

SOME EXAMPLES OF USING *MATHEMATICA* IN TEACHING GEOMETRY

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ABSTRACT

The paper outlines some thoughts on teaching geometry at technical faculties, as well as on various possibilities and advantages of the application of *Mathematica* in teaching. Some examples of using *Mathematica* in teaching geometry at the Faculty of Civil Engineering in Zagreb are given.

1 INTRODUCTION

At the technical faculties in Croatia geometry is primarily taught within the Descriptive geometry course in the first year. The way of teaching is very traditional. Computers are rarely used. There are many reasons for such situation which are probably similar to those in some other transitional countries: financial conditions, policy of the university, teaching staff etc.

Unwillingness to give up the traditional way of teaching has negative consequences. I would like to point out two consequences which I find the most significant:

- decreasing interest, lack of motivation of the students;
- an increasing gap between the subjects of geometry and mathematics.

(Majority of students do not realize that the contents dealt with in geometrical and mathematical subjects are often the same but treated by different methods - constructive and analytical.)

In my opinion, the main aims of our teaching are:

- developing cognitive and perceptive abilities and knowledge of the students in the field of the three-dimensional Euclidean space,
- acquiring knowledge of constructive methods necessary for the graphics communication of future engineers.

In the computerization of teaching, for which we are unfortunately not well equipped, the development of constructive methods and skills is no doubt connected with CAD pro-

grams. For that purpose *Mathematica* can not help. However, for teaching the geometry of Euclidean space it is very convenient because of its great graphics possibilities. Moreover, it is an ideal program for connecting the content of geometrical and mathematical subjects.

2 *MATHEMATICA* IN TEACHING GEOMETRY

Mathematica is a program which is used as a numerical and symbolical calculator, computer language, the system for visualisation of functions and data, the platform for designing packages for specific applications and creating interactive documents with text, animations and sound, etc. Such complex system with very sophisticated graphics can be applied creatively in teaching geometry. It is especially suitable for developing educational material. For example, for designing

- presentation notebooks with animations and sound,
- web applications for interactive learning on the internet,
- educational notebooks used in computer classrooms.

The usage of *Mathematica* depends on teaching conditions.

For example, I am one of the five members of a team who teach Descriptive and Applied Geometry in a very traditional way. We have only three computers for teachers and none (!) for students. Working in such conditions I have used *Mathematica* visualizations to clarify some topics which I have discovered to be the most difficult for students. I used it in some lectures and for designing internet pages. As mentor I encouraged some of my best students to explore issues beyond regular curriculum.

Although I am aware that the medium of writing can hardly replace *Mathematica* notebook I would like to present here some examples.

3 EXAMPLES

3.1 LECTURE - hyperbolic paraboloid

The following example is an extract from the lecture on algebraic ruled surfaces.

Because of their geometric properties ruled surfaces are suitable for roofing, therefore they are especially interesting for the students of civil engineering. The second degree surfaces (the hyperboloid of one sheet and the hyperbolic paraboloid) are the simplest. They have two systems of rulings, i.e. through each point on such surface two rulings pass - one from each system. All rulings of one system are skew and each of them cuts all rulings of the other system. Any three rulings of one system can be chosen as directing curves.¹

3.1.1 The generations of the hyperbolic paraboloid

The hyperbolic paraboloid \mathcal{HP} is a ruled quadric with the simplest parametrization $(u, v) \rightarrow (u, v, uv)$.

It is a conoidal surface with two lines at infinity. Each system of rulings has one ruling at infinity. The rulings of \mathcal{HP} can be constructed in the following ways:

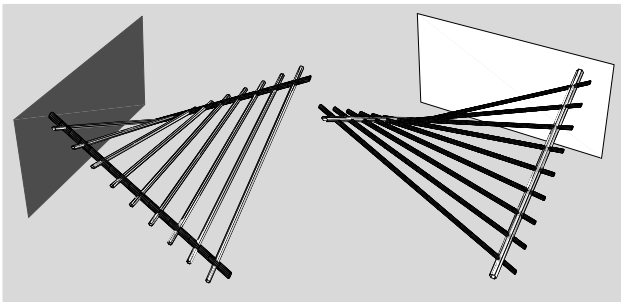


Figure 1: Each system of rulings of \mathcal{HP} can be constructed as a system of lines which cut two skew directing lines and which are parallel with a directing plane - the plane which contains the ruling at infinity. Directing lines are not parallel with the directing plane.

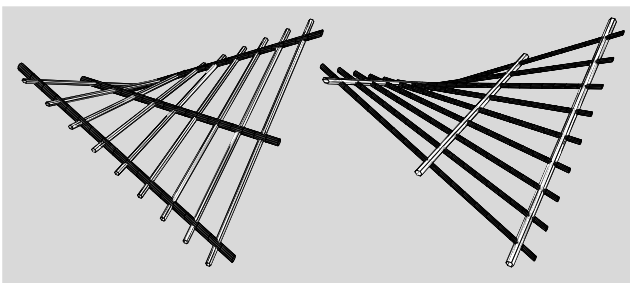


Figure 2: The directing curves are three skew lines parallel with the same plane.

¹[1], [7], [6]

The hyperbolic paraboloid is also a translation surface. It can be obtained by moving the parabola of one main section along the parabola of other main section.

The other very simple parametrization of \mathcal{HP} is $(u, v) \rightarrow (u, v, u^2 - v^2)$.

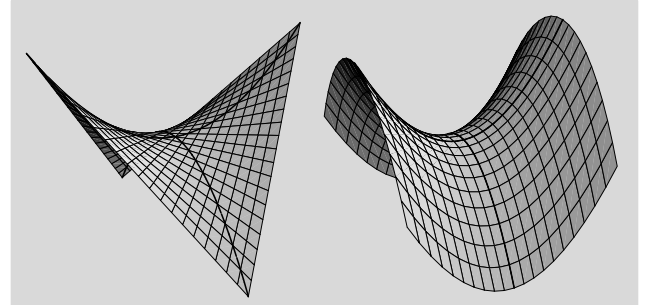


Figure 3: Two parabolas which generate \mathcal{HP} as a translate surface. Mathematica graphics are obtained by using two parametrizations above mentioned.

3.1.2 Intersections of the hyperbolic paraboloid and planes

The tangent plane cuts \mathcal{HP} into the pair of rulings (Fig. 4). Since the plane at infinity contains two rulings it is a tangent plane at the point of the intersection of directing planes.

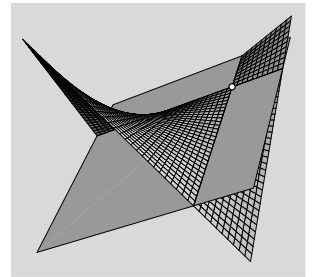


Figure 4

Since \mathcal{HP} has two real lines at infinity the conic intersection of that surface and a plane possesses two or one real point at infinity. Therefore, there are no ellipses on \mathcal{HP} . A plane cuts \mathcal{HP} into a parabola if it is parallel with the intersection of directing planes (Fig. 5). In all other cases, excluding tangent planes, the intersections of \mathcal{HP} are hyperbolas (Fig. 6).

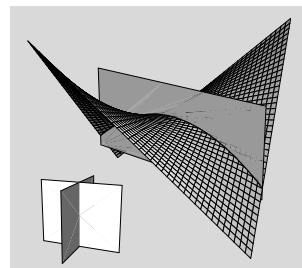


Figure 5

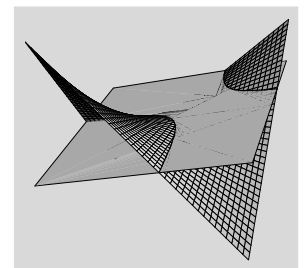


Figure 6

3.1.3 Mean and Gaussian curvature

The Gaussian and mean curvatures of regular surface are important functions in the differential geometry. \mathcal{HP} is convenient for some visualization connected with these functions.

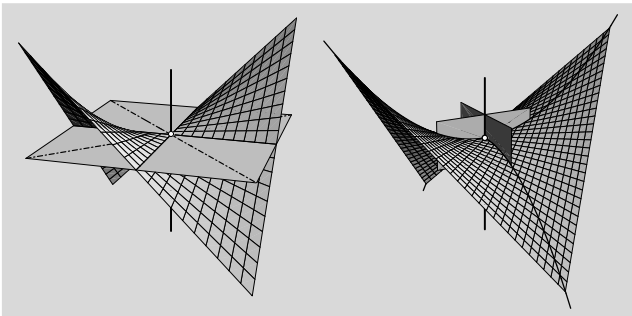
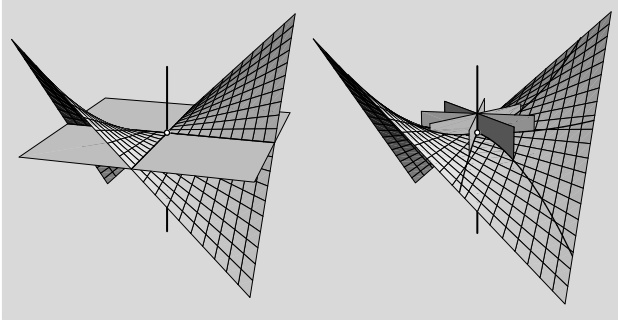


Figure 8: The maximum and minimum values of the normal curvature of \mathcal{HP} at each its point are *principal* curvatures. The principal directions are determined by the lines symmetric to the rulings in a tangent plane.

The Gaussian curvature is a product and a mean curvature is a half of sum of principal curvatures at the regular point of a surface. For computing we use the first and the second fundamental form.

In the book [3, pp. 394] we can find *Mathematica* definitions of the functions *mcurvature* and *gcurvature* which compute mean and Gaussian curvatures in each regular point of the surface given by parametric equations. We used that functions for drawing the following pictures.

²[4], [5]

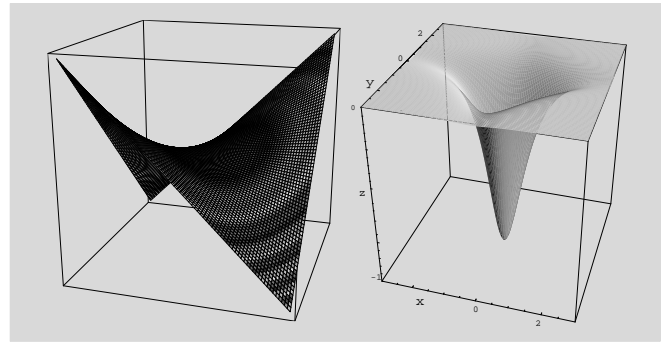


Figure 9: \mathcal{HP} colored with function of Gaussian curvature and the graph of the function *gcurvature* of \mathcal{HP} .

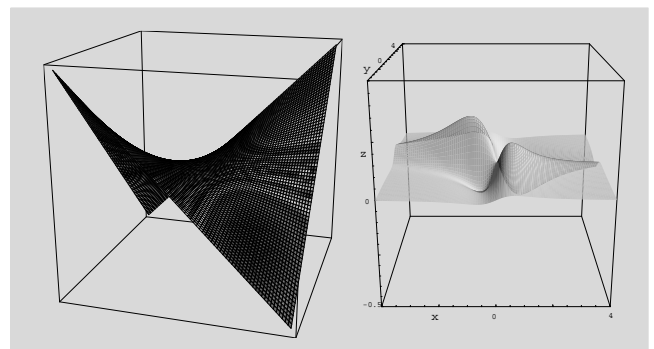


Figure 10: \mathcal{HP} colored with function of mean curvature and the graph of the function *mcurvature* of \mathcal{HP} .

3.2 INTERNET PAGE

At the address www.grad.hr/nastava/geometrija/ilustracije students can find *Mathematica* graphics and animations which visualize properties of ruled surfaces.²

Since we have just obtained *webMathematica* in Croatia, I hope that in the future this page will contain educational materials for interactive learning designed by that new technology.

3.3 STUDENTS' PROJECT

Last year I was a mentor to second-year students Sanja Filipan and Hrvoje Kvasnička who wrote the paper *Covering with Parabolic Conoid*. The paper has been selected for the Rector's Prize. I think the paper won the prize partly due to the usage of *Mathematica* for visualization. Certain parts of that paper, extended with visualization of Gaussian and mean curvatures can be found in [2].

In their paper the students treated the class of 3rd degree parabolic conoids with directing curves which can be connected with the Cartesian coordinate system in the following way:

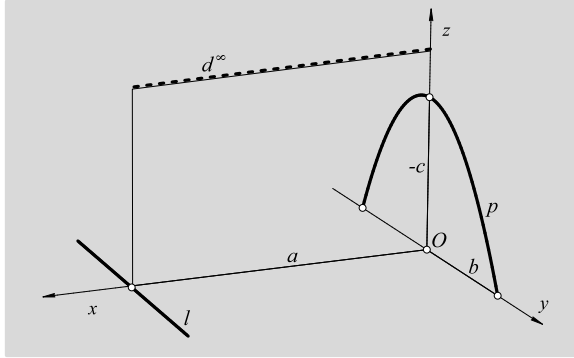


Figure 11: The directing curves of the parabolic conoid are the parabola p , the simple line l and the double line d^∞ which lies in the directing plane δ . They are given by the equations:

$$\begin{aligned} p \dots \begin{cases} x = 0 \\ z = \frac{c}{b^2}(y^2 - b^2), \quad b, c \in R, \quad b, c \neq 0 \end{cases} \\ l \dots \begin{cases} x = a \\ z = 0, \quad a \in R, \quad a \neq 0. \end{cases} \\ \delta \dots y = y_0, \quad y_0 \in R. \end{aligned} \quad (1)$$

The equation of the parabolic conoid which belongs to that class is:

$$z = \frac{c}{ab^2}(y^2 - b^2)(a - x), \quad a, b, c \in R, \quad a, b, c \neq 0 \quad (2)$$

The students defined, using *Mathematica* the functions for drawing parabolic conoids for each choice of the parameters a , b and c .

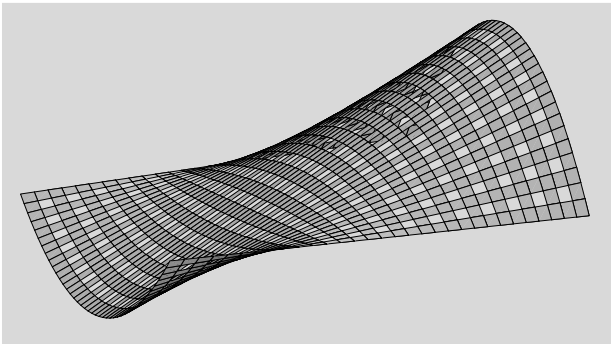
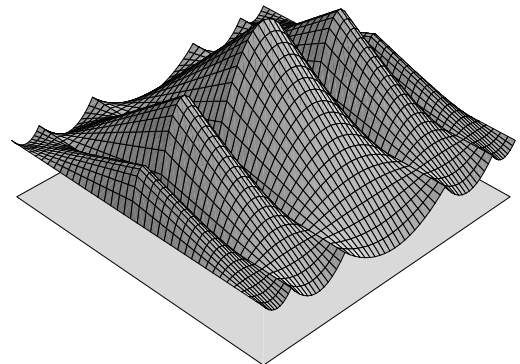
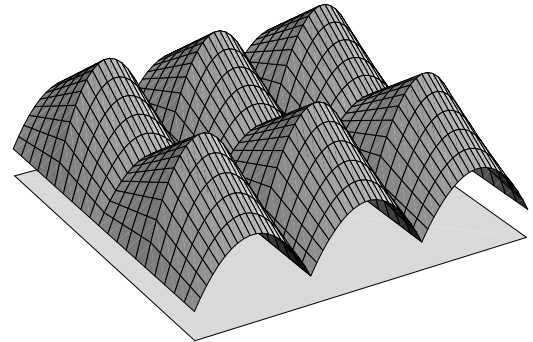
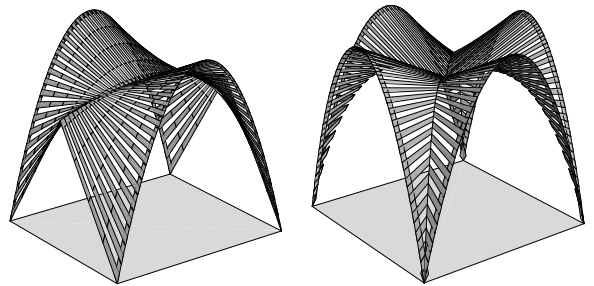
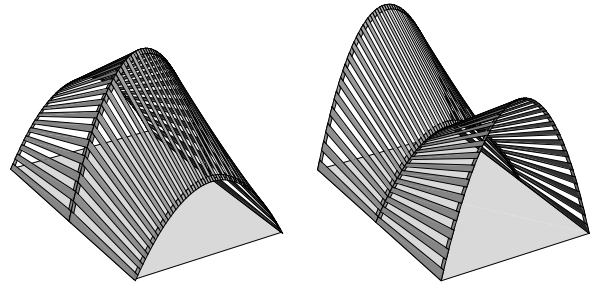
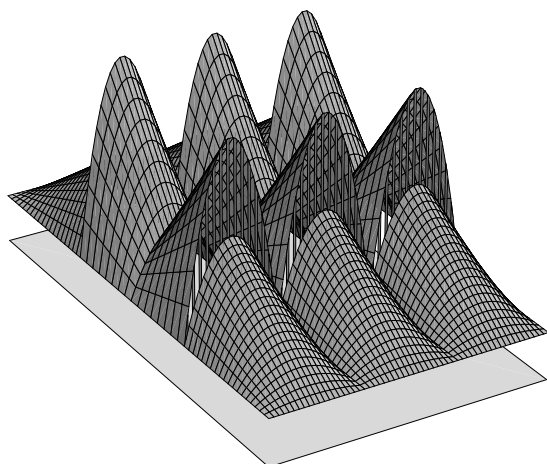
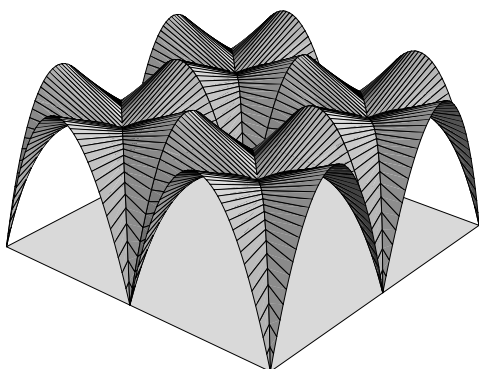
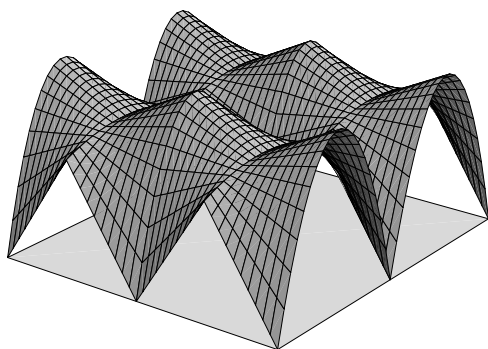


Figure 12: The parabolic conoid with the parameters $a = 2, b = 1, c = -1$.

Parabolic conoids can be used for covering in numerous ways. Various parts of conoids can be composing in simple series. In the following pictures some ideas for the covering over the rectangular plan are given.





4 CONCLUSION

In teaching geometry *Mathematica* can be used for designing diversified and high-quality educational material.

New technology *webMathematica* opens the door to interactive computing and visualization of data directly from the user's web provider.

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