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ABSTRACTS

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Contents

Plenary lectures

Mechanisms and their visualization

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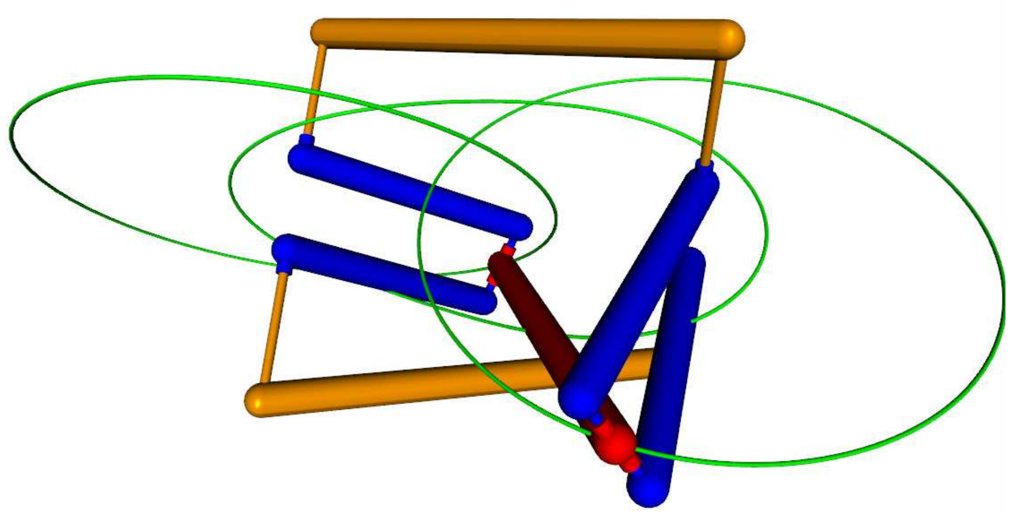
A mechanism is an assembly of rigid bodies whose movability is restricted by a set of joints like revolute, prismatic, universal and spherical joints. An industrial robot is a typical example. Some other prominent types are the Bennett-mechanism (see Figure below), the Schatz-mechanism or the mechanisms of Goldberg, each of them being movable despite having a theoretical degree of freedom (dof) ≤ 0.

It is a major issue in kinematics and robotics to visualize and/or animate mechanisms on a computer, for instance to be capable of predicting and estimating their behavior in reality. In my presentation I discuss how this can be done in different environments, like CAD-packages for mechanical engineers, CAS-systems like Maple or standard 3D-environments like VRML (**V**irtual **R**eality **M**odeling **L**anguage.)

I will also outline the geometric and kinematic background of certain mechanisms, emphasizing the fact that the knowledge of this background is inevitable for obtaining appealing results.

Key words: kinematics, robotics, mechanism, VRML, visualization, animation



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An interpolating trigonometric spline curve

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Nowadays, in CAD and CAGD the standard description form of curves is

$$\mathbf{c}(t) = \sum_{i=0}^{n} F_i(t)\,\mathbf{d}_i,\ \ t \in [a,b] \subset \mathbb{R},$$

where $\mathbf{d}_i \in \mathbb{R}^\delta$ $(\delta \geq 2)$ are called control points, and $F_i : [a,b] \to \mathbb{R}$ are sufficiently smooth combining functions. In general, such a curve does not pass through the control points, it just approximates the shape of the control polygon. However, if the combining functions are linearly independent, one can always solve the following interpolation problem. Given the sequence of data points $\left\{\mathbf{p}_i \in \mathbb{R}^\delta\right\}_{i=0}^{n}$ along with associated strictly monotone parameter values $\{t_i \in [a,b]\}_{i=0}^{n}$, find those control points for which equalities $\mathbf{c}(t_i) = \mathbf{p}_i$, $(i = 0, 1, \ldots, n)$ are satisfied. This problem can be reduced to the solution of a system of linear equations. A drawback of this method is that the interpolating curve will be globally controlled by data points, i.e., the displacement of any $\mathbf{p}_i$ results in the change of the shape of the whole curve, even if the combining functions are splines.

This disadvantage can be overcome by applying a local interpolation scheme, that was initiated by Overhauser [1]. He considered a sequence of data points $\mathbf{p}_i$ with associated parameter values t_i and constructed a C^1 cubic interpolating spline curve the arcs of which are linearly blended parabolas, i.e. each arc is a convex combination of two parabolic arcs. This blending concept has various generalizations. Their advantage is that there is no need for solving linear systems, the interpolating curve can be computed locally, therefore the spline curve is locally modifiable. A drawback is that, in general, we lose the control point representation when the arcs to be blended are specified by control points.

Our proposed method combines the advantages of the two options described above, i.e., it produces the interpolating spline locally and provides directly the control points of the interpolating arcs with arbitrary order of continuity at joints. We demonstrate the method for trigonometric curves, namely we consider the normalized B-basis of the vector space $\mathcal{F}_{2n}^{\alpha} = \operatorname{span}\{\cos(kt), \sin(kt) : t \in [0,\alpha]\}_{k=0}^{n}$ of trigonometric polynomials of degree at most n, define a proper blending function using which we can produce the arcs of a C^n interpolating spline curve of degree $n+1$, by blending two first degree trigonometric arcs, where α is a global shape parameter. Polynomial interpolation can be obtained as the special case $\alpha \to 0$.

Key words: Interpolation, trigonometric spline, blending, shape parameters

MSC 2010: 65D17

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Conics and quadrics for ever

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Though the history of conics started already about 300 BC, there is still a vivid interest and research on conics and quadrics. This can be seen from recent publications on Poncelet porisms, on billiard or on thread constructions of quadrics. The author himself is also involved in plans for a new book on conics and quadrics jointly written with Georg Glaeser and Boris Odehnal.

The lecture will pick out some of the raisins from the theory of these remarkable and decorative geometric objects. By comparing the new methods with classical results, which mainly originate from the 19th century, we will notice that both have their advantages. And it is always surprising how variations of classical results give still rise to new insights.

Finding principal axes and moments of inertia

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Inertia moment measurements on an irregular rigid body, *e.g.*, a crankshaft, about a unique axis has been treated in a recent machine dynamics text [1] and reference to inertia matrix and ellipsoid is found therein. Similarly, elementary engineering mechanics books [2] apply the inertia matrix in the study of rotational dynamics. No experimental method to measure the matrix elements has been found. Such a method, along with its underlying geometry, is proposed herein. It uses six optimally distributed rotation axes. The test mass is related to the inertia ellipsoid via the idea of an orthogonal triple mass dipole to be skew mounted in a six axis jig, each of its axes to be presented for rotation about a common axis. Diagonalization of the experimentally determined matrix establishes principal moments of inertia and their axes. One sees geometrically that what is commonly called "inertia ellipsoid" is actually a complex conic coplanar with the absolute conic $x^2 + y^2 + z^2 = 0$. Diagonals on the four points of intersection of these curves are lines on two pencils of parallel planes that section the ellipsoid in circles. A conclusion about how inertia matrix elements may, via the addition of radial support bearing force sensors, be reduced to a three rotation axis experiment is put forth.

Key words: inertia, moment, matrix, ellipsoid, measurement

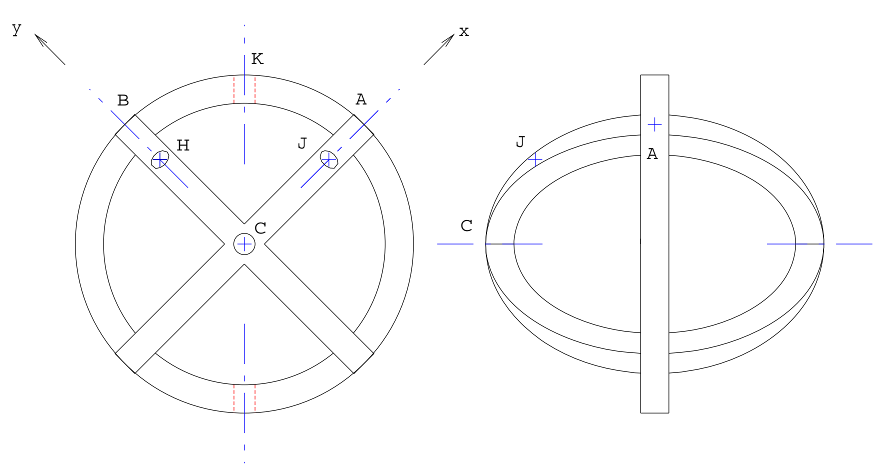

Figure 1: Jig showing six optimal rotation axes

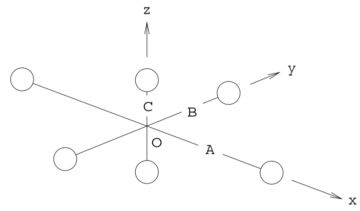

Figure 2: Six-mass triple dipole

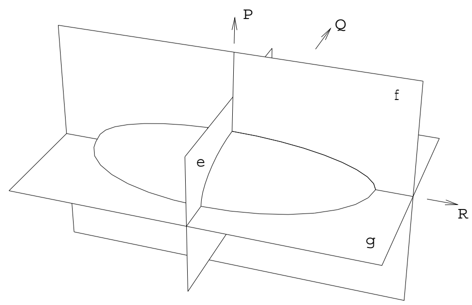

Figure 3: Inertia ellipsoid with principal planes and axes

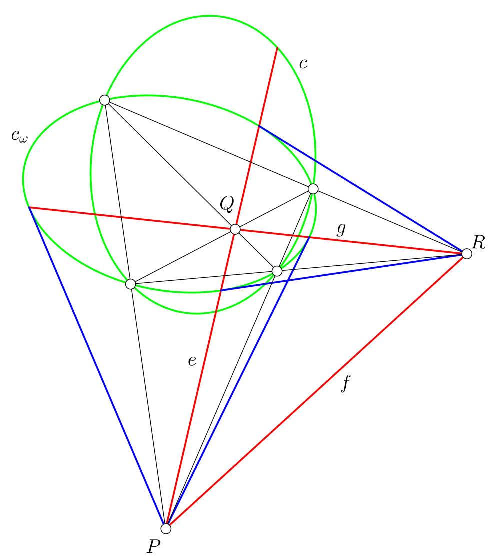

Figure 4: Pair of conics sharing a polar triangle

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Contributed talks

Virtual learning environments and online assessment of spatial abilities

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Visual skills development and assessment of spatial abilities have particular importance in different branches of science: they are measured by tests in psychology, and play significant role in didactics of geometry and visual studies.

Our objective was to develop virtual learning and controlling environments that fits the demands of training in engineering, and by which students acquire reliable spatial skills that can be used in practice.

In the first part of the presentation we introduce new tools for geometry education, which were developed at Ybl Miklós Faculty. These are based on digital course books [1,2], which help the individual understanding almost all basic chapters of Descriptive Geometry. Since our students have manifested poor performance in reading and comprehending mathematical texts, we had to involve digital contents instead. The web pages we created include 14 tasks, each: on the left you can find a blank problem sheet with the task; on the right there are videos with the solution. The concept of the digital coursebook was based upon the functionalities of a professional CAD system in the visualization of spatial problems.

In the second part of the talk we introduce a new, online spatial abilities assessment method. This research is a part of the *Developing Diagnostic Assessment* project at the University of Szeged, Center for Research on Learning and Instruction. We investigated four clusters of spatial skills: spatial positions, relations, directions - comprehension of structures of 3D shapes - spatial reconstructions - spatial transformations and manipulations. For all these groups we prepared multimodal tasks using both static and dynamic software [3]. We will present comparison of the two techniques and introduce the Electronic Diagnostic Assessment System (eDIA).

Key words: online testing, spatial abilities, virtual learning environment

MSC 2010: 51N05

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From the geometry of quasi-hyperbolic plane

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In this presetation the Euclidean model of the extended projective quasi-hyperbolic plane will be presented. This plane is dual to the pseudo-Euclidean plane. The absolute figure of the quasi-hyperbolic plane is a triple (j_1, j_2, F) where F is a real point and j_1, j_2 are pair of real lines through F. Basic elements and constructions will be shown, as well as duals of some well known theorems from triangle geometry and theory of conics.

Key words: quasi-hyperbolic plane (qh-plane), perpendicular points, qh-triangle lines, qh-conics, osculating circles

MSC 2010: 51A051, 51M15

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Education vs profession in architectural domain locating geometry in the digital delay

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The dialogue between "Scientific" and "Professional" worlds will be one of the most challenging tasks in the future, with refer to Geometry and Graphics.

About Geometry, it seems that it has never found a good location in the architectural curricula. In the analogue era, it was considered too much "scientific" and "difficult, while in the digital era it is often considered as something "historical", even "too humanistic", because of the big amount of digital modelers nowadays available. As a result, both Projective and Descriptive Geometry are disappearing from the architectural curricula, and the students only tend to use Geometry by trying and retrying through the menu of options displayed on their softwares, instead of going back to the disciplinary foundations.

Quite paradoxically, the young generations are ending up "using" geometry without "knowing" it, what is, on my opinion, so paradoxical as using a word processor without knowing a language.

How could this happen?

At least in my Country, one of the reasons may be that the architectural schools have neglected for a long time digital graphics, that have been developing as a 'stray' phenomenon, growing up outside the walls of the university. Thus, in the hand of the students the digital modelers and the books of geometry remained and still remain separated. Neither things seem to be better on the side of Graphics, where the "wild" growth, both of the number of softwares and of the number of self-taught users, has very quickly canceled and sometimes confused the basic rules of the graphic codes.

Therefore, also the professional world and the public administration have been affected by that, with bad consequences, over the times, on the quality of our built environment.

At this point the question is: catastrophe or opportunity?

It only will depend on us. As we know, the "digital era" has really been revolutionary and, as every revolution, it has implied disruption, similarly to what happened at the end of the Roman Empire, at the end of the Middle Age, at the end of the Renaissance, and between the XVIII and the XIX Centuries. But as we also know, in all these cases the re-birth only depended on "how" the reconstruction has been leading up.

In other words, while on the one hand the contribution of (our) University has been quite poor, on the other hand the amount of experiences accumulated in the real world is very rich, full of hybridizations among codes and languages from which is possible to take inspiration.

Of course there will be a lot of work to do, but we can be optimistic because, as Michel Foucault acutely predicted some decades ago, the disciplinary boundaries are becoming more and more "permeable" thanks to the new technologies and networks.

But most of all, nowadays the "knowledge management" is more and more "visual", what is matter of Geometry and Graphics, and of course and at last, it is in charge on us, researchers and educators.

Key words: geometry education, graphic education, education and profession

MSC 2010: 97Gxx

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Partridge number of a triangle is at least 4

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The p-number (Partridge number) of a shape S is the smallest value of $n > 1$ such, that 1 copy of S, 2 copies of S scaled by a factor of 2, up to n copies of S scaled by a factor of n, can be packed without overlap inside a copy of S scaled by a factor of $\frac{n(n+1)}{2}$. The problem of p-number stems from the identity $1^3 + 2^3 + ... + n^3 = (1 + 2 + ... + n)^2$ that tells us that 1 square of side 1, 2 squares of side 2, 3 squares of side 3, up to n squares of side n have the same total area as a square of side $(1 + 2 + ... + n)$.

It is known that the p-number of a square is 8, the p-number of an equilateral triangle is 9, the p-number of right-angled isosceles triangle is 8 and the p-number of a triangle with inner angles $(30°, 60°, 90°)$ is 4. From there we know, that p-number of the rectangle is at most 8, but there are known the p-numbers of some special rectangles and the p-numbers of some special trapezes too.

In this paper I show that the p-number of a general triangle is at least 4.

Key words: p-number, triangle, covering, packing, tiling

MSC 2010: 52C20, 05B45

Loci of centers of mass of plane figures cut off by a rotating line

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Let L be a plane figure with a piecewise smooth boundary and p a line in the same plane. Let p rotate around a fixed point $P \in p$. We investigate various properties of the trajectories of centers of mass of two segments of L cut by p as it rotates.

The sophisticated geometry of compound eyes and some applications in bionics

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Nature has evolved two major kinds of highly efficient eyes in the animal kingdom: Lens eyes and compound eyes. The latter are characteristic for insects (on land) and crustacean (in water). In general, the individual "ommatidia" (facets) of compound eyes are basically hexagonal conic frustums. Some families of crustacean (crayfish and some shrimps), however, have almost perfect quadratic prisms as ommatidia which turn out to be of special interest for applications in bionics.

We investigate both kinds of ommatidia and try to explain the effect of so-called "pseudo pupils". This effect already allows to classify the different types of compound eyes (Fig. 1, Fig. 2, [1]).

Surprisingly, crayfish-eyes work like sophisticated optical lenses [3, 5], although they use a completely different method: Light rays are not refracted but reflected several times at the reflecting faces of each ommatidium. Amazingly, a more or less large amount of all passing light rays is thus bundled on a concentric convex (basically spherical) shape (Fig. 2).

The method of focusing light by means of large series of reflecting quadratic prisms is currently being introduced to telescopes to exploit tiny amounts of rays (e.g., X-rays) from outer space [2], or even for new types of cameras [4].

Key words: multiple reflection, mirror optics, bionics

MSC 2010: 51N05, 51P05, 92B05

Figure 1: "Pseudo pupils" of insect eyes – Left: Superposition eye, middle: apposition eye, right: geometric model of compound eyes with hexagonal ommatidia.

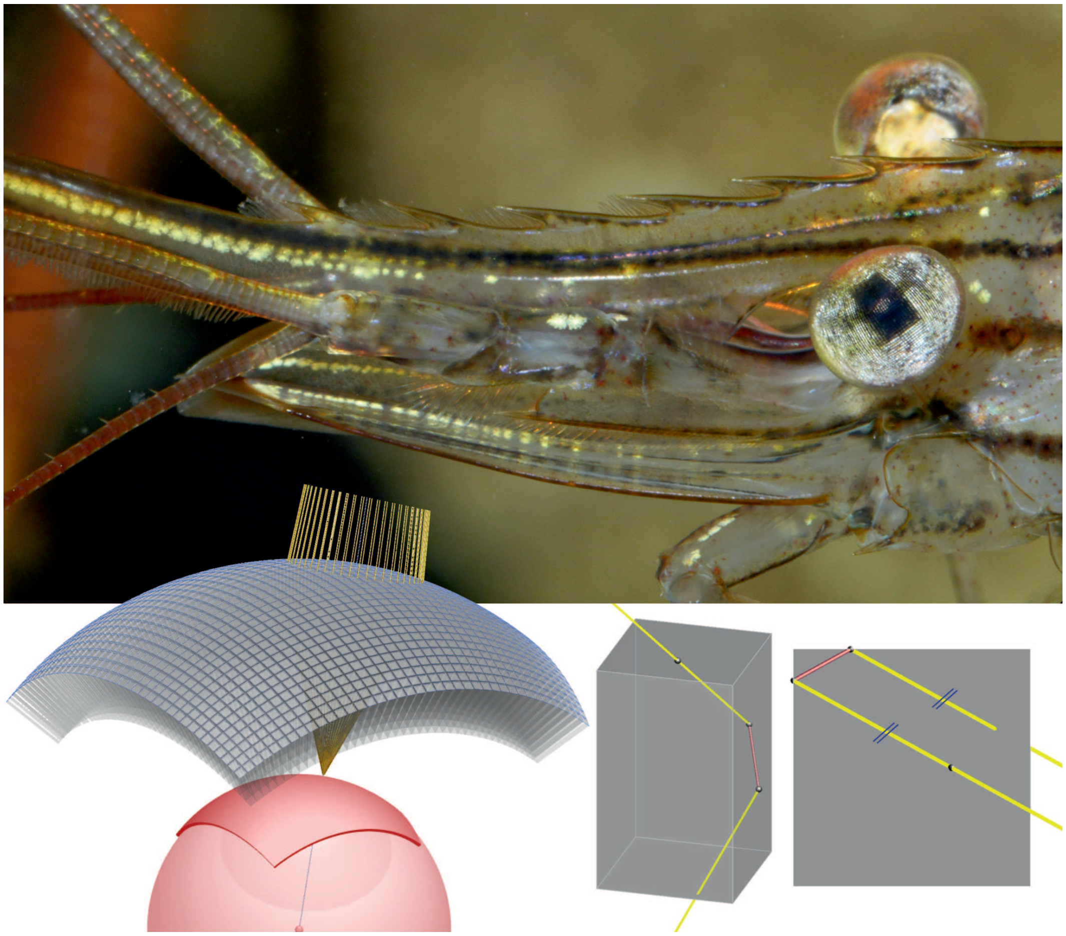

Figure 2: Multiple reflection in a quadratic prism. Certain rays are reflected twice as if they were reflected by a single virtual plane. If hundreds or thousands of such prisms are located on a sphere, incoming parallel light rays of this type are bundled on a concentric sphere. Such eyes are known with certain types of crustacean.

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Results of Project *Introducing 3D Modeling into Geometry Education at Technical Colleges*

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During the year 2012 nine members of the Croatian Society for Geometry and Graphics worked on the project *Introducing 3D Modeling into Geometry Education at Technical Colleges* supported by the Fund for the Development of the University of Zagreb. Four faculties were included: Faculty of Architecture, Faculty of Civil Engineering, Faculty of Geodesy and Faculty of Mining, Geology and Petroleum Engineering.

The goals of the project were strengthening the professional and scientific cooperation among the faculties in the area of technical sciences, developing teaching methodology for 3D computer modeling to enhance the geometry courses, and harmonization of educational material standards and their further implementation in the e-learning systems of the included faculties.

The focus of the project was creating a basic repository of educational materials related to common teaching topics and those customized to profiles of each faculty. The special emphasis was given to the materials connected to 3D computer modeling.

Until the academic year 2012/2013 *Descriptive geometry* as a course at the faculties of the University of Zagreb was mostly taught in the classical way by using rulers and compasses. Since this year the *Rhinoceros* program has also been included in the instruction of the aforementioned course at the Faculty of Civil Engineering and Faculty of Mining, Geology and Petroleum Engineering.

Here we will present some parts of the educational materials included in the repository.

Key words: geometry education, e-learning, 3D modeling, *Rhinoceros*

MSC 2010: 97G80, 97U50

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Computer graphics and descriptive geometry

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It is now known that the computer has changed the way of architectural representation. A lot of operations that the software makes are related to the theories of representation of the orthogonal projections and central projections. The process of the transformation of axonometric in perspective and vice versa is the subject of the article seeks to explain all operations to be made through the descriptive geometry.

Key words: Double central projections, axonometric transformation, perspective

MSC 2010: 51N05

Modeling some ruled surfaces using *Rhino* and *Grasshopper*

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Non-developable ruled surfaces (scrolls) can be constructed as a system of transversals of three space curves. Let the surface be given by three curves c_1, c_2, c_3 which are of order n_1, n_2, n_3 respectively. We denote with n_{ij} the number of common points of c_i and c_j, $i, j = 1, 2, 3$ and $i \neq j$. Then the order of the surface equals $2n_1n_2n_3 - (n_{23}n_1 + n_{13}n_2 + n_{12}n_3)$.

There are two distinct types of ruled surfaces of order 2: hyperboloid of one sheet and hyperbolic paraboloid. Surfaces of order 3 have three distinct types, and of order four there are 12 different types, due to R. Sturm.

We show how some ruled surfaces are constructed using *Rhinoceros* and *Grasshopper*.

Key words: ruled surface, *Rhinoceros 5.0*, *Grasshopper*

MSC 2010: 51M15

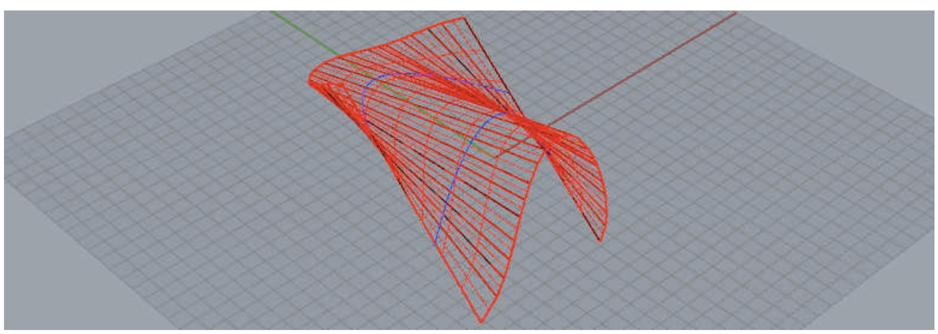

Figure 1: 4th degree conoid with two parabolas as directing lines.

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An improved method to construct intersection curves of skinning surfaces

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Joining of given geometric objects is an important part of surface modeling since in this way more complicated objects can be constructed than the starting forms. Therefore there is a persistent demand from both designers and users for sophisticated methods which provide more freedom throughout the designing process.

We developed an efficient algorithm [2] for joining skinning surfaces based on [1]. The algorithm gives visually much more satisfactory results than the presently available techniques.

We have improved our method and in this presentation we demonstrate a technique with which we can get better results at the construction of the intersection curve of the connecting surfaces.

Key words: skinning, spheres, interpolation, joining surfaces, intersection

MSC 2010: 65D17

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The generalization of Szabó's Theorem for rectangular cuboids with an example application

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Szabó's Theorem [1] provides a way to decide when the reference system of a central axonometric mapping is the central projection of a three dimensional cube. In most cases we have pictures or photos in which we can see only the image of a rectangular cuboid instead of a cube.

In this presentation we provide the criterion of when the central axonometry of a rectangular cuboid is the central projection of a rectangular cuboid. Then we demonstrate the new theorem's use in practice through an example application.

Key words: central projection, central axonometry, image processing, 3D reconstruction

MSC 2010: 51N05, 68U10

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Bonnet surfaces and harmonic evolutes of surfaces in the Minkowski 3-space

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A surface S in the Euclidean three-dimensional $\mathbf{R}^3$ space is called a Bonnet surface if it admits a non-trivial 1-parameter family of isometric deformations which preserve its mean curvature. Bonnet proved that any constant mean curvature surface which is not totally umbilical is such a surface. The result has been extended also to constant mean curvature surfaces in real space forms (i.e. in the completely simply connected Riemannian 3-manifold $\mathbf{R}^3(c)$ of constant curvature c, see [5]) and to indefinite space forms $\mathbf{R}^3_1(c)$ ([3], [4]). In this presentation we will give an overview of the results on Bonnet surfaces and also describe another special type of surfaces, so called harmonic evolutes. The harmonic evolute of a surface S is the locus of points which are harmonic conjugates of a point $p \in S$ with respect to centers of curvature p_1, p_2 of S. These points are therefore centers of the so called harmonic spheres, that is, spheres tangent to a surface and whose centers are exactly harmonic conjugates to points of tangency with respect to centers of curvatures. Harmonic evolutes of surfaces in Euclidean space have been studied in [1]. In this presentation we will present results on harmonic surfaces in a special indefinite space form – in the Minkowski 3-space, as in [7]. Furthermore, we will investigate their connection to Bonnet surfaces.

Key words: Bonnet surface, harmonic evolute, focal set, Minkowski 3-space

MSC 2010: 53A35, 53B30

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Densest geodesic ball packings by some $\sim\mathbf{SL_2R}$ space groups

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In recent works of the authors and other colleagues (Croatian colleagues as well) some classical Euclidean topics have been raised in the 8 Thurston geometries: $\mathbf{E^3}$, $\mathbf{S^3}$, $\mathbf{H^3}$, $\mathbf{S^2}\times\mathbf{R}$, $\mathbf{H^2}\times\mathbf{R}$, $\sim\mathbf{SL_2R}$, **Nil** and **Sol**.

In this presentation we overview some concepts of the $\sim\mathbf{SL_2R}$ geometry in its projective spherical $PS^3(\mathbf{V}^4, \boldsymbol{V}_4, \sim)$ interpretation by the real vector space $\mathbf{V}^4$ and its dual $\boldsymbol{V}_4$ with the usual positive real multiplicative equivalence $\sim$. Thus, the translation subgroup, the infinitesimal arc-length-square, the geodesic lines, their spheres and balls can be defined and explicitly determined. The volume of a geodesic ball is explicit, no more elementary, but straightforward for Maple computations. We have also attractive pictures by the projective model, so in the Euclidean screen of computer.

As a new initiative, we examine the classical ball packing problem for the $\sim\mathbf{SL_2R}$ space. In particular we consider a space group $\mathbf{pq2_1}$, generated by a p-rotation $\mathbf{a}$ of angle $2\pi/p$ and a q-rotation $\mathbf{b}$, so that their product will be a half-screw $\mathbf{h}=\mathbf{ab}$ (symbolically $\mathbf{2_1}$). This group can be expressed explicitly in $\sim\mathbf{SL_2R}$ with the defining relation $\mathbf{abab}=\mathbf{baba}=:\tau$, a translation. Then we can consider a point K on the p-rotation axis and its orbit under the above group $\mathbf{pq2_1}$. Moreover, we take the maximal balls centered in the orbit points, to form a packing with disjoint interiors of any two balls. We take the so-called Dirichlet-Voronoi cell (D-V cell) D_K for any ball B_K, consisting of points, not further from the given ball center K than to the others from the K-orbit under $\mathbf{pq2_1}$. The ratio $\delta=\mathrm{Vol}(B_K)/\mathrm{Vol}(D_K)$ is called the *density* of the above ball packing, since it is well-defined and can be computed by computer, of course, for given integers p,q, where $3\leq p$, $2p/p-2<q$ in $\sim\mathbf{SL_2R}$.

Similarly defined ball packing densities can be compared for different Thurston geometries and for their various discrete groups. *This topic is under our investigations with colleagues (you are also invited!).*

In this presentation we report new very dense ball packings of $\sim\mathbf{SL_2R}$ with equal geodesic and so-called translation balls, respectively. We look for some p,q at the above group $\mathbf{pq2_1}$, where the packing density is close to the density upper bound 0.85326 for the hyperbolic space $\mathbf{H^3}$.

Key words: Thurston geometries, discrete group in $\sim\mathbf{SL_2R}$, density of ball packing

A fast algorithm for finding special isoptic curve of Bézier surfaces

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In computer graphics finding isoptics is a difficult task. For two dimensional curves there is exact definition: For a given curve, consider the locus of points from where the tangents to the curve meet at a fixed given angle. From this there is a method for finding isoptics for Bézier curves [1].

In three dimensions there is no exact definition of isoptics, but we have a numerical method for finding an isoptic curve for a given surface [2]. The main problem with this approach is the efficiency. In this algorithm for one isoptic point we must scan the whole surface. If we want to determine the points of the isoptic curve more accurately, it slows down the search, since we do not use the fact that for close points the corresponding tangents are also close. The new algorithm we present here takes this into account and its speed is increased.

Key words: isoptic curve, Bézier surface

MSC 2010: 65D18

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Spherical conchoids

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We adapt the well-known concept of the construction of a conchoid curve for conchoids on the Euclidean unit sphere. Thus, the analoga to the conchoids of lines in the Euclidean plane are the conchoids of a great circle on the Euclidean unit sphere. Conchoids of circles on the sphere are then obtained by chosing a circle (different from a great circle) for a directrix. At hand of the parametrization as well as the equations of spherical conchoids we describe the algebraic properties of spherical conchoids. Especially the principal views, *i.e.*, the orthogonal projections of these spherical curves onto three mutually orthogonal planes of symmetry, are described in detail.

Key words: Spherical curves, conchoids, algebraic curves, principal views

MSC 2010: 51N20, 14H99, 70B99

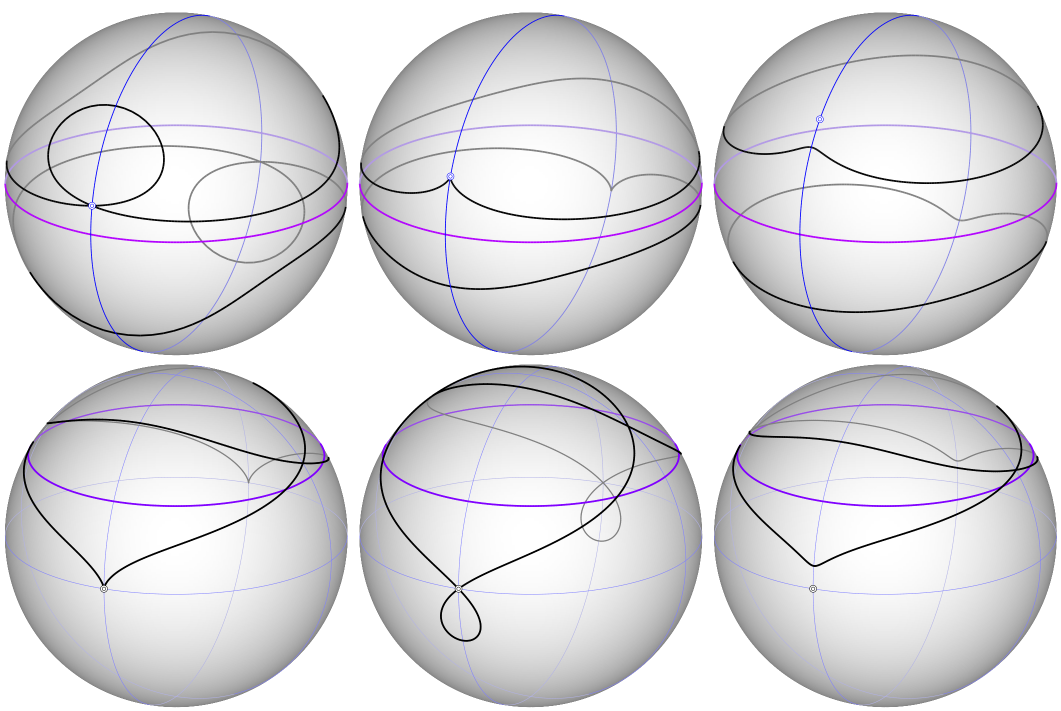

Figure 1: Top row: Spherical conchoids of a great circle. Bottom row: Spherical conchoids of a circle.

Modeling of higher order surfaces and project of e-course development

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This paper discusses the problem of accurate CAD modeling of ruled surfaces with three directing curves. Ruled quartics are classified by Sturm into twelve types [1]. Sturm's type VII ruled quartic surface is described in this paper. Its constructive generation is given in the 3D projective space: using directing cone of the surface and using new directing curves, appropriate for modeling techniques in any CAD-system.

Curvature analysis of the surface is given, using CAD software *Rhinoceros*. Paper can facilitate and encourage the introduction of new surfaces in design of architectural structures. It can improve CAD modeling in teaching engineering geometry.

Until year 1989 *Descriptive geometry* was taught classically at Civil Engineering Faculty in Rijeka. During the years the author worked on introducing 3D modeling in geometry courses at the faculty, without any support. Development of e-courses for geometry started in 2007. In year 2012/13 further development of e-course *Constructive geometry* was supported by University of Rijeka, within the project of e-course development. Educational materials were expanded with those made in *Rhinoceros*. Students are free to use any of the three software packages (*AutoCAD*, *Rhinoceros* and *DesignCAD*) that are suggested to them within e-course. Furthermore, links to trial versions of software are given in e-course.

Key words: modeling of ruled quartics, e-learning, 3D modeling, CAD

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On transition curves used in road design

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The paper deals with the area of *road design*, with special emphasis on curves used as *transition curves*. Specifically, these are the *clothoid*, the *lemniscate* and the *cubic parabola*. The transition curve as a ground element of the road indicates the gradual transition of the curvature, from that of the straight line to the curvature of the circle.

This paper links together areas of mathematics, geodesy and civil engineering. It demonstrates criteria that a particular curve should fulfill in order to become a transition curve. In addition, the paper gives the mathematical origin of each curve, its affiliation to a particular family of curves and the emergence of a concrete subgroup, which is applied as a transition curve.

Application. *Transportation* indicates displacement or, more precisely, the spatial movement of people, goods and information. Transportation means and transportation organization are needed in order for the transportation to function properly. Transportation means are qualified according to the medium in which the transportation is taking place. This can be on land, water or in air. Land transportation requires special infrastructure, and among others, it encompasses road and railroad transportation. Designing a road is a very complicated procedure, which is affected by various factors. One of the most important factors is the type of the road – whether a road or a railroad is in question. Particular parts of the process are similar, but in certain parts there are big differences. For example, when designing railroads different transition curves are used than when designing roads. With railroads the cubic parabola is most commonly used, while with roads it is the clothoid and the less frequently used lemniscate.

Key words: road design, transition curves, clothoid, lemniscate, cubic parabola

2-wire planar positioning of the 4-bar coupler curve via the kinematic mapping

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A particular direct kinematics problem that pertains to cable driven manipulators is resolved using a formulation based on the planar kinematic mapping of Blaschke and Grünwald. This problem has received considerable attention in engineering research using mechanical statics and optimization. Although the univariate polynomial of degree 12 has been revealed previous results fail to detect certain solutions that the geometric approach can handle. To complete the picture, the well known coupler curve of a planar 4-bar mechanism is derived with the kinematic mapping and advantages of a purely geometric image space formulation are discussed.

To illustrate the relation among the geometry of 2-wire manipulator kinematics, static equilibrium of concentrated forces in the plane and planar 4-bar linkage coupler curves, a set of six equilibrium poses of a coupler point G is shown in Fig. 1. The green arcs of circles centred at A and B are trajectories of 4-bar coupler pins, one moving on the red link anchored at A the other on the blue link anchored at B. If point G is taken as the coupler mass centre, at equilibrium it must remain in a line pencil wherein a vertical line on G must intersect lines of the cables AD and BE at a common point.

Key words: planar 4-bar, coupler curve, planar kinematic mapping

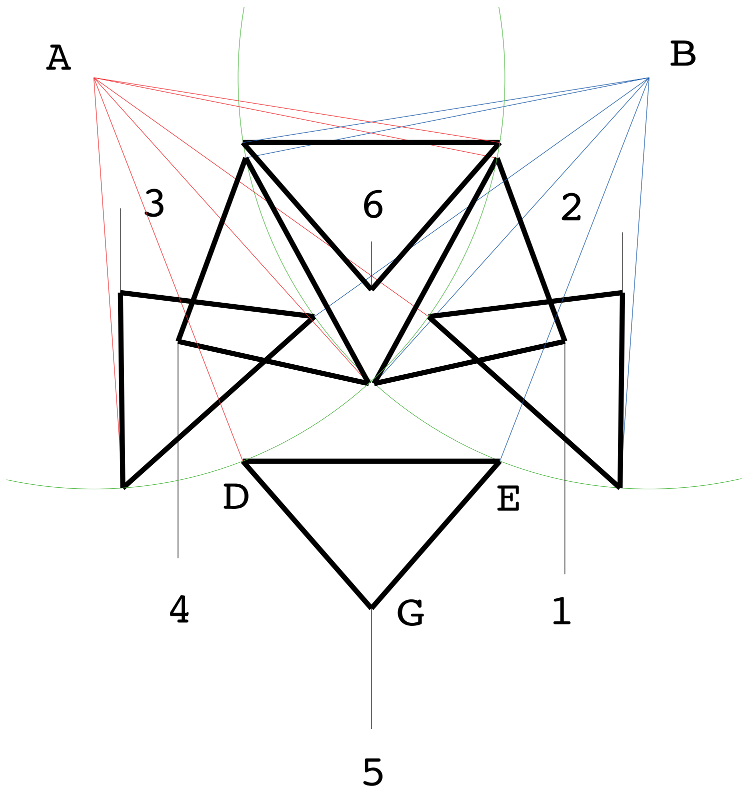

Figure 1: 6 possible solutions for the 2-wire 4-bar

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Approximation of B-spline curves or surfaces with third order continuity

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Our aim is to generate a B-spline curve for approximating given, separately created B-spline curves. The input curves and the resulting curve are represented by fourth degree, uniform B-splines. The input curves may join at their end points precisely or with gaps. The applied approximation technique is minimization of a target function expressed by squared differences in positions, first and second derivatives of the input and the resulting curves at their corresponding points. The variables in this function are the unknown control points of the approximating curve. In the case of uniform B-splines the coefficients of the basis functions are constant, therefore the minimization problem can be solved symbolically. In the solution the control points of the new approximating B-spline curve are expressed as linear combinations of the input control points. This method is basically different from the stitching method shown in [1] using interpolation and fairing.

We investigate the approximation error in dependence of the target function and the number of the considered control points in the region of a merging place. We apply the best satisfactory scheme to merge B-spline curves or B-spline surfaces. We show and analyze several solutions.

Key words: B-spline curves and surfaces, approximation

MSC 2010: 65D17, 65D07, 68U05

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On hypersphere packings in the 5-dimensional hyperbolic space

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The smallest three hyperbolic compact arithmetic 5-orbifolds can be derived from two compact Coxeter polytops which are combinatorially simplicial prisms (or complete orthoschemes of degree $d = 1$) in the five dimensional hyperbolic space $\mathbf{H}^5$. The corresponding hyperbolic tilings are generated by reflections through their delimiting hyperplanes that involve studying the related densest hyperball (hypersphere) packings with congruent hyperballs.

The analogous problem was discussed in [1] and [2] in the hyperbolic spaces $\mathbf{H}^n$ $(n = 3, 4)$. In this talk we extend this procedure to determine the optimal hyperball packings to the above 5-dimensional prism tilings. We compute their metric data and the densities of their optimal hyperball packings, moreover, we formulate a conjecture for the candidate of the densest hyperball packings in the 5-dimensional hyperbolic space $\mathbf{H}^5$.

Key words: n-dimensional hyperbolic space, hypersphere, packings

MSC 2010: 52C17, 52C22

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The experience of teaching in English at the Faculty of Architecture, University of Zagreb

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Geometry in Architecture has been for years an elective course within the programme of the Department of Mathematics, Descriptive Geometry and Perspective at the Faculty of Architecture, University of Zagreb. Since the academic year 2008/2009 it has been an elective course within the Master degree programme of Architecture and Urban Planning.
In academic year 2012/2013 we have registered for the *Application for the course in foreign language* announced by International Relations Office of University of Zagreb. We have been approved to present this elective course in English. In this talk we present our teaching experience gained during the course.

Key words: geometry in architecture, teaching in English

MSC 2010: 97B40

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Teaching experience on the course *Space Geometry with Computers* at Szent Istvan University

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At Ybl Faculty of Szent Istvan University, we renewed the course *Space Geometry with Computers* two years ago. This course is intended for students in Architecture, and this is an optional subject (only those students may sign on it who are interested in its topics).

The previous course programme was mainly an introduction to 3D solid modeling with AutoCAD, with an emphasys on such exercises where 3D navigation and 3D transformations are needed to construct the model.

Now the BSc programme in Architecture at Ybl Faculty includes a basic course on 2D and 3D AutoCAD. Therefore, it was possible to modify the course programme of the course *Space Geometry with Computers* by leaving out the basics of 3D AutoCAD. Instead, we are able to do the following:

- we introduce some more sophisticated features of AutoCAD to the students,
- we concentrate more on geometry and not on software, when working on 3D geometry modeling problems,
- there is enough time to work with other 3D geometry software as well, so the new course programme includes the use of three software: AutoCAD, Cabri 3D and GeoGebra 5 Beta.

The main course topics are the following:

- modeling convex polyhedra and star polyhedra,
- creating 3D curves with some applications,
- creating 3D surfaces,
- constructing elementary 3D objects from a given set of objects
- constructing complex 3D models,

with some flexibility of subtopics from term to term.

There is a special emphasys on describing the construction steps for the solution of a problem, in a step-by-step fashion, from the beginning to the end.

I report on my teaching experience and show some models/constructions of the students created in class and during homework.

Key words: mathematics education, space geometry, AutoCAD, Cabri 3D, GeoGebra

MSC 2010: 97D30

Minkowski triples of point sets

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Concept of Minkowski combinations of two point sets defined in [1] can be used for introducting a more general concept of Minkowski combinations of several point sets. Different combinations of set operations Minkowski sum and Minkowski product that are applied as modelling tool for generating of differentiable manifolds as described in [2], are used in their basic definitions. Three distinguished forms of multiple Minkowski set combinations of differentiable manifolds can be studied, while here wet restrict all considerations to smooth curves $\mathbb{E}^3$ as basic point sets, and definitions of different Minkowski triples of three curves, in particular.

Minkowski linear triple $A \oplus B \oplus C$, product triple $(A \otimes B) \otimes C$ and mixed triple $(A \oplus B) \otimes C$ of three smooth equally parameterized curves are smooth curves in the space. Considering different parameterisation of basic curves of 2 different real parameters t and u, generated differentiable manifolds are smooth surfaces in $\mathbb{E}^3$, while in the case of 3 different parameters t, u, and v solids in $\mathbb{E}^3$ are determined. Interesting geometric properties regarding the form of generated differentiable manifolds can be derived and demonstrated, based on the form of basic curves, their parameterization and mutual superposition, and positioning in the space.

Key words: Minkowski sum, Minkowski product, Minkowski set operators

MSC 2010: 51N25, 53A05

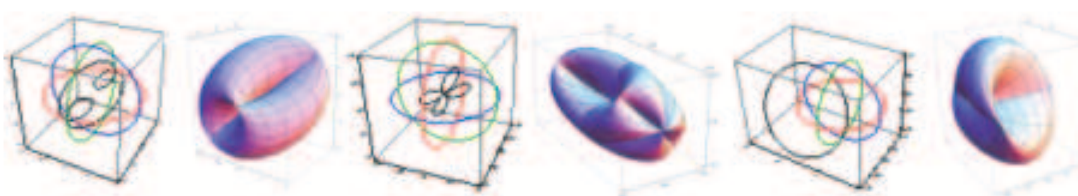

Figure 1: Minkowski linear triple (left), product triple (middle) and mixed triple (right) of three concentric circles in perpendicular planes

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New animations of symmetric patterns based on space-filling zonotopes

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The 3-dimensional framework (3-model) of any k-dimensional cube (k-cube) can be produced based on starting k edges arranged by rotational symmetry, whose Minkowski sum can be called zonotope. Combining $2 < j < k$ edges, we can build 3-models of j-cubes, as parts of a k-cube. The suitable combinations of these zonotope models can result in 3-dimensional space-filling mosaics. The investigated periodical tessellations (up to $k = 12$) always hold for the 3-model of the k-cube and necessary j-cubes derived from it. Such a space-filling mosaic can have a fractal structure as well, since we can replace it with a restructured one, built from multiplied solids. These are composed by addition of 3-models of k- and j-cubes and are similar to the original ones. The intersections of the mosaics with planes allow unlimited possibilities to produce periodical symmetric plane-tiling. Moving of intersection planes result in series of tessellations or grid-patterns transforming into each other which can be shown in varied animations.

Planar and spatial symmetry groups are the base of several works in different branches of art. Our symmetric models of the hypercube and the symmetrically arranged periodical tessellations offer several binding points to this field. The newer results can hopefully aid the correspondence between geometry, art and design.

Key words: constructive geometry, hypercube modeling, tessellation, fractal, design

MSC 2010: 52B10, 52B12, 52B15, 65D17

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