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ABSTRACTS

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Contents

Plenary lectures

Quality Culture in Higher Education in Croatia

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Quality in higher education (HE) can be defined in different ways, but in general it refers to "fitness for purpose"- meeting or conforming to generally accepted standards as defined by an accrediting or quality assurance body. Each institution of higher education (HEI) is responsible for establishing an organisational climate, so called **quality culture** in which groups of staff work together to relise their specific tasks. There is an expectation that an institution will have in place a plan to monitor and improve the quality of its programs. In most cases, quality assurance and accrediting agencies require that established procedures ensure that this is an ongoing process.

In May 2001 Croatia signed the Prague declaration and by this Croatian HE institutions have been officially included into the **Bologna process**.
Croatian Parliament adopted the new Act on Higher Education and Research in Croatia in 2003. The Act is based on Bologna Declaration and it promotes European dimension of HE and we highlight three innovations related to the Bologna process:

- The implementation of Bologna process in Croatia becomes a legal obligation, the deadline for the introduction of the appropriate study scheme being the academic year 2005/2006.
- The development of binary system (with vocational studies organized separately by higher schools and polytechnics) has to be completed by 2010.
- The development of the quality assurance (QA) including the Agency for HE, and the national network of QA offices and committees at HEIs.

Thus, recent development of quality culture at HEIs in Croatia can be seen as a top down process, triggered by Bologna process and accession process of Croatia towards EU, but it has been also motivated by bottom up forces arising from inner problems of higher education system in Croatia.

Among other problems in HE in Croatia there are the following: only 12% of the population over the age of 15 completed some higher education level; HE is not ready to meet the growth in labour demand in the intellectual service arena which includes proficiency in modern technology, developed generic skills and well organized life long learning support etc. At the top of that the system of HE in Croatia is seriously under-financed.

Further, drop-out rate in HE in Croatia in 2004 was 69.5%. There is the Croatian objective of decreasing the dropout rate in tertiary education to 50% that by 2010.

In the academic year 2005/2006 all Croatian HEIs were renovating their curricula according to the requirements of the Bologna process and the national Act.

The Croatian universities show different stages of progress towards Bologna, but generally the first phase of introducing the Bologna system has been started at all of them. Now is the proper time for making the shift in the focus of the Bologna process, since until this year the reform was concerned mainly with ECTS and structure of the study and just sporadically taking into account qualitative side of the reform.

Finally, it is also important to consider the question how teaching of mathematics is influenced by new (old?) paradigm of quality culture.

Osculating Tangents of Cayley's Ruled Surface and the Betten-Walker Spread

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We present a direct and self-contained approach to a spread of lines which was discovered independently, and approximately at the same time, by D. Betten (1973) and M. Walker (1976). We establish that this spread of lines is closely related with the set of osculating tangents of Cayley's ruled cubic surface.

This is joint work with Rolf Riesinger (Wien).

References:

[1] H. Havlicek, R. Riesinger: The Betten-Walker spread and Cayley's ruled cubic surface, *Beitr. Algebra Geom.*, (to appear).

Moving Central Axonometric Reference Systems

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Multiplication and square root of points of a triangle leads to a quadratic mapping with several interesting properties. Meanwhile this mapping has its own interest, some of its consequences yield a new condition, under which a central axonometric mapping is a central projection. This synthetic condition is applied for controlling the change of base points of a central axonometric reference system. Correction of a central axonometric system to be of central projection type is also discussed by the help of our condition.

Key words: multiplication of points, square root of a point, central projection, central axonometry

On Instantaneous Motions in Projective Kinematics

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By motivations of the classical Euclidean plane kinematics, we can follow a straightforward way which leads to some analogous, now "well-known" methods and results in projective plane kinematics, although these are not widely spread in the textbooks and in the scientific literature, in general. In this lecture a three- (and higher-) dimensional theory will be initiated and sketched in analogous way, in the framework of projective-spherical space $PS^3(\mathbf{V}^4, \boldsymbol{V}_4, \mathbf{R})$ of *points* (as rays of $\mathbf{V}^4$) and of *hyperplanes* (as rays of $\boldsymbol{V}_4$) namely by

1. Classification of projective-spherical motions in $PS^3(\mathbf{V}^4, \boldsymbol{V}_4, \mathbf{R})$ on the base of *Jordan normal form* of 4×4 matrices;

2. One-parameter group $G(A)$, generated by a fixed kollineation $A(\mathbf{A}, \boldsymbol{A}^{-1})$, and its orbit-space topology by the linear normalizer $N(A)$ *of* $G(A)$;

3. Classification of the above structure, as ***instantaneous motions***, and characterization with infinitesimal (Lie-algebra) elements;

4. Possible additional structures, e.g. projective metrics by form (hyperplane) $\to$ point (vector) polarity or scalar product on $\boldsymbol{V}_4$;

5. Discrete (discontinuous) group structures and their combinations to more generator discrete groups, etc.

This machinery, as a theory, can be applied to Euclidean and non-Euclidean (e.g. hyperbolic) kinematics as well, moreover to discrete groups of various spaces. In this lecture, however, it will be illustrated mainly with characteristic plane examples with invitation to further investigations.

Local Properties of More-parametric Equiform Spatial Motions

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We present an overview of techniques typically used in local kinematics. This will be done at the hand of the study of properties of $k-$parametric motions $M(u)$ within the group $AE(n)$ of equiform transformations of the $n-$ dimensional Euclidean space E_n. First order properties and standard representations of $M(u)$ at some moment u_0 are worked out by using Lie-group techniques. This allows to study local differential geometric properties of the point paths under the motion $M(u)$: We study the tangent spaces and describe pathologic cases, such as the manifolds of local spiral poles (in the general cases we gain rational $(k-1)-$dimensional manifolds).

Then the situation in the $3-$dimensional Euclidean space E_3 is being studied in detail: In the most interesting case of $k = 2$ the corresponding local manifold of spiral poles turns out to be a twisted cubic. Here we will go for second order properties, too. This includes the study of curvature properties of the corresponding point paths.

Remarks on Higher-order Rigidity

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There is no strict borderline between rigidity and continuous flexibility of structures, e.g., of frameworks. The intermediate case is that of "infinitesimal flexibility" which at real word applications results in slightly flexible models, so-called "shaky" structures. And here we can distinguish between different orders of infinitesimal flexibility. However, the proper definition of the order is problematic:

A framework F is called infinitesimally flexible of order n, if it admits a nontrivial n-th order flex $X_0 + tX_1 + \cdots + t^n X_n$. F is called infinitesimally rigid of order n, if any n-th order flex of F is trivial.

According to R. Connelly and H. Servatius [1] these definitions cause problems when a flex with trivial velocity distribution X_1 is already called trivial. But otherwise, any nontrivial first order flex $X_0 + tX_1$ of F gives rise to the nontrivial second order flex $X_0 + t^2 X_1$. How to escape this dilemma?

The lecture focusses on two items:

(1) It demonstrates how to identify nontrivial flexes.

(2) The representation of a nontrivial flex of F is not unique. As the definition of a flex of F is invariant under regular polynomial parameter transformations, the problem of "reducible" representations of an n-th order flex is adressed where the exponents of t have a common factor greater than 1.

As a conclusion, the definitions of n-th order infinitesimal flexibility and rigidity must be based on irreducible representations of flexes.

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[1] R. Connelly and H. Servatius: Higher-order rigidity - What is the proper definition? *Discrete Comput. Geom.*, **11**, no. 2 (1994), 193–200.

[2] T. Tarnai: Higher order infinitesimal mechanisms, *Acta Technica Acad. Sci. Hung.*, **102**, (3–4) (1989), 363–378.

About Curvatures on Triangle Meshes

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We present face-based curvature computations on triangle meshes. Our method characterises the triangular faces by curvature values, which can be applied for classifying elliptic, flat and hyperbolic regions of the mesh. In the examples we compare these results with widely used vertex-based curvature values.

Frolicking Visual Communication in the Edo Era

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Abstract will be available at the conference.

Generalised Surfaces of Euler Type without Singularities

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Special type of two-axial surfaces of revolution, generalised surfaces of Euler type created by the Euclidean metric transformation of a composite revolution about two skew axis are presented. Different groups of surfaces (ruled, cyclical and non-specified) are defined on the base of specifying the type of the surface basic figure. Special subgroups of surfaces are specified within each of the three different groups based on the superposition of the basic figure and the two skew axes of revolutions. Surfaces without singularities are determined by derived parametric equations of the defined types of two-axial surfaces of revolution, and some of their geometric properties are discussed. Several illustrations of the specific subgroup representatives are presented utilising the programme system MAPLE.

Contributred talks

Blocking Sets and Nuclei

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A *t-fold blocking set* $\mathbf{B}$ in a $PG(2,q)$ is a set of points such that every line of $PG(2,q)$ intersects $\mathbf{B}$ in at least t points.
A point $P \notin \mathbf{S}$ is a *t-fold nucleus* of $\mathbf{S} \subset PG(2,q)$ if every line through P meets at least t points of $\mathbf{S}$.
Certain lacunary polynomials (fully reducible polynomials with a gap between its degree and second degree) over finite field are associated with a *t-fold blocking sets* and possible number of *t-fold nuclei* to a set of points in $PG(2,q)$.

References:

[1] A. Blokhuis, S. Ball: Polynomials in Finite Geometry, *Quaderni di matematica*, vol.5, Univ. Napoli (1999)

[2] L. Rédei: *Lacunary Polynomials over Finite Fields* Akadémiai Kiadó, Budapest (1973)

[3] A. Blokhuis, L. Storme: Lacunary polynomials, multiple blocking sets and Baer subplanes, submitted to *J. London Math. Soc.*, **60**, vol. 2 (1999)

Reformed Teaching of Descriptive Geometry According to the Bologna Process at the Civil Engineering Dept of Polytechnics of Zagreb

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"Descriptive Geometry in Civil Engineering" is the new title of the course given in two semesters. Reduced number of lectures and exercises hours did not have a major influence on reducing the course contents, thanks to the new way of teaching. Also the number of students in a group is reduced and new system of tests is organized (one test in two weeks) to activate the students. Students are required to collect prescribed number of points necessary to get the teacher's signature, and besides good results partly exempt them from the written part of exam. In this way, the students are encouraged to work continuously during the whole academic year. The analysis shows that the students are more successful in exams. This way of teaching demands a great effort both from the students and from the teachers.

On Gergonne Point of the Triangle in Isotropic Plane

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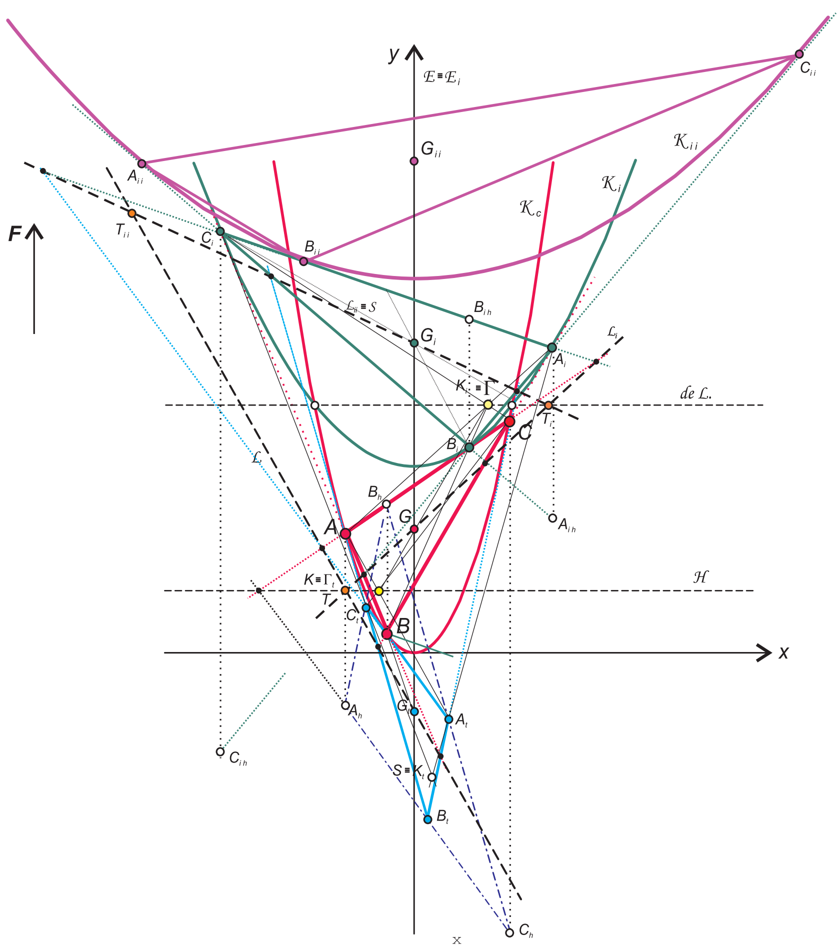
Using the standard position of the allowable triangle in the Isotropic plane relationships between this triangle and its contact and tangential triangle are studied. Thereby different properties of the symmedian center, the Gergonne point, the Lemoine line and the de Longchamps line of these triangles are obtained.

Key words: isotropic plane, standard triangle, contact triangle, Gergonne point, tangential triangle, Lemoine line



The discussed elements

Basic of Engineering Informatics - New Curriculum at the Faculty of Civil Engineering in Zagreb

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Within the framework of the Bologna process which brought changes in curricula, we have changed introductory informatical subject at the Faculty of Civil Engineering in Zagreb. The presentation gives an overview of the content of the subject with emphasis on the variety of informatical areas included in it (AutoCAD, the part of ECDL modules, Mathematica, programming). The presentation also describes the method of teaching which was pretty demanding because of the complexity of the content and a great number of students (more than 400) who needed computers for their exercises. The list of preliminary exam results shows that students were exceptionally successful.

Matrix Young Inequalities

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Operator and matrix versions of classical inequalities are of considerable interest in mathematics. A fundamental inequality among positive real numbers is the *arithmetic-geometric mean inequality* whose generalisation is the most important case of the *Young inequalities*.
It says that if $\frac{1}{p}+\frac{1}{q}=1$ with $p, q>0$, then

$$|ab| \leq \frac{|a|^p}{p}+\frac{|b|^q}{q}, \quad \text{for} \quad a, b \in \mathbb{C}.$$

A direct matrix generalisation of the upper inequality does not hold in general, however, it was proved (*Bhatia-Kittaneh*) that for every pair A, B of $n \times n$ complex matrices there is a unitary matrix U, depending on A, B, such that

$$U^*|AB^*|U \leq \frac{|A|^2}{2}+\frac{|B|^2}{2}.$$

The extension of this proof to the case of general p (*T. Ando*) is based on pinching inequalities for the mapping $X \geq 0 \mapsto X^r, r>0$ and is related to the theorem which says that if $\frac{1}{p}+\frac{1}{q}=1$ with $p, q>0$, then for any pair A, B of $n \times n$ complex matrices there is a unitary matrix U depending on A, B, such that

$$U^*|AB^*|U \leq \frac{|A|^p}{p}+\frac{|B|^q}{q}.$$

References:

[1] T. Ando: *Matrix Young Inequalities* Operator Theory: Advances and Applications, vol. 75, 1995 Birkhauser Verlag Basel/Switzerland

[2] F. Hansen: An Operator Inequality, *Mathematische Annalen*, **246** (1980), 363–378.

Geodesics and Translation Curves in Sol Geometry

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The homogeneous geometries have main roles in the modern theory of three-manifolds, regardless being isotropic or not. It is well-known, that there exist only three homogeneous two-dimensional geometries, namely the elliptic, the Euclidean and the hyperbolic geometry. They are isotropic geometries, as well. Homogeneous, but non isotropic geometries can be found in three or higher dimensional manifolds. (Note, that the word geometry means the family of geometries.)

Homogeneous Riemannian manifolds play central role in topology, e.g. in the famous Thurston conjecture. The conjecture states that a three-manifold with a given topology has a canonical decomposition into a connected sum of "simple three-manifolds", each of which admits one, and only one, of eight homogeneous geometries: H^3, S^3, E^3, $S^2 \times S^1$, $H^2 \times S^1$, Sol, Nil and SL(2, R).[1]

The illustration of Sol geometry is really a hard task, but E. Molnar elaborated the projective interpretations of the eight homogeneous geometries, e.g. of the Sol geometry.[2]

In this presentation we give the complete classification of the geodesics based on the starting velocity vector in the origin. Their curvature and the torsion are given as well with the help of our previously computed formulas. On the other hand one of the authors and Emil Molnar introduced the new concept of translation curves. Translations in 3-spaces are meant in a natural way. Consider a unit vector at the origin and a geodesic curve starting in the direction of this vector by arc-length parameter. If a curve has just this translated tangent vector in each point then the curve is called translation curve. This assumption leads to a first order differential equation, thus translation curves are simpler than the geodesics and differ from them in Sol.[3]

Key words: differential geometry, Sol geometry

References:

[1] W. P. Thurston: Three dimensional manifolds, Kleinian groups and hyperbolic geometry, *Bull. Amer. Math. Soc.*, **6** (1982), 357–381.

[2] E. Molnar: The Projective Interpretation of eight 3-dimensional Homogeneous Geometries, *Beitrage zur algebra und Geometrie*, **38** (1997), 261–288.

[3] E. Molnar, B. Szilagyi: Translation curves and their spheres in homogeneous geometries, (to appear).

Quality Assurance System for Teaching and Learning Process

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In this presentation, the survey on the quality of lecturing process, carried out among students at the Faculty of Civil Engineering and Architecture in Split, is presented, as well as the results and experiences gained from the survey. The survey was carried out in order to find out the students' opinion on the quality of the lecturing process and on the lecturers, and to serve as the basis for the future similar surveys. Finally, some positive and negative experiences are presented, and recommendations are given for the future surveys.

2-Associative and 2-Coassociative Hopf Algebra Structures

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A possibility of endowing 2-coassociative colagebra to 2-coassociative Hopf algebra will be proved and some applications of mentioned Hopf algebra will be showed.

Special Sextics with Quadruple Line

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In this paper we describe a special class of 6th order surfaces in 3-dimensional Euclidean space which are the pedal surfaces of the 1st order and 4th class congruence C_4^1 with respect to the pole P. The directing lines of C_4^1 are the Viviani's curve and a straight line which cuts it in two points where one of the intersection points is the double point of the Viviani's curve. These surfaces belong to the class of nth order surfaces with $(n-2)$-ple straight line.

We derived the parametric and implicit equations of these sextics with a quadruple straight line. They anabled graphical representation in *Mathematica* and *webMathematica*, proving some properties and the classification of the surfaces according to the type of singular points on a quadruple line.

References:

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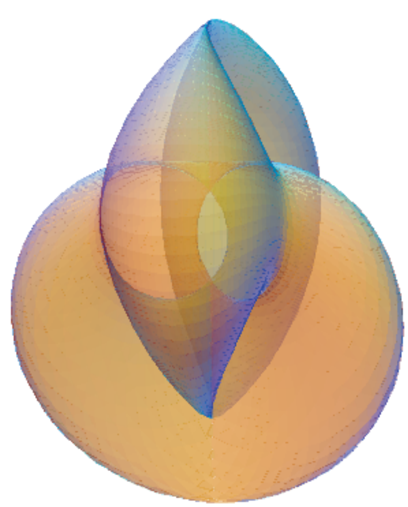
[2] R. Sturm: *Die Lehre von den geometrischen Verwandtschaften, Band IV.* B. G. Taubner, Leipzig-Berlin, 1909.



The Efficient Design of Developable Surfaces

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Developable surfaces (torses) are special ruled surfaces with many remarkable properties. Models of such surfaces can be produced easily by means of paper strips. In avantgarde architecture, free form surfaces are of high interest. In general, however, the costs to build such surfaces are extreme. These costs decrease dramatically, when the surfaces are developable. We present a program that provides the artist with tools to design such surfaces very intuitively. Due to the geometric properties, the corresponding algorithms all work in real time.

Key words: developable surfaces, torses, ruled surfaces, paper strip, free form surfaces

Evolutoides

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The aim of this work is to show the way how we can form associated curves by computer program, especially evolutoides. The evolute of Braude has been defined and genarated by computer program. Examples have been given in order to motivate undergraduate and graduate students for an autonomuos research of plane curves at the Faculties of Engineering.

Snails in the Hyperbolic Plane

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This work has two start points. The first is curve known as the limaçon of Pascal and the second is classification of the entirely circular curves of fourth order in the hyperbolic plane.

Properties of the limaçon of Pascal in the Euclidean plane are well known. Our aim is to obtain curves in the hyperbolic plane that have similar properties. That curves we have named hyperbolic snails and defined as circle pedal curves.

It is shown that only some of them are entirely circular, but all of them are at least bicircular.

Key words: limaçon of Pascal, hyperbolic plane, entirely circular 4-order curves

The Importance of the Development Spatial Ability at the Process of Creating the Quality Living Environment in Slovenia

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This report is taken from wider research with the title: Process of creating the quality living Environment in Slovenia. The most important fact for the architectural and urban planning design is good spatial ability. Due to it's importance/influence and useful on the other fields of human life, more institutional care for developing it is expected. Present situation in Slovenia and the role, which the spatial ability has in the educational system in Slovenia, is presented.

Geometry in Elementary School Geography

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Following conclusions can be drawn on the basis of conducted research:

- There is a large number of elementary school textbooks for geography in Croatia. Five different textbooks and corresponding workbooks for the 5th grade were analysed.
- Newer textbooks are more attractive and more sought.
- Introduction of geographic coordinates is a special problem, as at this age, students do not know the trigonometric functions. Moreover, the definition of an angle between a straight line and a plane, and the definition of an angle between two planes is not in the curriculum.
- The following terms should be excluded from the 5th grade: ellipse, ellipsoid, geoid, angular distance, and neighbouring meridian. The first three are too difficult and unnecessary for the age, and the last two make no sense.
- It would be nice if students after finishing elementary school had knowledge about following terms: proportionality, linear scale, plane coordinate system, Pythagoras' theorem, equality of angles with perpendicular legs, circle perimeter and circle area, sphere area and volume, ellipse and rotational ellipsoid basics, geographical coordinate system, map projections basics.
- All terms should be introduced correctly, and not in a distorted or even wrong way.
- Authors of geography textbooks should be assisted by geodesists and cartographers while processing geodetic and cartographic themes.

Minimal Surfaces in Minkowski Space

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Minimal surfaces in Lorentz - Minkowski 3-space R_1^3 share many properties with minimal surfaces in euclidean 3 - space. Many differences result from splitting in spacelike and timelike surfaces in Minkowski space.

In case of spacelike surfaces the induced metric is Riemannian and isothermic coordinates lead to harmonic coordinate functions, so as in euclidean case we have the machinery of complex analysis. O.Kobayashi called spacelike minmal surfaces *Maximal surfaces*, because they maximize the surface area locally.

Timelike minimal surfaces can be generated locally as translation surfaces with nullcurves as generating curves. They neither minimize nor maximize the surface area.

In the lecture we discusss examples of ruled surfaces, rotational surfaces and screw surfaces which turn out to be the associates of rotational surfaces. Furthermore we point out some relations concerning quantities of affine differential geometry.

Many of the results can be found in the book "An Introduction to Lorentz Surfaces" be Tilla Weinstein (Tilla Milnor Klotz) (De Gruyter Expositions in Mathematics 22, 1996).

Visualisation and Remarks on Equiform Motions with Three Circular Paths

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Plane equiform kinematics was developed by R. Müller and M. Krause in 1910 - 1920. Now computer software put us in a position to visualize equiform motions and observe some of their properties. As an example the equiform motions determined by three circular paths is presented.

Modelling Ross Business Rules

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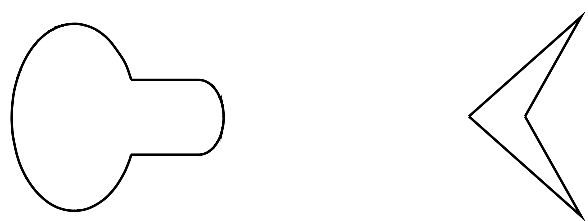
In this paper we represent modelling of business rules. This radically new approach in information systems was introduced by F.G. Ross.
Method is based on this facts:

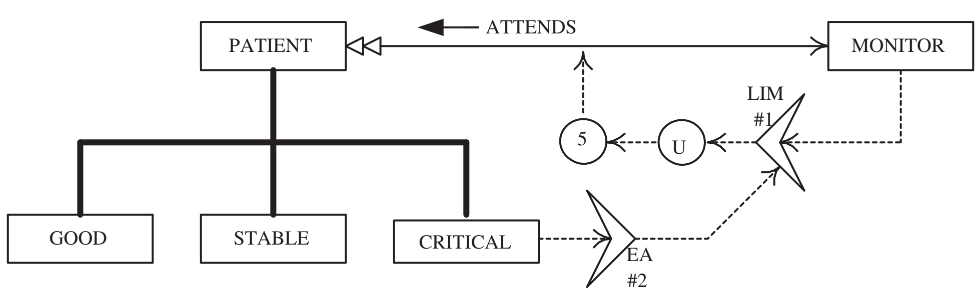
1. The eneterprise has hunderds or thousands specific rules or business policies that govern its behaviour and distinguish it from others.
2. Rules are data-based.
3. Business rule performs specific computation or test. Thats why business rule can be atomic or derivative type.

Atomic rule types can be equivalent to the atomic elements in Periodic Table used in chemistry. They can be combined in building-block fashion as compound rules.

Business rules can be classified in families: atomic rules - 7 families, derivative rules - 12 families with various types. Every rule may appear in two forms as condition or as integrity constraint:



Every rule has a graph and explanation.



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The Orthogonal Axonometry Related to the Perspective

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The relationship between two perspective drawings of one object, projected from two different station points on the same plane, is considered. The properties of this relationship make it possible not only to replace a perspective drawing with a better one, but also to replace an axonometric drawing with a perspective drawing and vice versa. This property is valid for the perspective with horizontal axis of view as well as for the perspective with inclined axis. Among other possible projections, this property is a basis for the constructions of the stereoscopic drawings, but here the emphasis is on the constructions of the orthogonal axonometric drawing from the perspective drawing.

Key words: orthogonal axonometric drawing, perspective drawing

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[1] F. Hohenberg: *Konstruktivna geometrija u tehnici*, Građevinska knjiga, Beograd, 1966. (translated by V. Niče)

[2] P. Kurilj, N. Sudeta, M. Šimić: *Perspektiva*, Golden marketing-Tehnička knjiga, 2005.

[3] V. Niče: *Deskriptivna geometrija*, 2. svezak, Školska knjiga, Zagreb, 1980.

Determining the Optimal Hyperball Packings to the Regular Prism Tilings in the Hyperbolic n-Space

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In this talk we investigate the n-dimensional, $n \geq 3$, hyperbolic prism honeycombs, which are generated by the "inscribed hyperspheres".

The 3-dimensional prism tilings (mosaics) were classified by I. Vermes in [2] and [3]. He found infinitely many prism tilings, whose optimal hyperball packings and metric data are determined in [1].

In the hyperbolic 4-space there are only 2 analogous honeycombs whose metric data and the densities of their optimal hyperball packings are determined in this paper. In the hyperbolic 5-space there are 3 types of these mosaics, whose analogous problems will be discusse elsewhere. In the hyperbolic n-space, $n > 5$, there are no such regular prism tilings. Our method and computations are based on the projective interpretation of the hyperbolic geometry.

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Theorems on Foci and Asymptotes of Conics in the Isotropic Plane

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Every conic with foci in the isotropic plane can be represented by the equation of the form $y^2 = \epsilon x^2 + x$, where $\epsilon \in \{-1, 0, 1\}$ for an ellipse, a parabola and a hyperbola with foci respectively. Using this equation the important properties of the foci are being proved. According to duality the properties of asymptotes of the hyperbola in the isotropic plane are valid as well.

Key words: isotropic plane, conic section, focus.

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Some Ways to Construct a Symmetric 3-Dimensional Model of the Hypercube

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Lift the outer endpoints of each radius of a regular k-sided polygon to an angle identical to the plane of the polygon. These sections are the edges conjoining in the starting vertex of the 3-dimensional model of the k-cube (further: 3-model) [2], [4]. The model's construction with the well known sliding-method creates a lattice structure, whose outer bars equal a zonotope's [3] edges. This structure keeps the 3-cube's central symmetry and rotational symmetry related to the main diagonal joining the starting vertex referred to any $j < k$ dimensioned elementgroup.

It is also possible to get to the endpoint of the main diagonal from the starting point along easily recognizable barchains, whose binding points (the outer vertices of the model) join on one helix each. The common lead of these helixes is the main diagonal. According to these attributes, such a barchain can, distributed around the main diagonal in the number equal to the number of his elements and the given distribution mirrored to the centre point of the common lead, generate the outer edges of the k-cube's 3-model.

Increasing the sections of the barchains infinitely, a continuous helix is created, whose polar distribution means, according to the procedure so far, the rotation around the lead, therefore the shell of the n-cube's 3-model is generated as the surface of a solid of rotation in which any k-dimensional cube's 3-model can be constructed. The height of the solid (i.e. of the full model) is also determinable, as it holds the 3-cubes 3-model with the proportions of the normal cube.

According to these our lattice 3-model of any k-cube can also be generated whether as ray-groups of the edges or as sequences of barchains originated from a seperate helix.

The creation of the constructions and figures required for the paper was aided by the AutoCAD program and the self developed Autolisp routines.

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New Results on the Number of Points at Distance at Least 1 in the Unit d-Cube

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Let C be a cube having edge length 1 in $\mathbb{R}^d$. Denote by $\mathrm{n}(d)$ the maximum number of points that can be placed in C in such a way that the distance between any two points is at least 1. New upper bounds will be presented for $\mathrm{n}(d)$ when $d \geq 6$. We show the universal upper bound

$$\mathrm{n}(d) \leq 2 \cdot \left(\sqrt{\frac{2d}{3}} + 1\right)^d$$

for $d \geq 6$ with some further improvements for $6 \leq d \leq 10$, and we also obtain the asymptotic bounds

$$d^{d/2} 0.428^d e^{o(d)} \leq \mathrm{n}(d) \leq d^{d/2} 0.567^d e^{o(d)}.$$

This is a joint work with V. Bálint and A. Joós.

Geometry of Real Time Shadows

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Shadows provide important visual cues about the spatial relationship among objects. Shadow volumes are one way to generate sophisticated shadows for use in real time environments. This paper focuses on the geometric aspects which are involved in the creation of the shadow volume. Speed up techniques like shaders and dual space approaches for silhouette determination are discussed. Finally the application of the described methods in a software for shadow profile calculation is addressed.

Key words: shadow volumes, dual space, silhouette determination, shader, real time

Posters

Geometry, Dimensions and Illusions

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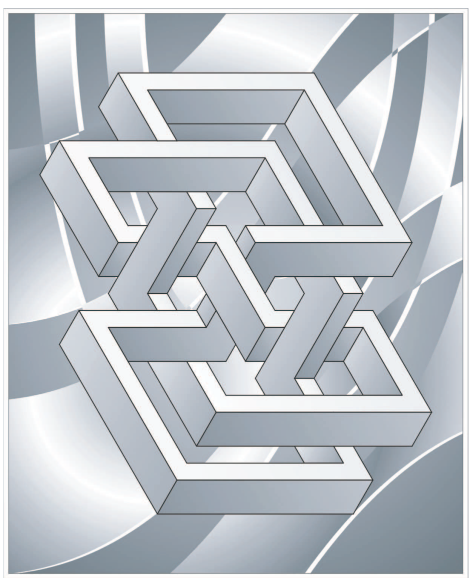
My lecture gives an inside view into a specific world of shapes. These shapes are composed from planar, spatial and multi dimensional units. This modular architecture helps in teaching and developing spatial view. It also gives ideas for natural sciences which use and study extra ordinary forms and connections of modular elements.



The Translation Curves in Homogeneous Geometries

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In this paper four 3-dimensional homogeneous geometries Sol, Nil and product geometries ($H^2 \times R, S^2 \times R$) are discussed. We introduce a seemingly new family of curves, called translation curves. These curves seem to be more natural in these geometries, than their geodesic lines.

Translations in Riemannian 3-spaces can be introduced in natural way. Consider a unit vector at the origin and a geodesic curve starting in the direction of this vector by arc length parameter. Any other vector at the origin can be translated along the above geodesic curve and so geodesic translation as local isometries can be defined along geodesic curves. But a Riemann necessarily homogeneous. In a homogeneous space (above) there are postulated isometries, mapping each point to any point. Specific translations can be introduced in a natural way, involving an invariant Riemann metric. However, its geodesic translations will be different from the special translations. Again, consider a unit vector at the origin. Translations, postulated at the beginning carry this vector to any point by its tangent mapping. If a curve has just this translated tangent vector in each point, then the curve is called *translation curve*. This assumption leads to a first order differential equation, thus translation curves are simpler than geodesics.

Similarly to geodesic sphere, translation sphere can be defined and visualised in our axonometric figures as usual in their projective affine model in E^3.

Key words: homogeneous spaces, geodesic, translation curve, translation sphere