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ABSTRACTS

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Contents

Plenary lectures

On Some Structures of Finite Geometry and Their Geometries

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A symmetric $2-(v,k,\lambda)$ design **D** is a set **P** of v points together with a collection **B** of v k-subsets of **P** called blocks (lines) such that every two points are contained in exactly λ blocks. The difference n of the parameters k and λ, (i.e. $k-\lambda=n$) is the order of symmetric design **D**.

The parameters v,k,λ of a symmetric design **D** have to satisfy some necessary conditions like the Bruck-Ryser-Chowla Theorem, and the design can be constructed with the help of some chosen groups of collineations operating on it.

On the other hand, a symmetric $2-(v,k,\lambda)$ design **D** can be described in terms of its incidence matrix, being a square v by v (0,1)-matrix with a constant row and column sum equal to k, and a constant scalar product of pairs of rows equal to λ.

We will speak about some symmetric designs, especially the cases where the order n is a square.

Key words: symmetric design, automorphism group, tactical decomposition

MSC 2000: 05B05

Material Models of Surfaces by 3D Printing

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Material models of mathematical objects can be very useful as a tool to improve the understanding of Geometry and to popularize Mathematics.

3D printers are computer-driven machines that allow to produce real copies of 3D models that can be held in one's hands and carefully observed in every detail.

We defined a procedure to realize models of parametrized surfaces by means of 3D printing techniques: from the parametrization of a surface we obtain a thickened 3D model of it, solve possible problems due to self-intersections of the representation, get a suitable triangular mesh and send it as input file to a 3D printer.

A basic collection of remarkable models (pseudosphere, Costa's surface, Boy surface, various versions of the Klein bottle) has been realized as a set of test cases.

Key words: Surfaces, models of surfaces, 3D printing.

Minimal Surfaces in 3-Space: From Minkowski to Euclidean via Isotropic

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We consider Minimal surfaces in 3-space with respect to geometries ruled by a scalar product

$$^{\sigma}\langle X, Y\rangle := x_1y_1 + x_2y_2 + \sigma x_3y_3, \quad \sigma \in \mathbb{R}.$$

So in case of $\sigma = -1$, $\sigma = 1$ and $\sigma = 0$ we have Minkowski geometry, Euclidean geometry and the geometry of the (simple) isotropic space respectively.

Clearly looking for minimal surfaces with certain properties in case of different values of σ we get results which are affinly equivalent when the sign of σ is the same. Nevertheless it turns out that many constructions and results are analogous even in case of different sign of σ. Minimal surfaces in Isotropic space ($\sigma = 0$) which are well known to be power surfaces have a special position because of parallel normals.

After some basic structural observations we will discuss the analogues to the surfaces of Bonnet and Thomsen, especially geometric properties of the Enneper surfaces.

The Thomsen surfaces are the minimal surfaces which are affinminimal at the same time. It turns out that in all three cases these surfaces can be characterized by the spherical images of their osculating lines (orthogonal system of circles). The results in the euclidean and the isotropic case are due to Thomsen and Strubecker respectively.

The surfaces of Bonnet are the minimal surfaces with plane lines of curvature. We show that as in Euclidean and Isotropic geometry the Bonnet surfaces are exactly the adjoined surfaces to the Thomsen surfaces.

In the Euclidean case the Enneper surface is well known to be enveloped by the symmetry planes of two focal parabolas (G.Darboux). We show that is true also in Minkowski geometry. By a result of K.Strubecker in the isotropic case the Enneper surface is enveloped by the osculating planes of the orbits when applying a certain Euclidean screw motion to a suitable hyperbolic paraboloid. The idea behind is a general result of Strubecker concerning power surfaces. Another geometric construction of the Enneper surface due to H.Jonas (1953) has an analog in Minkowski geometry too.

Some Classes of Idempotent Medial Quasigroups in Geometry

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It will be given a survey of geometrical notions and properties of some classes of idempotent medial quasigroups, especially the IMC-quasigroups, Vakarelov's quasigroups, hexagonal quasigroups, quadratical quasigroups, rot-quasigroups, GS-quasigroups, G_2-quasigroups and plastic quasigroups. Furthermore, the geomatrical concepts of parallelograms, midpoints and some other concepts can be defined in these classes of idempotent medial quasigroups.

Contributed talks

Distance From Point to Line and From Point to Plane Or If We Had More Time

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Here we deal with different approaches in solving the following two tasks *distance from a point to a line* and *distance from a point to a plane*.

The first approach supposes the ability to find the derivative of a function of one and several variables, and a necessary and sufficient condition for extrema. Another approach is based on one lemma and one needs to be acquainted with the equation of a line in parametric form. A third way supposes the knowledge of a scalar product and a definition of a projection of a vector to the axis determined by the vector. Tasks without using formulas can be solved constructively, using methods of descriptive geometry in space. As the fifth way the article by Antun Ivankovic from the journal *MIŠ* (35) can be used, where the task is solved analytically, etc.

The students of the fourth grade of high school should be able to use each of the mentioned approaches to determine/solve *the distance from a point to a line*. This applies also to the students in the first year of any technical faculty in determing/solving *the distance from a point to a plane*.

However, is it really so? From experience, we can say, no. Why is this the case?

The reason is that some approaches to solving problems are tightly connected to the areas of mathematics which are dealt with in certain classes in high school, or some courses at the faculties. For example, the first approach of solving problems at the faculties is connected to the course of Mathematical Analysis; the second and third to the course of Analytical Geometry and Linear Algebra; the fourth one to Descriptive Geometry, etc.

If we had more time and will, and with more knowledge, perhaps we could use more of a "vertical/horizontal" approach to problems. This way, not a lot of material could be covered within the class, but it could save time for some other themes/courses. With proper distribution of materials, the teaching program would be covered.

And most importantly, we think that a goal which is often missed could be reached i.e., instead of using prepared recipes the students will be able to understand the principles of solving the problems/tasks and with a specific problem/task they would apply a broader idea. That way they should be able to use previously learned materials. Different approaches to solving the mentioned tasks could be developed and defended by students within projects, thus improving the dynamics of the teaching class. That would be a motivation for their individual and team work.

Singular Points on Surfaces $\mathcal{P}_4^6$

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The pedal surfaces $\mathcal{P}_4^6$ with respect to any pole P and one special 1st order 4th class congruence $\mathcal{C}_4^1$ are 6th order surfaces with a quadruple line. The highest singularity which these surfaces can possess is a quintuple point. The quintuple points on $\mathcal{P}_4^6$ are classified according to the type of their 5th order tangent cone – six types are obtained.

Points on the quadruple line of $\mathcal{P}_4^6$ are quadriplanar. We distinguish nine types of these points and six of them are the types of pinch-points.

Except the singular points on a quadruple line surface $\mathcal{P}_4^6$ has at least one real double point iff pole P lies on one 5th degree ruled surface (see Fig. 1) and exactly two real double points iff it lies on one parabola.

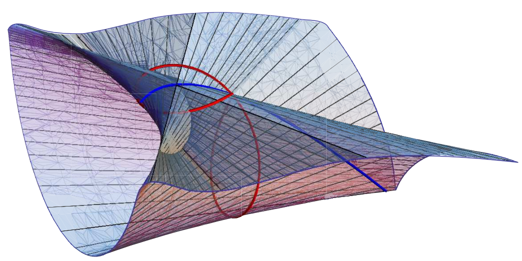

Figure 1

Key words: quintuple point, quadruple point, tangent cone

MSC 2000: 14J17, 51N20

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Učiti na drugačiji način - upotreba interaktivnih i multimedijskih obrazovnih materijala

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Osuvremenjivanje te podizanje kvalitete nastave u visokoškolskom obrazovanju stvara obavezu, kako Sveučilištu, fakultetima (profesorima), tako i studentima da aktivno sudjeluju u navedenom procesu, prihvaćajući izazove.

Potpisivanjem *Bolonjske deklaracije* 2001. godine Hrvatska je dala do znanja da želi unaprijediti obrazovanje te postati konkurentna u Europskom sustavu visokog obrazovanja.

Budući da s vremenom dolazi do promjena u obrazovnom sustavu, tako i sada, uz nužno osiguranje tradicionalnih vrijednosti, potrebno je unošenje noviteta u razvoju nastavnih programa, ispravno koristeći mogućnosti koje nudi današnja tehnologija.

Nezaobilazna u procesu obrazovanja postaje upotreba ICT-a (informacijsko-komunikacijske tehnologije), temeljena na mikroeletronici, kao dodatna kompetencija za učenje i poučavanje i kao ključni element uspješnog učenja i osnove za stvaranje i primjenu znanja.

Upravo je uz pomoć ICT-a zaživio pojam *e-učenja* u hrvatskom obrazovnom sustavu koji uvođenjem i aktivnom primjenom na Sveučilištu u Zagrebu obećava već spomenuto podizanje kvalitete i dostupnost obrazovanju.

Kako bi profesori i studenti aktivno surađivali sa svrhom postizanja zadanih obrazovnih ciljeva, potrebno je pri tome intenzivno koristiti informacijsku i komunikacijsku tehnologiju za stvaranje prilagodljivog virtualnog okruženja u kojem se razvijaju i koriste multimedijski interaktivni obrazovni materijali.

Stoga u ovom radu i koristimo programe *Mathematica* i *webMathematica* (veza između web servera i programa *Mathematica*) uz pomoć kojih provodimo interaktivne numeričke proračune i matematičke vizualizacije, te kao multimedijsku komponentu koristimo program koji pripada skupini *Video screen recorders*.

Pri izboru tema usmjerili smo se na nastavu na našem matičnom Geodetskom fakultetu. Kao prvo iznosimo teorijsku podlogu određenog problema a zatim, ukoliko je moguće, slijedi njegova vizualizacija. Na kraju video zapisom dajemo sažeti prikaz problema, od njegovog postavljanja do konačnog rješanja.

Smatramo da je ovaj drugačiji oblik učenja vrlo pristupačan, kako nastavnicima tako i studentima, jer uz činjenicu da nastavni sadržaj ostaje sačuvan, nadograđuje se vizualizacijama i interaktivnošću koji olakšavaju i unapređuju nastavu te je čine dostupnom i tako doprinose kvaliteti i širenju obrazovanja kojem svi težimo.

The Golden Section in Paintings - is the Cat there?

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The use and role of the Golden Section in works of art has been hotly discussed over the course of several decades. In this work we statistically examine if the Golden Section is present in a representative sample of European paintings. Our findings support the conclusion that the Golden Section is there, but not quite in the place where it was expected.

Images and Mathematics

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Mathematical and geometrical concepts can be used to create beautiful images or even art. Such images must not necessarily be an end in themselves because they can help to convey mathematical concepts to a not so mathematically oriented audience or can be used to gain new insights to otherwise intractable problems.

In this talk we will present some images which all either represent some mathematical concept in a visual way or which were created with the help of mathematics and/or geometry. The examples – to name a few – reach from 4D hypercubes, archimedean honeycombs, flow visualizations and geodetic lines on curved surfaces to Mandelbrot orbits, the Riemann Zeta function and the traveling salesman problem. The mathematical and geometrical background of these images is discussed and the necessary algorithms, to create them on a computer, are presented.

Key words: Mathematical Images, Educational Illustrations, Visual Proofs.

MSC 2000: 00A06

Experiences in Applying Bologna Process to Descriptive Geometry Teaching

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This paper presents the teaching results referring to the success of the first-year students enrolled in the course Descriptive Geometry and Perspective after the Bologna reform has been carried out for the last three years at the Faculty of Architecture. The results are compared with results obtained at the School of Design, Faculty of Architecture.The results of the student survey are also available thus allowing teachers to get a feedback about their work.

Key words: Bologna process, teaching Descriptive geometry, e-learning, students" pole, School of Design.

MSC 2000: 97B40, 97C90

Generalized Fibonacci Numbers and Metallic Shofars

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It is well-known that Fibonacci numbers $F_n = F_{n-1} + F_{n-2}$, $(F_0 = 0, F_1 = 1)$ are closely related to the Golden Mean: according to Binet's formulae

$$F_{2k+1} = \frac{\tau^{2k+1} + \tau^{-(2k+1)}}{\sqrt{5}} \qquad F_{2k} = \frac{\tau^{2k} - \tau^{-2k}}{\sqrt{5}}$$

where $\tau = (1+\sqrt{5})/2$. In a recent paper [1] a nice surface is established through the points (x, F_x) in the following way. Consider the continuous functions through the odd and even Fibonacci values respectively as

$$cF(x) = \frac{\tau^x + \tau^{-x}}{\sqrt{5}} \qquad sF(x) = \frac{\tau^x - \tau^{-x}}{\sqrt{5}}.$$

The surface passing through these curves is defined as

$$\left(y - \frac{\tau^x}{\sqrt{5}}\right)^2 + z^2 = \left(\frac{\tau^{-x}}{\sqrt{5}}\right)^2$$

and called the Golden Shofar, due to its horn-like shape and close relation to the Golden Mean.

In this presentation we generalize this surfaces according to the generalizations of the Fibonacci sequence [2] and the Golden Mean [3]. Fig. 1. shows the Golden Shofar and some of its generalizations, which can be called Metallic Shofars.

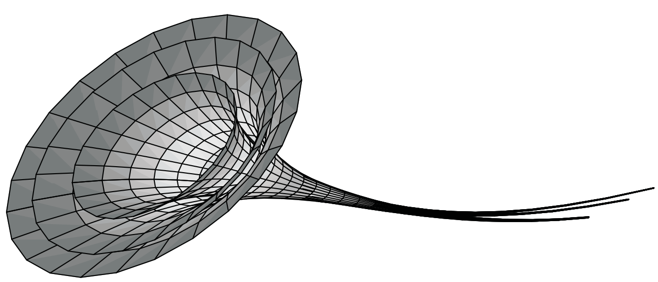

Figure 1: Generalized Shofars

Key words: Fibonacci numbers, Golden Mean, Metallic Means

MSC 2000: 11B39, 53A05, 51M09

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Animation of Isooptic Curves

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In this paper we investigate isooptic curves associated to second order curves and some special less known curves. The code for generation of these curves is also given.

Key words: animation, isooptic curves

MSC 2000: 53A04

Circular Quartics in Isotropic Plane Obtained as Pedal Curves of Conics

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The problem will be studied on the projective model of an isotropic plane with the absolute figure (f, F), F incident with f.

A curve in the isotropic plane is circular if it passes through the absolute point F. Its degree of circularity is defined as the number of its intersection points with the absolute line f falling into the absolute point F.

The pedal curve k_N of a given curve k with respect to a conic q is the locus of the foot of the perpendicular to the tangent of the curve k from the pole of the tangent with respect to the conic q.

There are four types of the pedal transformation. The conditions that the generating conic has to fulfill in order to obtain a circular quartic of certain type will be determined for each type by using the synthetic (constructive) method. It will be shown that it is possible to get only 2-, 3- and 4-circular quartics by pedal transformation.

Key words: circular curve, quartic, isotropic plane, pedal curve

MSC 2000: 51M15

Some Planimetric Tasks in Isotropic Plane

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For constructive geometry of isotropic plane we chose a projective model given the absolute figure (f, F) consisting of a straight line f and its finite point F. Defining the measure of a line segment, its midpoint and the measure of an angle in this model we may constructively solve different planimetric tasks in the isotropic plane. We show some known theorems constructively and the most interesting is the construction of the center of 2nd grade curve given by its five tangents.

Key words: isotropic plane, isotropic line segment measure, isotropic angle measure, conic center.

MSC 2000: 51N05, 51N99, 51M15

Some Interesting Properties of Heptagonal Triangle

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The triangle with the angles $\frac{\pi}{7}, \frac{2\pi}{7}, \frac{4\pi}{7}$ is called a heptagonal triangle (according to Bankoff and Garfunkel). This triangle has many interesting properties and some of them will be presented in this talk. The relationship of the heptagonal triangle with its successive orthic triangles will be considered. The meaning of the heptagonal triangle, in the known Dixmier–Kahane–Nicholas inequality, will also be investigated.

Division of the Wheel Windows of Churches of St. Savoir in Dubrovnik and Sta. Maria's in Zadar and Portals of Church in Osor

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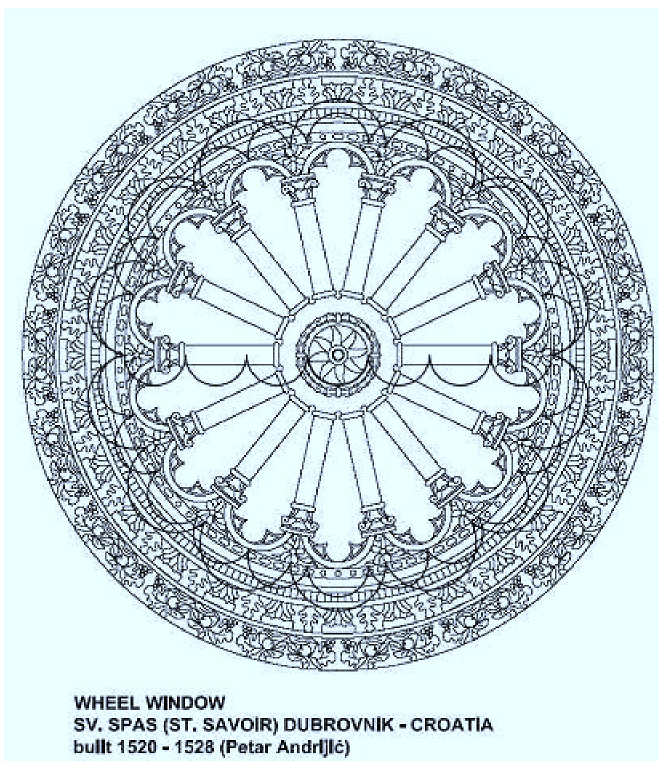
Relationship between circumference and diameter of circle is one of the most historical geometric problems. This problem was solved on theoretical level by introduction of irrational number π. But the solution wasn't practical. That's why early in the history of architecture already come across experiments, that they would express this relationship with the help of relationship between little whole numbers. Probably the oldest written fact concerning this is in the Bible that speaks about dimensions of the cleaning water reservoir in the Jerusalem's temple. More accurate approximations were discovered and used afterward. Article is presenting three cases of divisions of wheel windows (rosettes) in Croatia namely Dubrovnik, Zadar and of the web portal of church in Osor. All three examples show very high level of geometric knowledge.

Key words: wheel windows, number π, geometry, architecture



Construction of Some Triangles in the Beltrami-Klein Model of the Hyperbolic Plane

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Constructions in the real projective plane can be interpreted as constructions in the Beltrami-Klein model of hyperbolic geometry and vice versa. Let c_1 and c_2 be two conics, each of them having double contact to another conic c. Hence, we have a common pair of pole and polar with respect to c_1 and c_2. So we can construct the common points and tangents of c_1 and c_2 in the real projective plane (embedded into the complex projective plane) with compass and ruler. This result can be interpreted as the construction of a hyperbolic triangle with three given side lengths or with three given angle measures in the Beltrami-Klein model. In the sense of complex projective geometry the constructions are perfectly dual. However, in the real part of the complex plane the two siblings appear pretty different.

Key words: Hyperbolic Plane, Beltrami-Klein Model, Projective Geometry

MSC 2000: 51M10, 51M15

Cube Orbifolds and Manifolds with Various Metric Structure

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Joint work with István Prok

In earlier works (e.g. [2] and [3]) István Prok determined by computer all the face pairings of a combinatorial cube, up to cube isomorphisms, which can generate a combinatorial space tiling. Such generators to a face pair will be denoted by $\mathbf{g} : g^{-1} \mapsto g$ and $\mathbf{g^{-1}} : g \mapsto g^{-1}$ ($\mathbf{g}$, $\mathbf{g^{-1}}$ are in I; the cubes C and C^g have g as common face). Thus we get the fundamental group of the cube tiling. This group $G(I, n(e_i))$ is generated by the pairing I, as PL (piecewise linear) simplicial maps by the barycentric subdivision of the cube. In addition, free natural exponents can be chosen, *as free rotational order* $n(e_i)$, to each induced edge equivalence class ei. Thus, I. Prok enumerated, e.g., the 298 cube tilings of the Euclidean 3-space $\mathbb{E}^3$ under 130 crystallographic groups. The general polyhedron algorithm had been described in [1] by the speaker.

In this lecture we concentrate on hyperbolic cube tilings in Bolyai-Lobachevskian space $\mathbb{H}^3$ where the 12 cube edges fall either into 2 equivalence classes, 6-6 edges in each, or all the 12 edges are in one class of equivalence, induced by fixed point free face pairing generators, now, with trivial exponents $n(e_i) = 1$.

Thus we get hyperbolic cube manifolds: Either with a cusp to each of the *3 manifolds in the first case*, i.e. one vertex equivalence class at the absolute with a non-compact cube of finite volume where all the dihedral angles are 60°, so that $6 \times 60° = 360°$ to both edge classes. Or a vertex class lies out of the absolute, since each dihedral angle is equal to 30° (i.e. $12 \times 30° = 360°$). Thus we get *10 infinite cube manifolds in this second case.*

In the second case we can truncate the vertices by their polar planes, in addition, so that we obtain a space filler Archimedean solid with Schläfli symbol $\{8, 8, 3\}$, and we can attempt to get compact hyperbolic manifolds from these 10 cube manifolds by additional fixed point free pairing of the 8 triangle faces of above $\{8, 8, 3\}$ solids. Since this $\{8, 8, 3\}$ has rectangles at the 24 edges of the 8 truncating triangles, this additional triangle pairing has to induce 6 triangle edge classes, 4 edges in each class (so that $4 \times 90° = 360°$).

In this manner 1 above Archimedean solid provides 1 new compact hyperbolic manifold, another 1 provides 3 new ones. The *other* 8 $\{8, 8, 3\}$ solids serve orbifolds only, with fixed points. Our method promises further various new results, e.g. for *combinatorial octahedron.*

Key words: Polyhedral manifolds and orbifolds

MSC 2000: 57S30, 51M10, 51M20

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Quartics Deduced by Pencil of Polarities in $\mathbb{P}_2$

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Each pencil of conics defines one polar-system, as one correlative involutive transformation. The 4^{th} degree curves in the projective $\mathbb{P}_2$ space can be deduced as the result of that transformation, which transforms the common point Y into Y', conjugated with respect to all conics of the pencil. The conic of centres in the pencil form the locus of points that transform into the line at infinity. This quadratic transformation preserves the genus of a curve. Deduced quartic have singularities which depend on conic intersections with auto-polar triangle of the pencil.

Based on classification of conic pencils, from different starting conics we can deduce different classes of quartic curves.

Key words: curves of order four, polarity, pencil of conics, construction of curve

MSC 2000: 14H50

On the Logarithmic Mean in n Variables

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The logarithmic mean, defined for two positive real numbers by

$$L(x,y)=\begin{cases}\frac{y-x}{\ln y-\ln x} & , \quad \text{for } x\neq y\\ x & , \quad \text{for } x=y\end{cases}$$

doesn't give insights about possible generalizations to several variables.

However, if we write this mean in integral form, we get the idea for generalization using the standard notion of Euclidean simplex.

In this talk we consider two such generalizations given by A.O. Pittenger and E. Neuman. We also show some other ideas for generalizing the logarithmic mean on n variables using some different methods, and we show that these methods bring us again to the main two generalizations of Pittenger and Neuman.

At the end we also mention some further generalizations of such means.

Key words: logarithmic mean, Euclidean simplex.

MSC 2000: 26E60

Curves with Constant Geodesic Curvature with respect to Two Cylinders

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We study a curve $\vec{k}(s) \in C^3$ of the 3− dimensional Euclidean space E_3 lying on two cylinders Φ_1, Φ_2. Let $\vec{k}(s)$ be parametrized by its arc length s and have constant geodesic curvature κ_1, κ_2 on two different cylinders Φ_1, Φ_2. The two cylinders Φ_1, Φ_2 are defined by $\vec{k}(s)$ and their generator directions $\vec{e}_1, \vec{e}_2$. In order to determine such curves we study the spherical image $\vec{k}'(s)$.

We demonstrate that constant geodesic curvature of $\vec{k}(s)$ with respect to the cylinders Φ_1, Φ_2 is characterized by some 'angular condition': For the angles $u_i(s) := \angle(\vec{k}'(s), \vec{e}_i)$ we have $u_i(s) = A_i - s\kappa_i$ with real constants A_1, A_2. This result yields properties of the spherical image $\vec{k}'(s)$ and the original curve $\vec{k}(s)$ as well. Some interesting examples are being presented in the lecture.

Key words: Constant geodesic curvature, curves on cylinders

MSC 2000: 53A04, 53A05

Osculating Circles of Conics in the Isotropic Plane

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In the isotropic plane there are seven types of conics relative to their position to the absolute figure (F, f). The paper shows how to construct an osculating circle in any point of a real conic. In the Euclidean plane every conic has at least one real vertex, i.e.a point where hyperosculating circle exists. In the isotropic plane only real ellipses and hyperbolas of the 2nd type have two real vertices. For those types of conics are the hyperosculating circles constructed as perspective collinear images of a given conic with appropiately selected elements of collineation.

Key words: isotropic plane, osculating circle, vertex of a conic, perspective collineation.

MSC 2000: 51N05, 51N99, 51M15

Oskulacijske kružnice konika u izotropnoj ravnini

Konike u izotropnoj ravnini razvrstane su u sedam tipova s obzirom na njihov položaj prema apsolutnoj figuri (F, f). Za konstruktivnu geometriju odabran je model izotropne ravnine s apsolutnom figurom u konačnosti. Na takvom se modelu konstruira oskulacijska krunica u općoj točki realne konike. Pokazuje se, da od svih konika u izotropnoj ravnini po dva realna tjemena imaju samo realne elipse i hiperbole druge vrste. Uz pogodno odabrane elemente perspektivne kolineacije u tjemenima se takvih tipova konika konstruiraju hiperoskulacijske kružnice.

Ključne riječi: izotropna ravnina, oskulacijska kružnica, tjeme konike, perspektivna kolineacija.

On Simplices with Prescribed Areas of 2-facets in Euclidean Spaces

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Due to the Cayley-Menger-formula the volume of any simplex can be expressed in terms of its edge lengths. In dimension four the edges are dual to the 2-faces. Hence there might be a 'dual' formula expressing the simplex volume by the areas of its ten 2-faces. The aim is to discuss the uniqueness of a simplex with given 2-areas. In 3-space each tetrahedron admits a 2-parametric area-preserving flex. In Euclidean 4-space the question for an equi-areal counterpart to a given simplex leads to a rather complex algebraic problem.

Geometric Problems in Manufacturing

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Computer-aided manufacturing applications, such as NC machining require efficient computations of specific curves on complex sculptured surfaces. Many methods have been developed for tool path generation of milling tools on free-form surfaces. The used numerical methods depend on the mathematical description of surfaces, which can be analytic or discrete. The use of triangulated surface format for discrete representation of a CAD model has been widely accepted in industry.

We have developed intersection, offset and curvature estimating algorithms for triangular meshes. On the base of those methods we present a strategy for planning the moving direction of a cutting tool on a triangulated surface such that the machining process is efficient, and the error is kept within a given tolerance.

MSC 2000: 68U05, 68U07, 65L50

Lattices in the Nil and Sol Spaces

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The **Nil** and **Sol** geometries are two of eight homogeneous Thurston 3-geometries

$$\mathbf{E}^3, \mathbf{S}^3, \mathbf{H}^3, \mathbf{S}^2 \times \mathbf{R}, \mathbf{H}^2 \times \mathbf{R}, \widetilde{\mathbf{SL}_2\mathbf{R}}, \mathbf{Nil}, \mathbf{Sol}.$$

In [3] we have determined the densest lattice-like translation ball packing to a type of **Nil** lattices. In [4] we have considered the densest lattice-like translation ball packing to the fundamental Sol lattices. The notions of lattices in **Sol** and **Nil** spaces are introduced by P. Scott in [2].

In this talk we investigate the lattice types in the **Nil** and **Sol** spaces, introduce the notions of the lattices and parallelepipeds. Moreover, in **Sol** geometry we study the relation between **Sol** lattices and lattices of the pseudoeuclidean (or Minkowskian) plane. We are going to use the affine model of the **Nil** and **Sol** spaces through affine-projective homogeneous coordinates [1] that gives us a way of investigating and visualizing homogeneous spaces.

Key words: Lattice, Non-Euclidean Geometry, Discrete Geometry

MSC 2000: 52B20, 53A35, 53A20, 05B40

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Further on Properties of a Non Tangential Quadrilateral

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More on properties of the non tangential quadrilateral $\mathcal{ABCD}$ in the isotropic plane is given in this talk. After a standard quadrilateral and its diagonal triangle are introduced, properties of the non tangential quadrilateral related to foci of quadrilaterals formed by two diagonals and some two sides of the non tangential quadrilateral $\mathcal{ABCD}$ are studied.

Key words: isotropic plane, non tangential quadrilateral, diagonal triangle

MSC 2000: 51N25

Geometric Aspects of Cartographic Projections

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Geodesy and Cartography are using in large extent geometric knowledge. In this paper we discuss application of geometric methods in mathematical cartography. We describe several types of cartographic projections, the criteria of choice of projection from the different aspects, like a shape of projected areas and their location on the reference surface. We formulate equations of cartographic projections, geometric properties of the image of latitude and longitude frame, we characterize distortion of length, area and angle. We show our calculations and graphic presentations in the Mathematica software environment.

Geometry in Science and Education at Slovak Universities

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The presentation gives overview about the pedagogical and scientific activities related to promotion of geometry and geometric methods at the universities in the Slovak Republic. Brief information about geometric subjects lectured at the technical universities in Slovakia are given, namely Slovak University of Technology in Bratislava, Technical University in Zvolen, University of Žilina, and Technical University in Košice. Some interesting achievements in the theoretical geometric research are mentioned, together with the main scientific orientation of the Slovak geometry at the Comenius University in Bratislava and Slovak University of Technology in Bratislava.

Regular and Semi Regular Solids Related to the 3-Dimensional Models of the Hypercube

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The lecturer has already experimented with the application of the groups of edges obtained from combinations of platonic solids and common parts of these on the occasion of 3-dimensional modelling of the hypercube. The hull of these models correspond in several cases with the surface of some Archimedean solids.

The present proves shows that by parallel sliding of edges in case of three Archimedean solids we obtain special 3-dimensional models of the 6-, 9- and 15-dimensional cubes inside these solids. The edges of the platonic and Archimedean solids (excepting the snub cube and the snub dodecahedron) join the grids of edges in these models. Thus the adequate sides of these solids can be fitted to each other and the suitable combinations of the solids can result space-filling arrangements similar to the spatial mosaics based on the 3-dimensional models of the hypercube and parts of these described by the lecturer before. All this interpret the already known tessellations too partially in other sense. It can be expected additional outcome from the investigations of the Catalan solids, dual pairs of the Archimedean solids.

The sections of the tessellations yield unlimited possibilities to produce plane-tiling. The moved sectional plane result in series of tiling transforming into each other. These can be assembled in animations.

Key words: polyhedra, constructive geometry, 3-dimensional models of the hypercube, plane-tiling, space-filling

MSC 2000: 51M20, 68U07

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