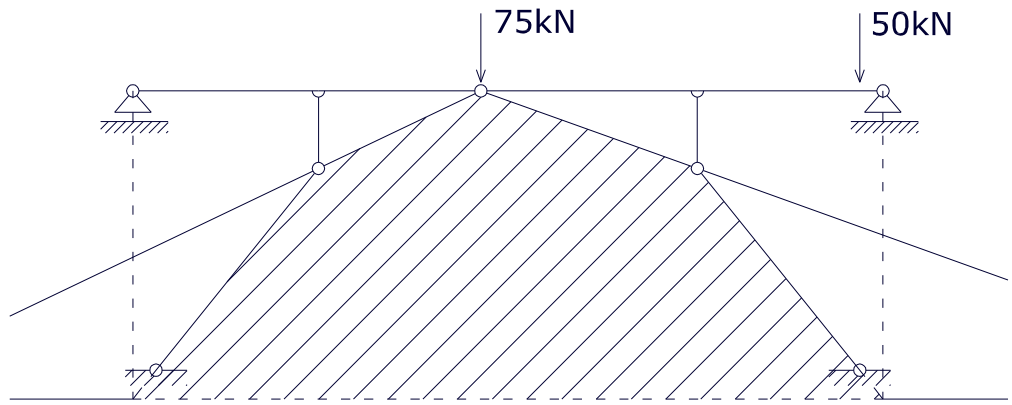
$$l_5 = l_6 = 1,0 \text{ m}$$

b) greda-  $E_2 = 3 \cdot 10^7 \text{ kN/m}^2$

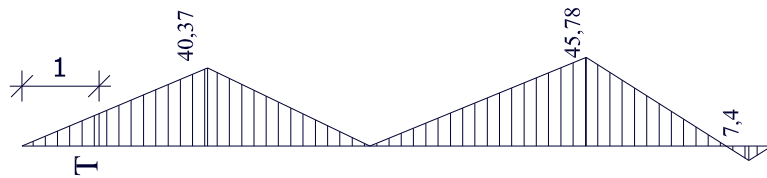
$$b_2 / h_2 = 30 / 60 \text{ cm}$$

$$I = \frac{bh^3}{12} = \frac{0,3 \cdot 0,6^3}{12} = 5,4 \cdot 10^{-3} \text{ m}^4$$

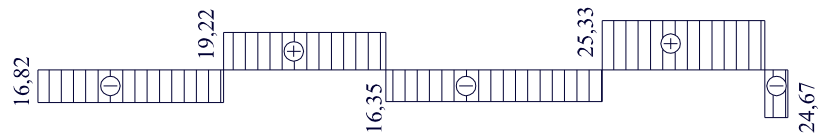
**DIJAGRAMI M,T,N**



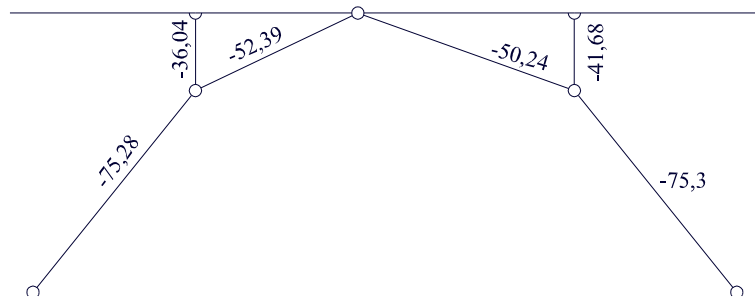
**M**



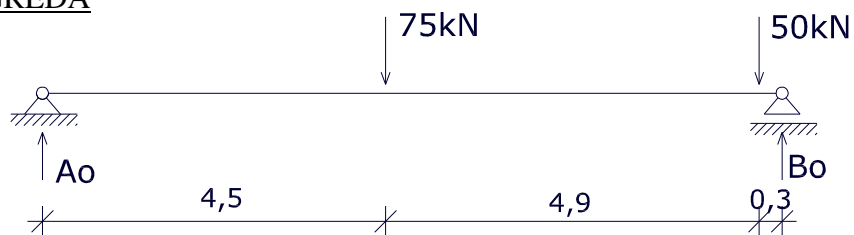
**T**



**N**



## PROSTA GREDA

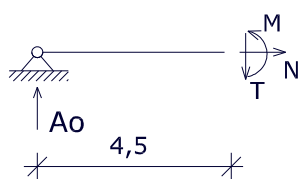


$$\sum M_A = 0 \Rightarrow B_0 = (75 \cdot 4,5 + 50 \cdot 9,4) / 9,7 = 83,247 \text{ kN}$$

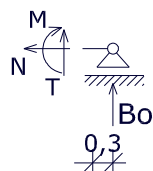
$$\sum M_B = 0 \Rightarrow A_0 = (75 \cdot 5,2 + 50 \cdot 0,3) / 9,7 = 41,753 \text{ kN}$$

$$\text{Kontrola: } \sum F_y = 0 \Rightarrow 41,753 - 75 - 50 + 83,247 = 0 \Rightarrow 0 = 0$$

Momenti u karakterističnim točkama:



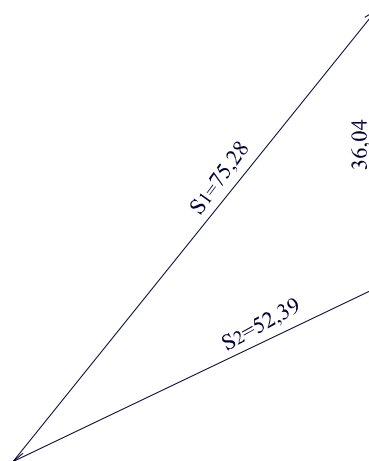
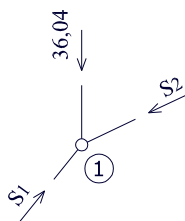
$$M = A_0 \cdot 4,5 = 41,753 \cdot 4,5 = 187,89 \text{ kNm}$$



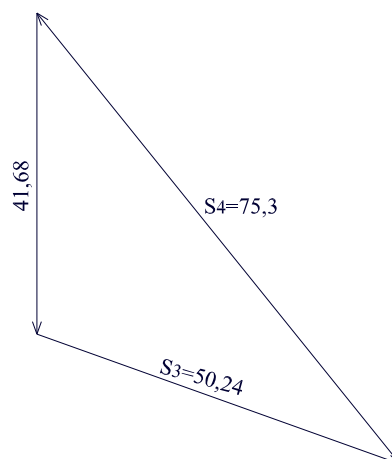
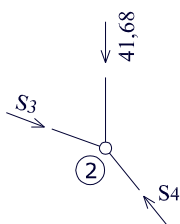
$$M = B_0 \cdot 0,3 = 83,247 \cdot 0,3 = 24,97 \text{ kNm}$$

## RAVNOTEŽA ČVOROVA

ČVOR 1

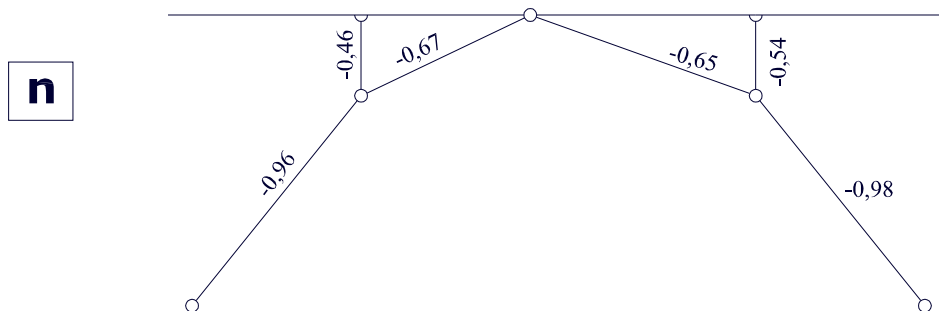
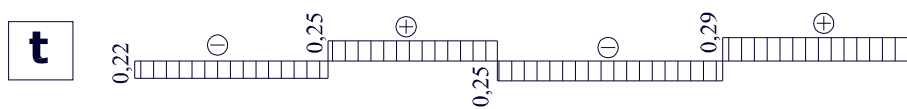
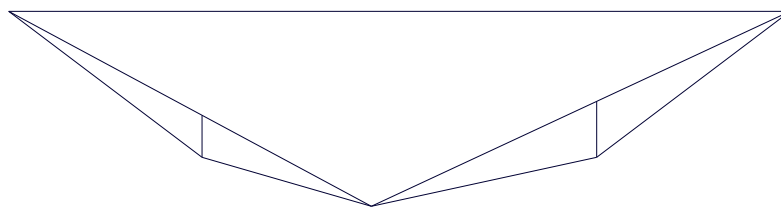
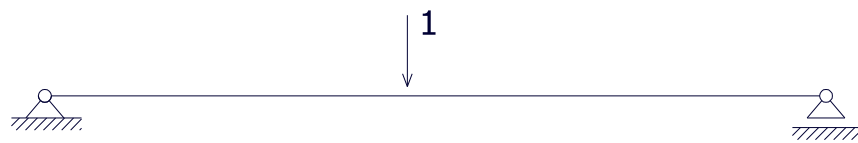
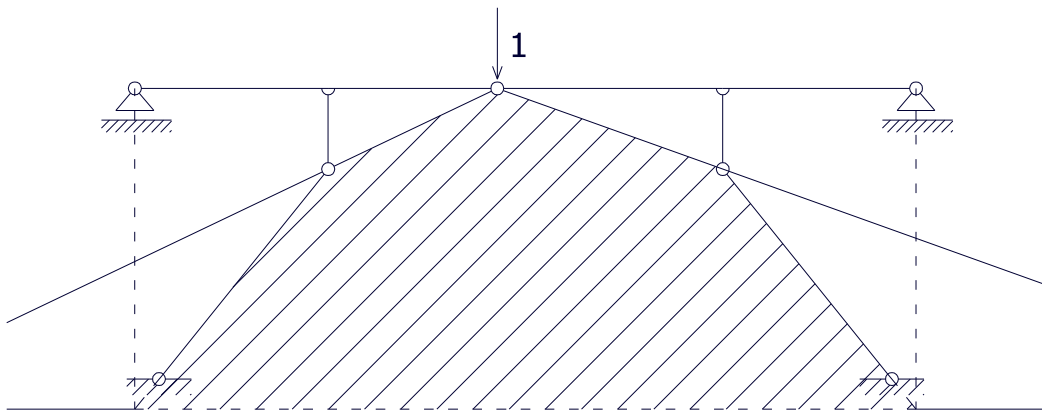


ČVOR 2

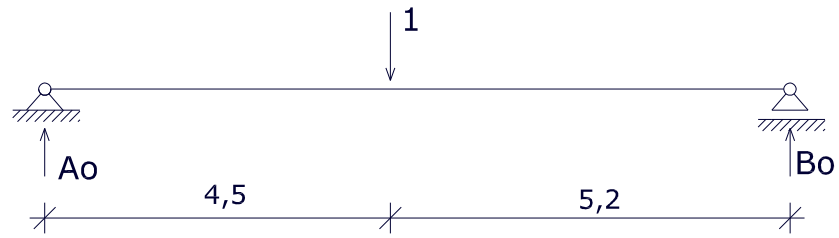


# VERTIKALNI POMAK TOČKE C

DIJAGRAMI m, t, n



## PROSTA GREDA

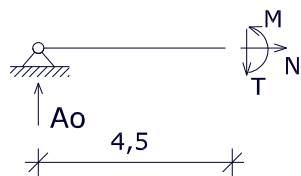


$$\sum M_A = 0 \Rightarrow B_0 = (1 \cdot 4,5) / 9,7 = 0,46 \text{ kN}$$

$$\sum M_B = 0 \Rightarrow A_0 = (1 \cdot 5,2) / 9,7 = 0,54 \text{ kN}$$

$$\text{Kontrola: } \sum F_y = 0 \Rightarrow 0,46 + 0,54 - 1 = 0 \Rightarrow 0 = 0$$

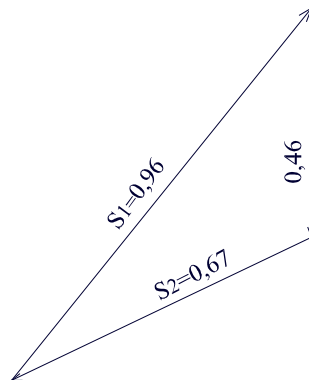
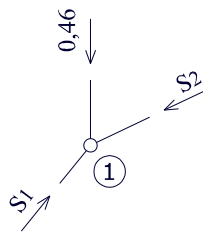
Momenti u karakterističnim točkama:



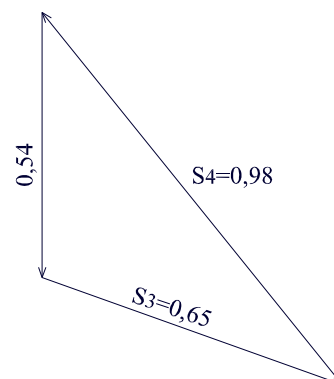
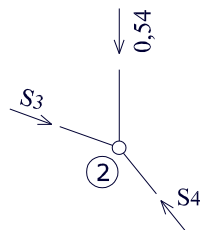
$$M = A_0 \cdot 4,5 = 0,54 \cdot 4,5 = 2,43 \text{ kNm}$$

## RAVNOTEŽA ČVOROVA

### ČVOR 1

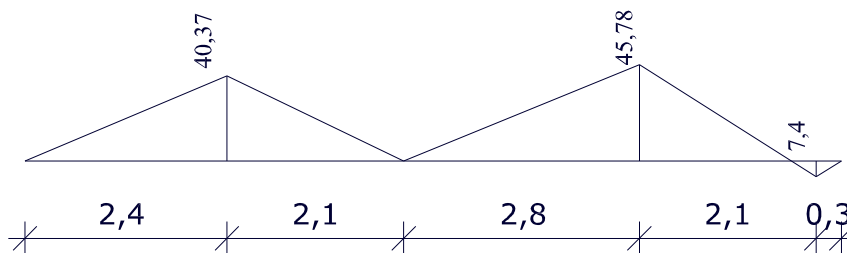


### ČVOR 2

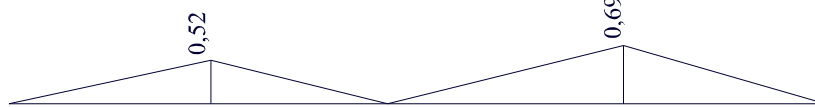


## IZRAČUNAVANJE POMAKA

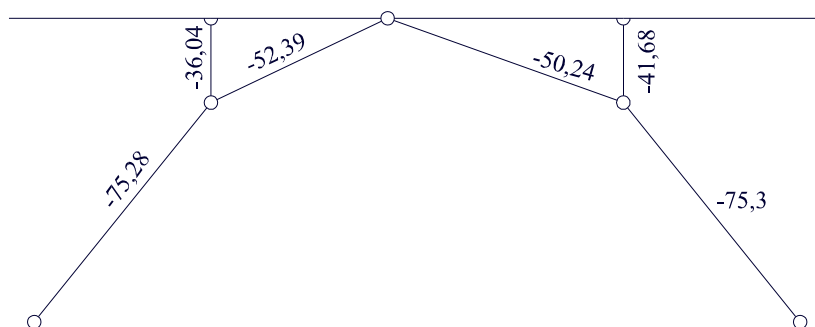
**M**



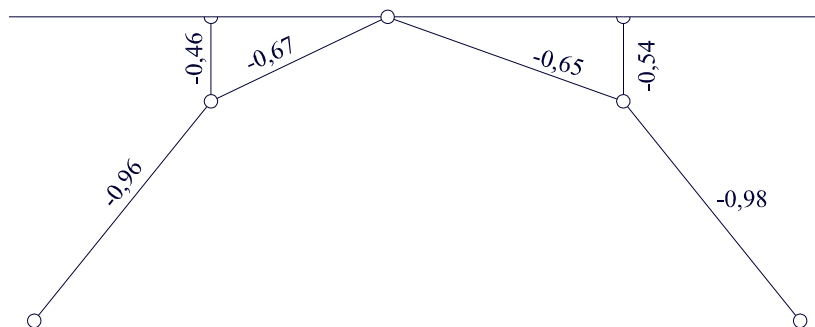
**m**



**N**



**n**



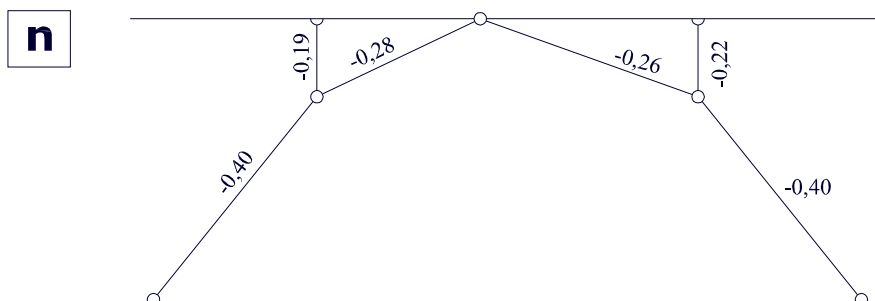
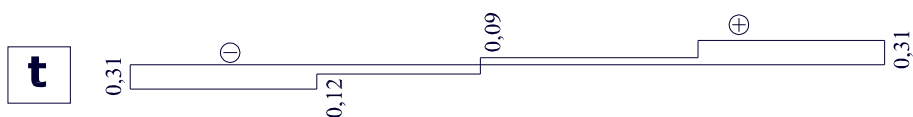
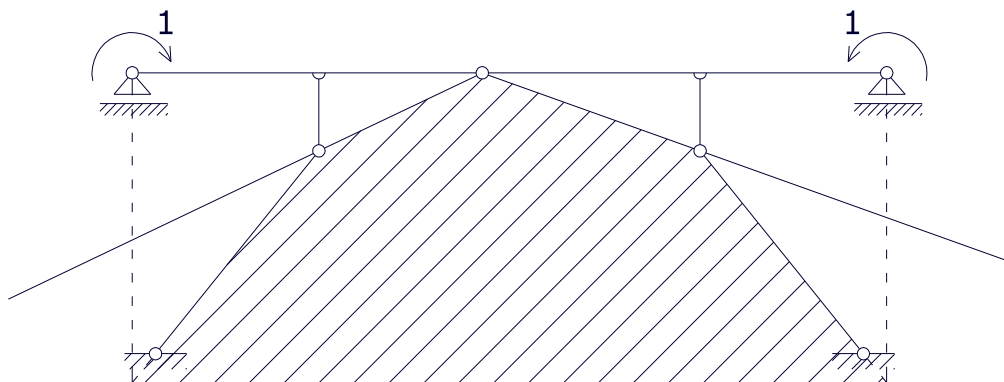
$$w_C = \int_0^l \frac{M(x)m(x)}{EJ} dx + \sum \int_0^{l_{el}} \frac{N(x_{el})n(x_{el})}{EA} dx$$

$$w_C = \frac{1}{EJ} \left[ \left( \frac{40,37 \cdot 2,4}{2} \right) \left( \frac{0,52 \cdot 2}{\cdot 3} \right) + \left( \frac{40,37 \cdot 2,1}{2} \right) \left( \frac{0,52 \cdot 2}{3} \right) + \left( \frac{45,78 \cdot 2,8}{2} \right) \left( \frac{0,69 \cdot 2}{\cdot 3} \right) \right. \\ \left. + \left( \frac{45,78 \cdot 1,8}{2} \right) \left( \frac{0,69 \cdot 1,5}{2,4} \right) - \left( \frac{7,4 \cdot 0,3}{2} \right) \left( \frac{0,69 \cdot 0,4}{2,4} \right) - \left( \frac{7,4 \cdot 0,3}{2} \right) \left( \frac{0,69 \cdot 0,2}{2,4} \right) \right] \\ + \frac{1}{EA} [75,28 \cdot 3,34 \cdot 0,96 + 52,39 \cdot 2,33 \cdot 0,67 + 50,24 \cdot 2,97 \cdot 0,65 \\ + 75,3 \cdot 3,34 \cdot 0,98 + 36,04 \cdot 1 \cdot 0,46 + 41,68 \cdot 1 \cdot 0,54] = (0,48486 + 1,4114) 10^{-3}$$

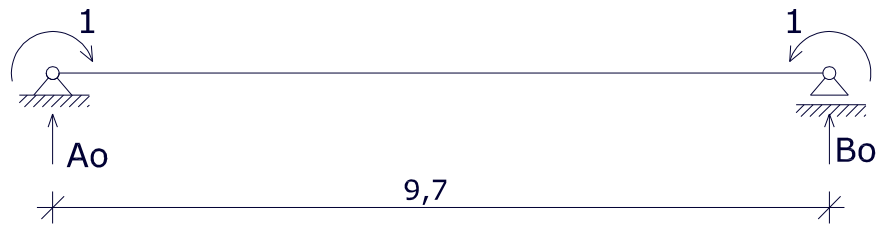
$$w_C = 1,90 \cdot 10^{-3} m = \mathbf{1,90 cm}$$

# RELATIVNI KUT ZAOKRETA ELASTIČNE LINIJE U TOČKAMA A,B

DIJAGRAMI m, t, n



## PROSTA GREDA

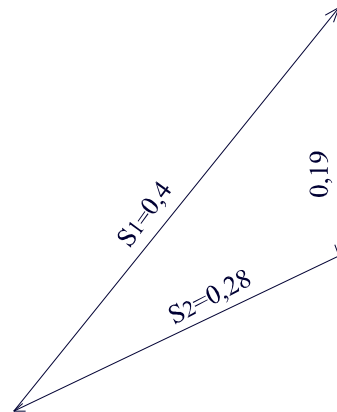
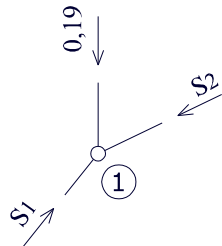


$$\sum M_A = 0 \Rightarrow B_0 = 1 - 1 = 0,0 \text{ kN}$$

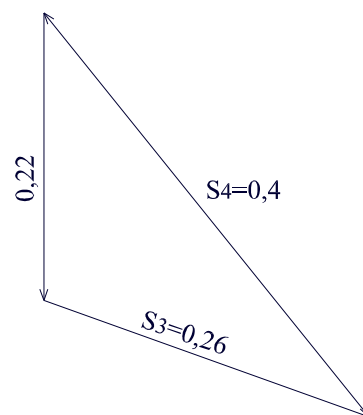
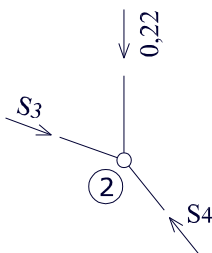
$$\sum M_B = 0 \Rightarrow A_0 = 1 - 1 = 0,0 \text{ kN}$$

## RAVNOTEŽA ČVOROVA

ČVOR 1

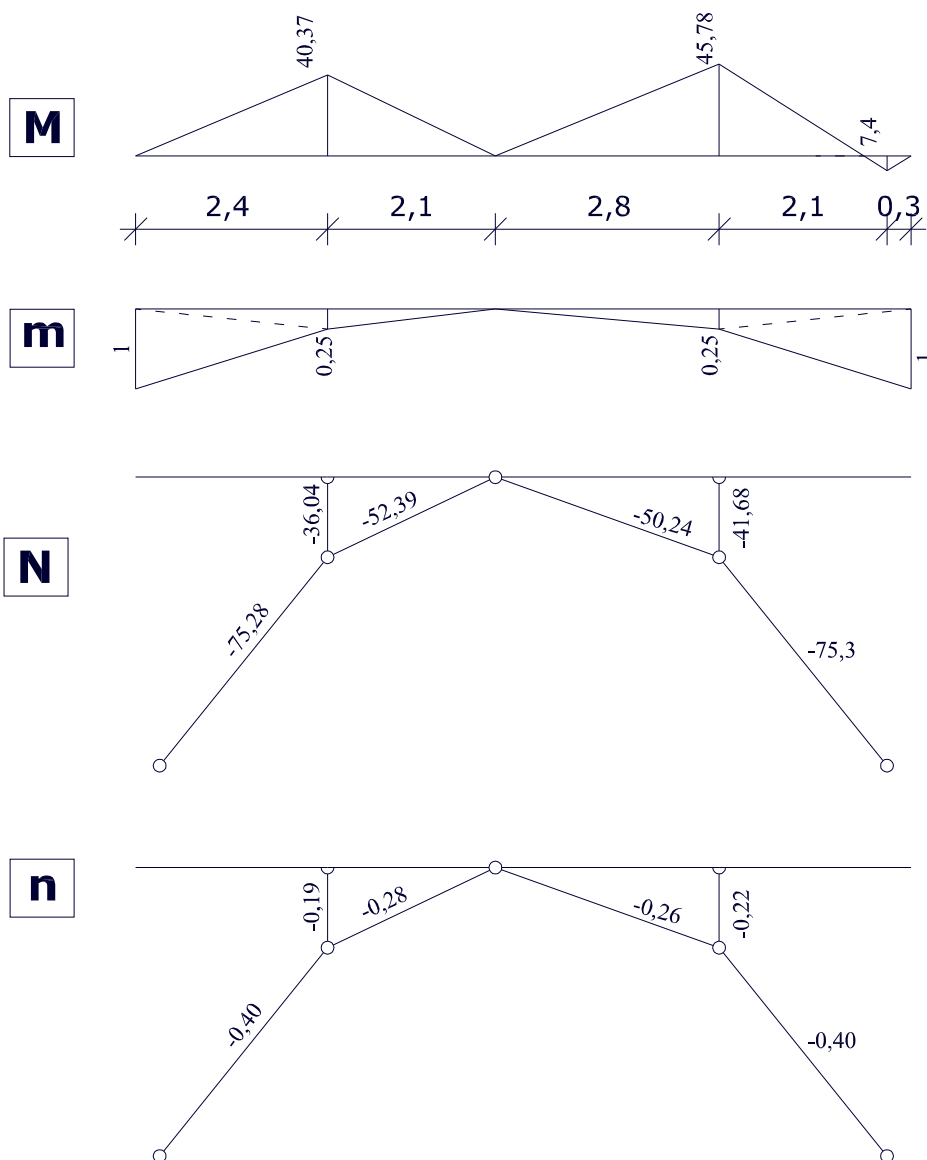


ČVOR 2





## IZRAČUNAVANJE RELATIVNOG ZAKRETA



$$\Delta\varphi_{AB} = \int_0^l \frac{M(x)m(x)}{EJ} dx + \sum \int_0^{l_{el}} \frac{N(x_{el})n(x_{el})}{EA} dx$$

$$\Delta\varphi_{AB} = \frac{1}{EJ} \left[ \left( \frac{40,37 \cdot 2,4}{2} \right) \left( \frac{-1 \cdot 1}{3} - \frac{0,25 \cdot 2}{3} \right) + \left( \frac{40,37 \cdot 2,1}{2} \right) \left( \frac{-0,25 \cdot 2}{3} \right) + \right.$$

$$+ \left( \frac{45,78 \cdot 2,8}{2} \right) \left( \frac{-0,25 \cdot 2}{3} \right) + \left( \frac{45,78 \cdot 1,8}{2} \right) \left( \frac{-0,25 \cdot 1,2}{2,4} - \frac{1 \cdot 1,2}{2,4} \right) +$$

$$+ \left( \frac{7,4 \cdot 0,3}{2} \right) \left( \frac{0,25 \cdot 0,4}{2,4} + \frac{1 \cdot 2}{2,4} \right) + \left( \frac{7,4 \cdot 0,3}{2} \right) \left( \frac{0,25 \cdot 0,2}{2,4} + \frac{1 \cdot 2,2}{2,4} \right) +$$

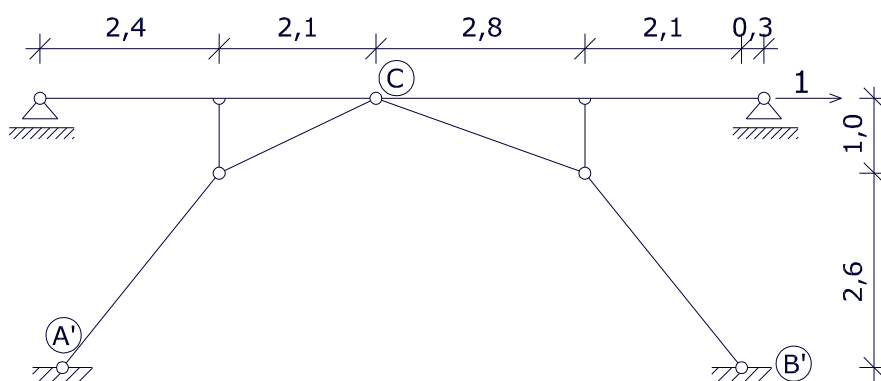
$$+ \frac{1}{EA} [75,28 \cdot 3,34 \cdot 0,4 + 52,39 \cdot 2,33 \cdot 0,28 + 50,24 \cdot 2,97 \cdot 0,26$$

$$+ 75,3 \cdot 3,34 \cdot 0,4 + 36,04 \cdot 1 \cdot 0,19 + 41,68 \cdot 1 \cdot 0,22] = (-4,0561 + 5,8033) 10^{-4}$$

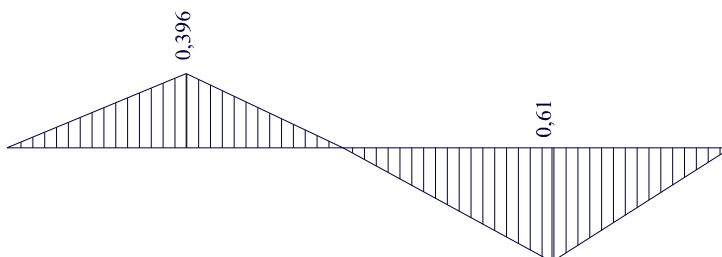
$$\Delta\varphi_{AB} = \mathbf{1,747 \times 10^{-4} m^0}$$

# HORIZONTALNI POMAK TOČKE B

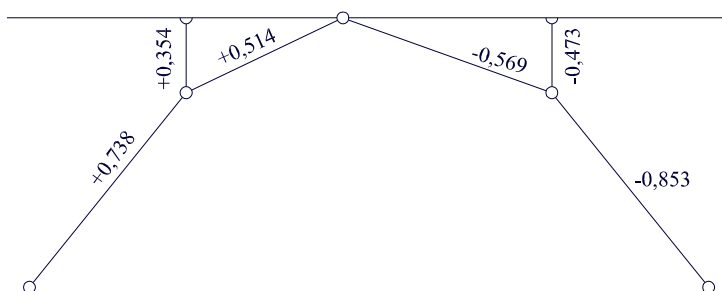
## DIJAGRAMI m, t, n



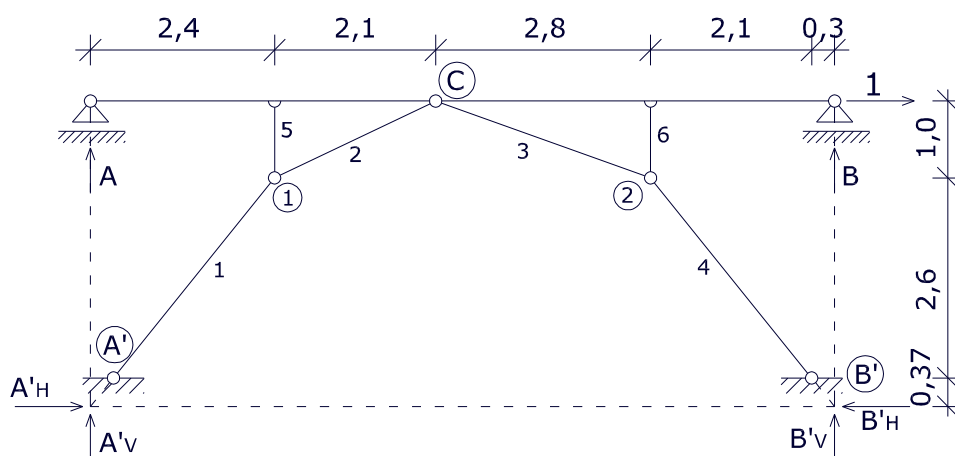
**m**



**n**



## REAKCIJE



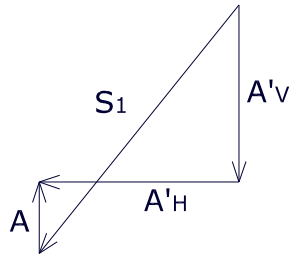
$$\sum_{\text{cijeli}} M_{A'} = 0 \Rightarrow B'_V = (1 \cdot 3,97) / 9,7 = 0,4093 \text{ kN}$$

$$\sum_{\text{cijeli}} M_{B'} = 0 \Rightarrow A'_V = (-1 \cdot 3,97) / 9,7 = -0,4093 \text{ kN}$$

$$\sum_{\text{desno}} M_C = 0 \Rightarrow B'_H = (0,4093 \cdot 5,2) / 3,97 = 0,5361 \text{ kN}$$

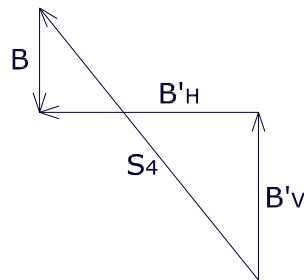
$$\sum_{\text{ljevo}} M_C = 0 \Rightarrow A'_H = (-0,4093 \cdot 4,5) / 3,97 = -0,4639 \text{ kN}$$

$$\text{Kontrola: } \sum F_x = 0 \Rightarrow 0,4639 + 0,5361 - 1 = 0 \Rightarrow 0 = 0$$



$$A'_H = \frac{2,1}{3,34} S_1 \Rightarrow S_1 = 0,7383 \text{ kN (vlak)}$$

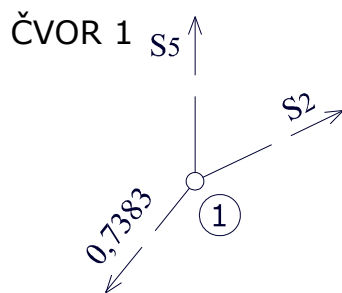
$$A = \frac{2,6}{3,34} 0,7383 - 0,4093 = 0,1649 \text{ kN}$$



$$S_4 = \frac{3,34}{2,1} 0,5361 = 0,8532 \text{ kN (tlak)}$$

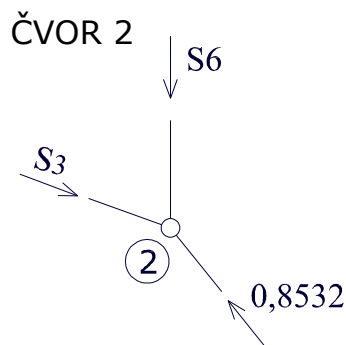
$$B = \frac{2,6}{3,34} 0,8532 - 0,4093 = 0,2543 \text{ kN}$$

### RAVNOTEŽA ČVOROVA



$$S_2 = \frac{2,33 \cdot 2,1}{2,1 \cdot 3,34} 0,7383 = 0,5138 \text{ kN (vlak)}$$

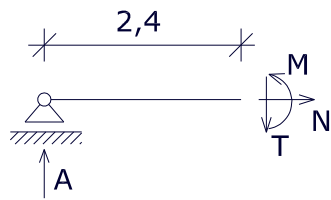
$$S_5 = \frac{2,6}{3,34} 0,7383 - \frac{0,5138}{2,33} = 0,3535 \text{ kN (vlak)}$$



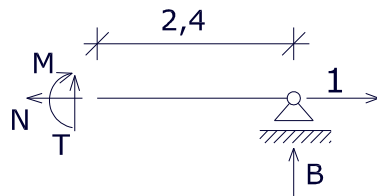
$$S_3 = \frac{2,1 \cdot 2,97}{3,34 \cdot 2,8} 0,8532 = 0,5692 \text{ kN (tlak)}$$

$$S_6 = \frac{2,6}{3,34} 0,8532 - \frac{0,5692}{2,33} = 0,4723 \text{ kN (tlak)}$$

## MOMENTI U KARAKTERISTIČNIM TOČKAMA



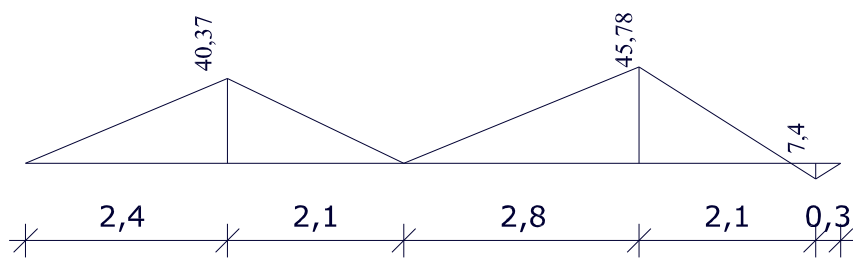
$$M = 2,4 \cdot 0,1649 = 0,3958 \text{ kNm}$$



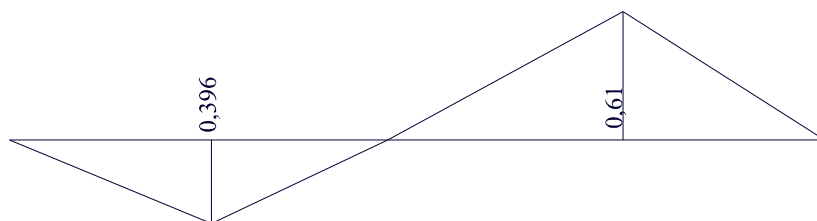
$$M = -0,2543 \cdot 2,4 = -0,6103 \text{ kNm}$$

## IZRAČUNAVANJE POMAKA

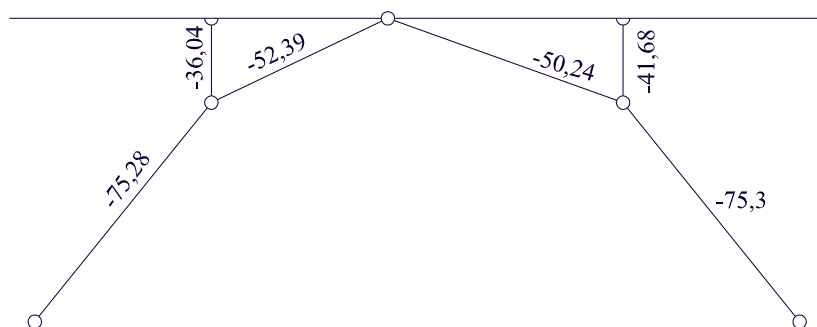
**M**



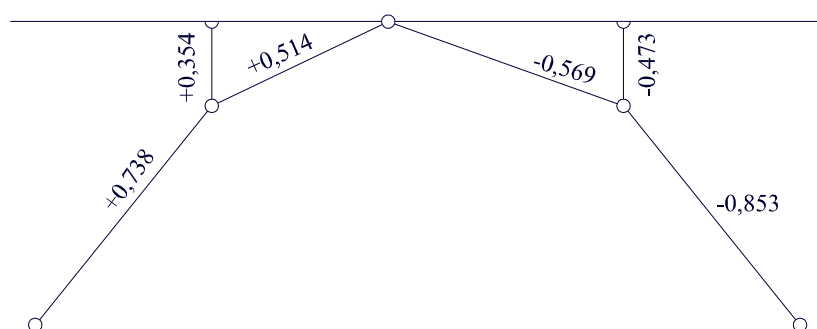
**m**



**N**



**n**



$$\begin{aligned}
 u_B = \frac{1}{EJ} & \left[ \left( \frac{40,37 \cdot 2,4}{2} \right) \left( \frac{-0,396 \cdot 2}{3} \right) + \left( \frac{40,37 \cdot 2,1}{2} \right) \left( \frac{-0,396 \cdot 2}{3} \right) + \left( \frac{45,78 \cdot 2,8 \cdot 0,61}{3} \right) \right. \\
 & \left. + \left( \frac{45,78 \cdot 1,8}{2} \right) \left( \frac{0,61 \cdot 1,2}{2,4} \right) + \left( \frac{-7,4 \cdot 0,3}{2} \right) \left( \frac{0,61 \cdot 0,4}{2,4} \right) + \left( \frac{-7,4 \cdot 0,3}{2} \right) \left( \frac{0,61 \cdot 0,2}{2,4} \right) \right] \\
 & + \frac{1}{EA} \left[ -75,28 \cdot 3,34 \cdot 0,738 - 52,39 \cdot 2,33 \cdot 0,514 + 50,24 \cdot 2,97 \cdot 0,569 \right. \\
 & \left. + 75,3 \cdot 3,34 \cdot 0,853 - 36,04 \cdot 1 \cdot 0,354 + 41,68 \cdot 1 \cdot 0,473 \right] = (0,8939 + 1,1617) 10^{-4}
 \end{aligned}$$

$$u_B = 2,06 \times 10^{-4} \text{ m} = 2,06 \text{ mm}$$