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ABSTRACTS

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Contents

Plenary lectures

3D homogeneous geometries and some special surfaces

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The Riemannian manifold (M, g) is called homogeneous if for any $x, y \in M$ there exists an isometry $\Phi : M \to M$ such that $y = \Phi(x)$.

In 1982 W. Thurston conjectured that any maximal, simply connected, three-dimensional geometry which admits a compact quotient is equivalent to one of the eight homogeneous geometries

$$E^3, S^3, H^3, S^2 \times \mathbb{R}, H^2 \times \mathbb{R}, \widetilde{SL(2,\mathbb{R})}, \mathrm{Nil}, \mathrm{Sol}.$$

In 2003 G. Perelman sketched a proof of the Thurston geometrization conjecture using Ricci flow with surgery.

In this lecture we will briefly describe each of the eight homogeneous geometries considering their basic properties. We will devote special attention to understanding of isometry groups of these geometries. We will discuss homogeneous geometries in light of Cayley's famous phrase "projective geometry is all geometry".

The lecture will also outline few important examples of surfaces in twisted product homogeneous geometries $(\widetilde{SL(2,\mathbb{R})}, \mathrm{Nil}, \mathrm{Sol})$ e.g. minimal surfaces, CMC surfaces, parallel surfaces etc.

Key words: homogeneous geometry, isometry group, minimal surface

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Structural and geometric concepts for architectural design processes

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The actual trends in architecture show more and more complex, irregular and seemingly "non-geometric" forms. It seems that the digital tools seduce the users to create anything possible. The more spectacular a building appears, the better and more innovative it is evaluated. Therefore we are asking for fundamentals for design processes, in order to escape from an arbitrary design and finding criteria for design processes.

When we look back in the history of architecture, we can find the background of geometric structures as important fundaments for design, for example in symmetry concepts or using transformations like perspective transformations. There is a tradition of using structural thinking for design disciplines referring to a mathematical-geometric basis. Mathematics had been developed in the 1930s as a general structural science, based on the notions of set, relation, and transformation, whereby an universal applicability got possible, thus also for designing.

There had been efforts in the structuralism and cybernetic school of thought to follow rational methods also in design processes. It was the initial point of structural thinking to look for the relations between elements of a system not for the elements itself, to find out the rules of their combinations in a system. Later the technological developments influenced the structural science. The first computer experiments in art and designing in the 1960s had strong relationships to this rational mathematical background. Aesthetics with these characteristics become principles of order and tools for structuring the world, therefore a fundament for designing.

With our digital tools today we have appropriate possibilities for referring to a rule-based parametric design, finding the relations between the various parameters for the design and representing the developed structure by suitable codes. This way gives the chance to create a dynamic architectural design process, working with the formulated relations and interactions between geometry, material, construction, and other components, also social-cultural matters, in multidisciplinary interrelation with an integrative role for geometry. This theoretical background for architectural design processes will be illustrated by examples and appropriate approaches in geometric-architectural education shown with some experiments of our students.

Results of the international Summer Schools in frame of the Erasmus Intensive Programme "Structural Architectures - Geometry, Code and Design" 2011 and 2012 will provide insight into such architectural design processes in international and interdisciplinary projects.

Key words: design processes, structural thinking, geometry, parametric design, aesthetics

MSC 2010: 51A05, 51M10, 51M15

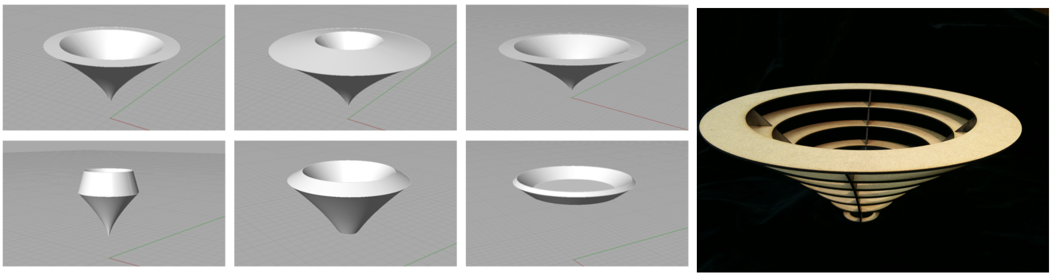

Figure 1: Variations of parameters for a rotation solid by the evolute of a parabola and physical model by student Hanno Katschinski, TU Kaiserslautern 2013.

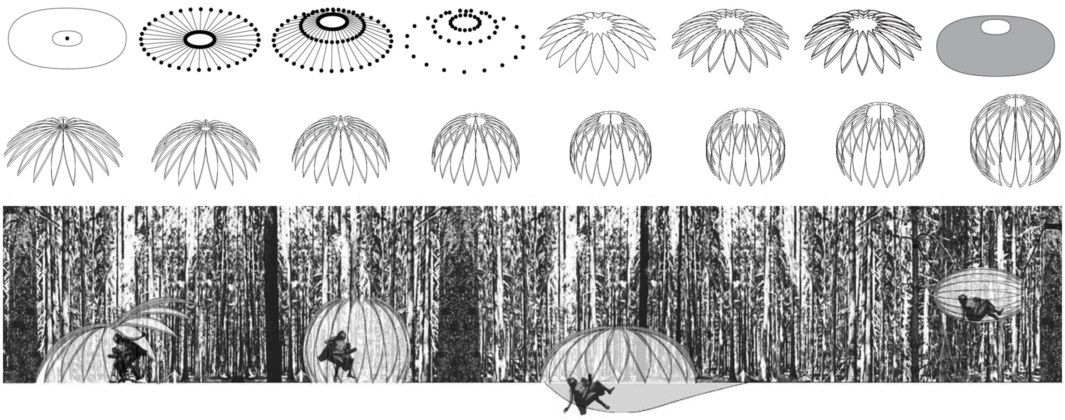

Figure 2: Erasmus Intensive Programme "Structural Architectures Geometry, Code and Design II", Kaiserslautern 2012. A hermit's cabin - design project example.

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Planar kinematics and overconstrained spatial mechanisms

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It is the aim of this presentation to give an overview of last years' development in the field of overconstrained mechanisms. Based on the famous Heureka mechanism and its generalisations (see H. Stachel [8]) the so-called Fulleroid mechanisms were studied in the last few years by G. Kiper et al. ([1], [2], [3]), K. Wohlhart ([9], [10], [11]) and O. Röschel ([4], [5], [6], [7]). All these mechanisms are highly overconstrained and consist of rigid bodies linked by $1R$- or spherical $2R$-joints (a $2R$-joint is called "spherical" if its two rotary axes intersect in a point).

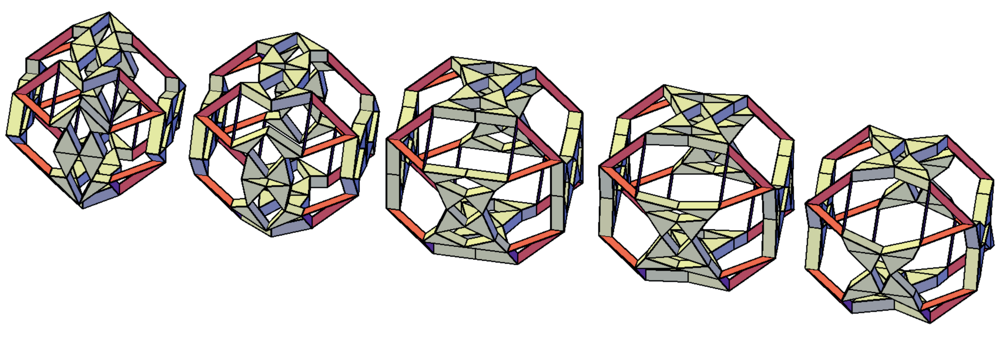
Many of these generalisations are based on observations on special planar mechanisms sharing an interesting property: Some coupler points (connected with the rigid parts of these planar mechanisms) describe polygons P homothetic to a prototype polygon all throughout the corresponding self-motion. We present a few examples of such planar mechanisms.

Then we demonstrate how these planar results can be embedded into the faces of a polyhedron. This procedure yields further and new examples and generalisations of Fulleroid linkages. We present a range of new examples of this new type of overconstrained mechanisms. The figure below displays some positions of the self-motion of an example based on this idea (see [7]).

According to the Chebyshev-Grübler-Kutzbach formula we compute the theoretical degree of freedom of these linkages. Our construction uses planar sub-mechanisms - this formula can be specified in order to deliver a more precise value in these specific cases. This yields a modified Chebyshev-Grübler-Kutzbach formula.

Key words: Kinematics, robotics, Fulleroid-mechanism, self-motion, generalisations of Fulleroid-mechanism, modified Chebyshev-Grübler-Kutzbach formula

MSC 2010: 53A17



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Three-bar curves, asymmetric propellers and idempotent medial quasigroups

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The three-bar curve (invented by James Watt) is generated by a linkwork of three jointed rods va and wb, the two legs each having a pivot point at fixed points v and w. The tracing point p is attached to the traversing bar ab by a rigid triangular plate abp. The resulting three-bar curve is a sextic with triple points at two imaginary absolute points of Euclidean plane and with three (real or imaginary) double points.

S. Roberts (1876) made the discovery that the same three-bar curve can be generated by three different linkages of the same type with directly similar triangular plates. The basis for this result is the following theorem.

Theorem 1 *If pab, dpc, efp are directly similar triangles and if $dpeu$, $fpav$, $bpcw$ are parallelograms, then the triangle uvw is directly similar to the given triangles.*

In the triple generation of S. Roberts, the pivot points u, v, w are fixed. So, there are three linkages $uefv$, $udcw$, $vabw$ with pivot points u, v; u, w; v, w and with triangular plates abp, cdp, efp, respectively.

L. Bankoff, P. Erdös and M. S. Klamkin (1973) proved the generalized asymmetric propeller theorem with the statement:

If pqr, pab, dqc, efr are directly similar triangles, then u', v', w', the midpoints of de, fa, bc, are the vertices of a triangle $u'v'w'$ directly similar to the first four triangles.

In Euclidean plane this statement is equivalent to the statement of the following theorem.

Theorem 2 *If pab, dqc, efr are directly similar triangles and if u', v', w', the midpoints of de, fa, bc, are the vertices of a triangle $u'v'w'$ directly similar to these three triangles, then all four triangles are directly similar to the triangle pqr.*

If $p = q = r$ in Theorem 2, and if the points u', v', w' are the midpoints of the segments pu, pv, pw in Theorem 1, then it is obvious that Theorem 2 implies Theorem1.

If the triangles pqr, pab, dqc, efr are equilateral, then we have the classical case of asymmetric propeller.

Here we shall study the algebraic background of Theorem 1 and Theorem 1. This background is an idempotent medial quasigroup, i.e. a quasigroup $(Q, \cdot)$ with the identities $aa = a$ and $ab \cdot cd = ac \cdot bd$.

Key words: three-bar curves, idempotent medial quasigroup, asymmetric propeller

MSC 2010: 14H50, 51N20, 20N05

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Contributed talks

Solving one maximization problem using a computer

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Malfatti's Problem (1803): Maximize the sum of areas for the packing of three circles into a given triangle without overlapping.

Complete solution was published by V. A. Zalgaller and V. A. Los [6] in 1994: the maximum is given by the greedy algorithm, i.e. one must inscribe in each step the largest possible circle.

Dual Malfatti's Problem (see [1]): Maximize the sum of areas for the packing of three triangles into a given circle without overlapping.

Greedy algorithm does not give the maximum for this problem - it is sufficient to consider the regular 5-gon.

Theorem. Let k be a unit circle. Let no two of the three triangles $T_1, T_2, T_3 \subset k$ have a common inner point. Let all vertices of all packed triangles lie on the given circle k. If the sum of areas of those triangles is maximal, then their union $\bigcup_{i=1}^{3} T_i$ is the regular 5-gon.

For solving this problem the authors of [1], [2], [3] used the assumption "if all vertices of all packed triangles lie on the given circle". Bezdek & Fodor [3]: "The dual Malfatti's problem for $n \geq 3$ triangles seems hopelessly difficult: even the case of three triangles is still unsolved."

In the presentation we show the outline of the computer aided proof of the Theorem without the annoying restriction "if all vertices of all packed triangles lie on the given circle".

Key words: Malfatti's problem, computer aided proof

MSC 2010: 52C17

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Surfing projective space homology in graphic education

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Talking about *homology* might appear a very old fashion matter nowadays, especially in relation to university education: sometimes freshmen coming from high school are already skilled in the basic use of computer graphics, while more advanced students are normally excellent self-directed learners in the field of digital modelling.

This fact could largely encourage graphic educators to retire, as *computer can do everything*.

Unfortunately things work a bit differently, since a digital modeller is not enough to make you an excellent architectural designer, as well as a word processor is not enough to make a talented writer. In fact, we know that many students are often incapable of solving simple unexpected spatial problems in spite of the most performing software, as well as many of them can not write a good paper in spite of the use of the latest release of a writing program.

What has that got to do with *homology*? Since our core business consists of relating three dimensional space and bi-dimensional visualizations, we cannot forget that the search for this relation has been since the beginning quite consubstantial with the invention/discovery of perspective, the early projective method, as confirmed by the well-known *Fig. XVa*, included in the Piero Della Francesca's treatise *De Prospectiva Pingendi* (about 1475), showing in the same image the foreshortened view of a tiled ground geometrically related to its true shape.

Over times, significant enhancements took place starting from this *incunabulum*, until the *homology* was found as the mother of the projective transformations, able to connect in one image various projections of the same configuration: in this way, the projective image overcame its bi-dimensional limits, allowing to *explore* the geometrical properties of the represented spaces in spite of the apparent distortions, not only by conic but also by cylindrical projections.

Moreover, besides the investigation of the true properties of the usual spatial structures, *homology* also deserves stimulating intellectual experiences when dealing with the *apparent properties* of less usual illusory configurations, like *anamorphoses* and *perspective-reliefs*, either theoretically described, i.e. by Jean Franois Niceron in the basic treatise *La Perspective Curieuse, ou magie artificielle des effets merveilleux de l'optique, de la catoptrique ed de la dioptrique* (Paris 1638), or physically realized, as in the perspective gallery of *Palazzo Spada* (1652) by Francesco Borromini in Rome.

However, the real power and the wide role of the *homology* in the modern era can also be easily understood looking at the history of technical drawing from Gaspard Monge on, while traces of its heritage still remain in the use of displaying several viewports during the digital modelling activity. But in this latter case, only the related views are shown, not the projective connections between them. Therefore, in spite of the great intuitive visual help, the deeper logical connections provided by projective geometry remain secretly kept into the algorithmic body of the machines, and students end up using them without seriously knowing and understanding them, that is to say, without really taking advantage of them to improve their spatial cognition.

Aiming to avoid either academic anachronism as well as ingenuous avantgardism, in order to renew graphic education after about twenty year from the wide diffusion of computer graphics, it seems that we still need to complete the integration between traditional knowledge and new technologies, that is to say, the interaction between their educational powers, which means much more than their mere “addition”.

Key words: projective geometry, descriptive geometry, digital graphics, graphic education, homology, anamorphosis, perspective-relief, visualization

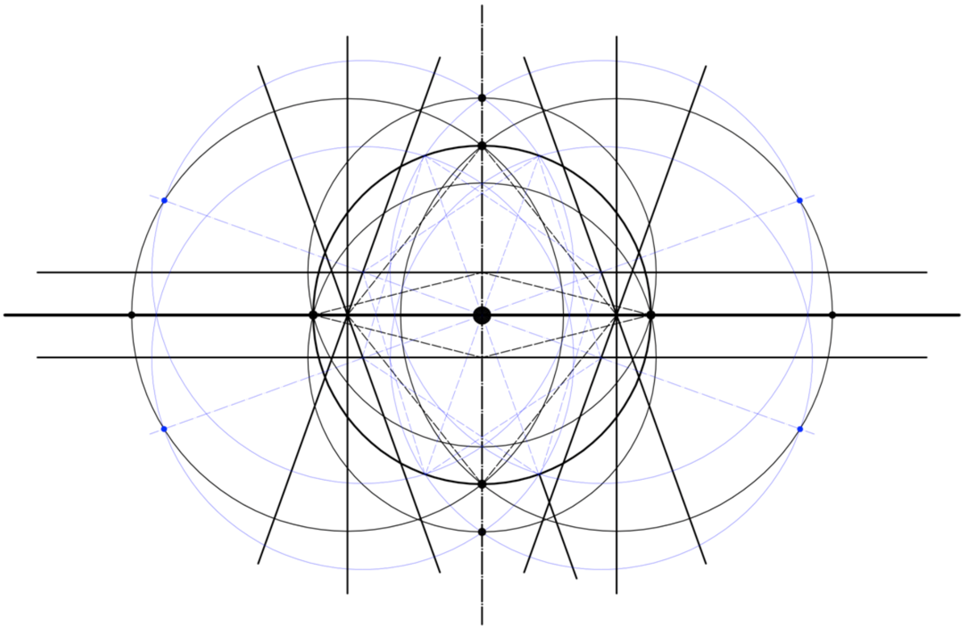

Figure 1: Space surfing in action in a central projection; “excursions” of the centre of homology on the picture plane according to the viewing distance (drawn by the author).

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Introduction to orthogonal axonometry via GeoGebra

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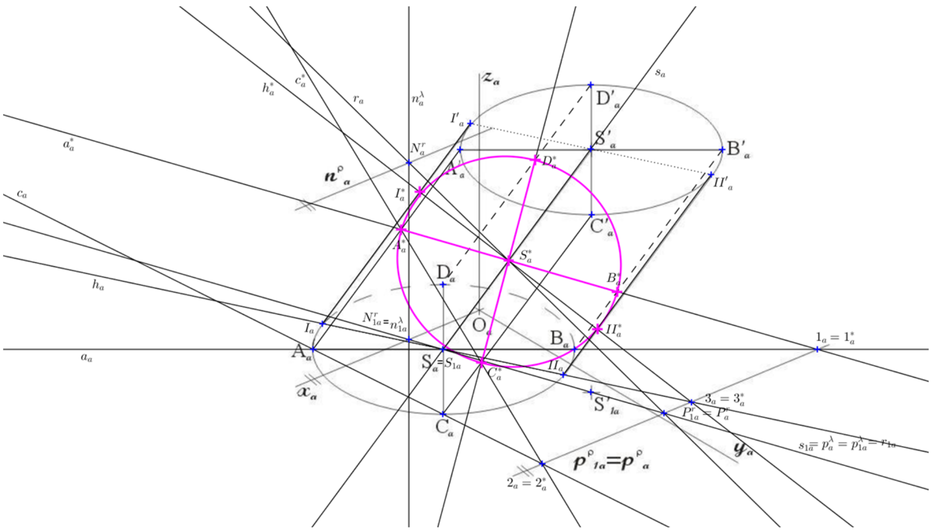
GeoGebra is free multi-platform dynamic mathematical software. It is usable for all levels of education from primary school to university study. GeoGebra joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package. Now a new version of GeoGebra with 3D view is developing. The utility of GeoGebra in teaching and learning descriptive geometry, especially orthogonal axonometry, is presented.

Key words: GeoGebra, dynamic software, education, geometry, axonometry

MSC 2010: 51N05, 97G80, 97U50



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Combinatorial denaturalization of the Kepler-Bouwkamp constant

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The Kepler-Bouwkamp constant is defined as the limit of radii of a sequence of concentric circles that are simultaneously inscribed to a regular n-gon and circumscribed to a regular $(n+1)$-gon for $n \geq 3$. The outermost circle, circumscribed to an equilateral triangle, has radius 1. We investigate what happens when the number of sides of regular polygons from the definition is given by a sequence different from the sequence of natural numbers.

Key words: Kepler-Bouwkamp constant, integer sequences, infinite product

MSC 2010: 51M04, 51M25, 40A20, 40A05

Geometry and kinematics of a convertible's rear side window

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The side windows of cars with retractable roofs (convertibles) have to be retractable themselves. This can be an issue for the rear side windows as the space between the door and the wheel arches is cramped. A simple motion (translation, rotation, screw motion) will most probably be inappropriate as the window pane S will have to assume a couple of prescribed positions. Additionally S has to move through the sealing slit along the daylight line d. These are two nasty constraints for the desired motion.

A solution to this motion interpolation problem will be presented. The input consists of the window surface S and a couple of prescribed positions $S_0, \dots, S_n$ of S. In a first step a suitable motion μ is being constructed and – as a by-product – a new surface S_d emerges which is pretty close to S and perfectly moves through the given sealing slit d. Additionally, we provide a set of blending surfaces S_β which are even closer to S but still more appropriate in terms of a smooth motion through d.

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Nature and numbers – But where are the numbers?

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Mathematicians tend to model natural shapes which can be put into a certain unambiguous category. The crystal structure of a diamond, for instance, is perfectly tetrahedral. However, this is only hard to prove photographically. On the other hand, there exist plenty of less perfect crystals, and insofar as their crystalline structure is visible to the naked eye, it is hardly perfect.

Nature is a pragmatic mechanism and accepts many supposedly imperfect solutions (see the attached figures), which emerge by means of selection or random chance, insofar as they improve an organism's reproductive success. If they are advantageous or more optimal, new forms are always ready to be accepted. This holds equally true for the development of life as for the emergence of shapes and patterns. The digital age has provided mathematicians with unprecedented possibilities, allowing them to visualize ideas which used to be unreachable. It is especially practical for the simulation of natural processes. Here, computer-aided mathematics allows for a free experimentation with parameters – a legitimate and indispensable method in order to achieve results more efficiently.

Solving a problem in this way might entail a comprehension of how the various mechanisms of nature proceed on their own and intertwine among each other. It is remarkable to recognize that many such processes are very simple, but only if considered locally. The complexity of the mechanism as a whole often escapes immediate explanation. This principle may lie at the heart of using mathematics to understand nature successfully. After all, infinitesimal calculus uses a similar approach, focusing on increasingly tiny localities in which certain properties hold true. Integration is then used to ascertain the big picture. In the modeling of dynamic processes, the smallest change can affect the whole result, and yet, nobody will deny that weather forecasts today are many times more accurate than a few decades ago. Still, there are so many parameters at play that certain degrees of inaccuracy are unavoidable.

Key words: evolution, bionics

MSC 2010: 51N05, 51P05, 92B05

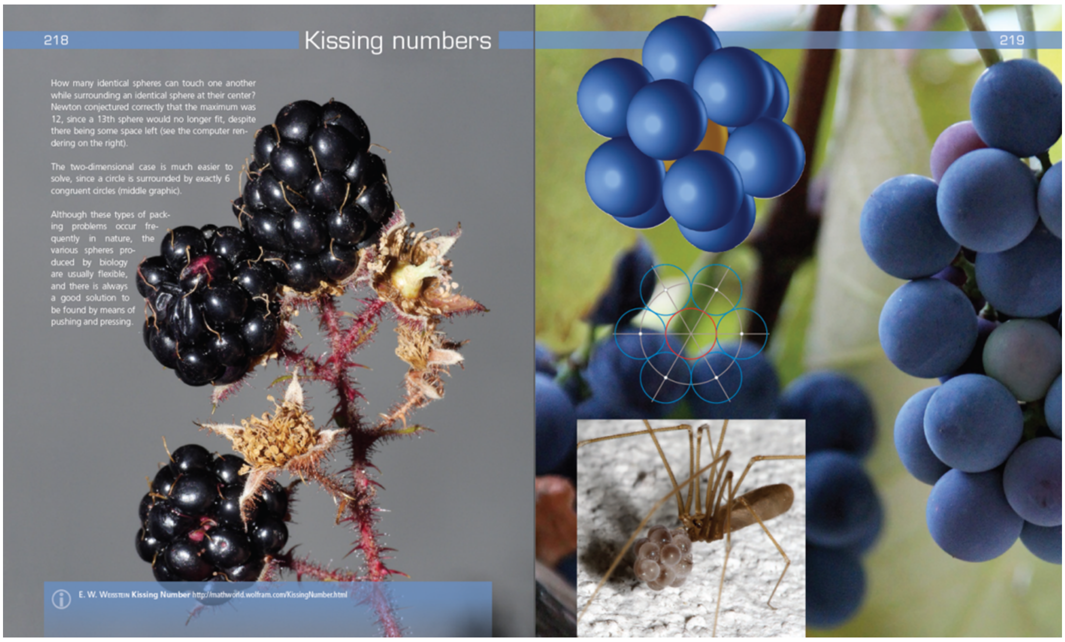

Figure 1: Packing of spheres ("kissing numbers")

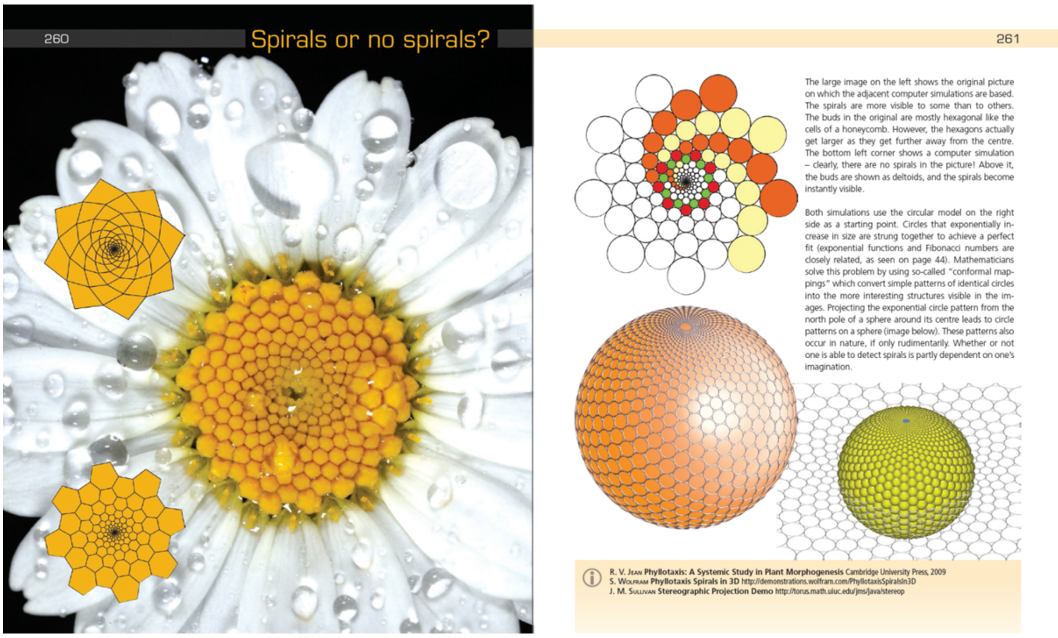

Figure 2: Fibonacci numbers, alleged spirals, packing problems on the sphere

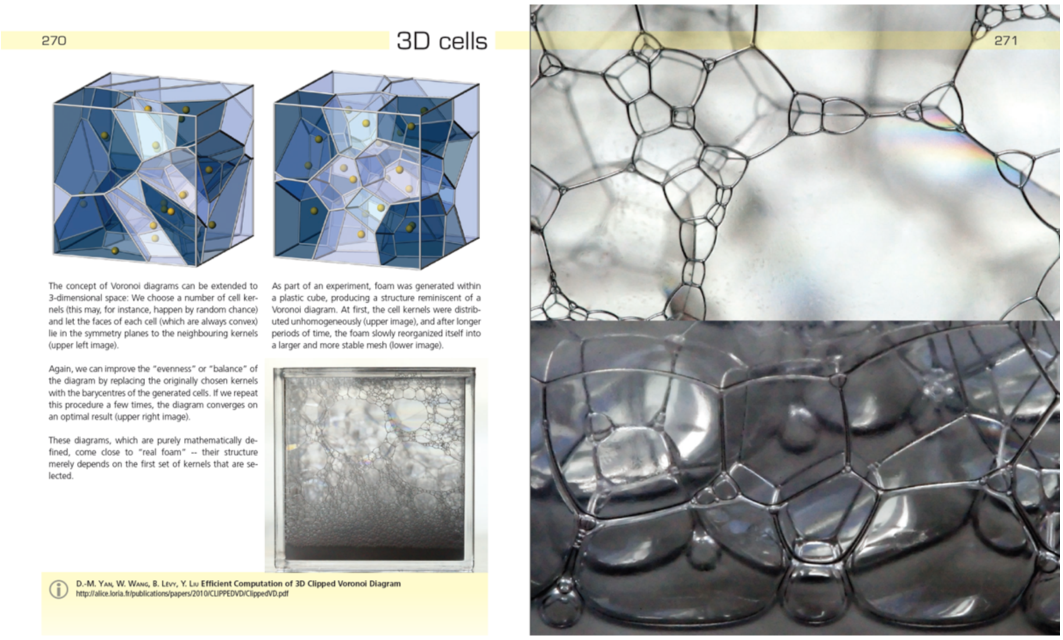

Figure 3: Voronoi-Diagrams and "something similar" in 3-space

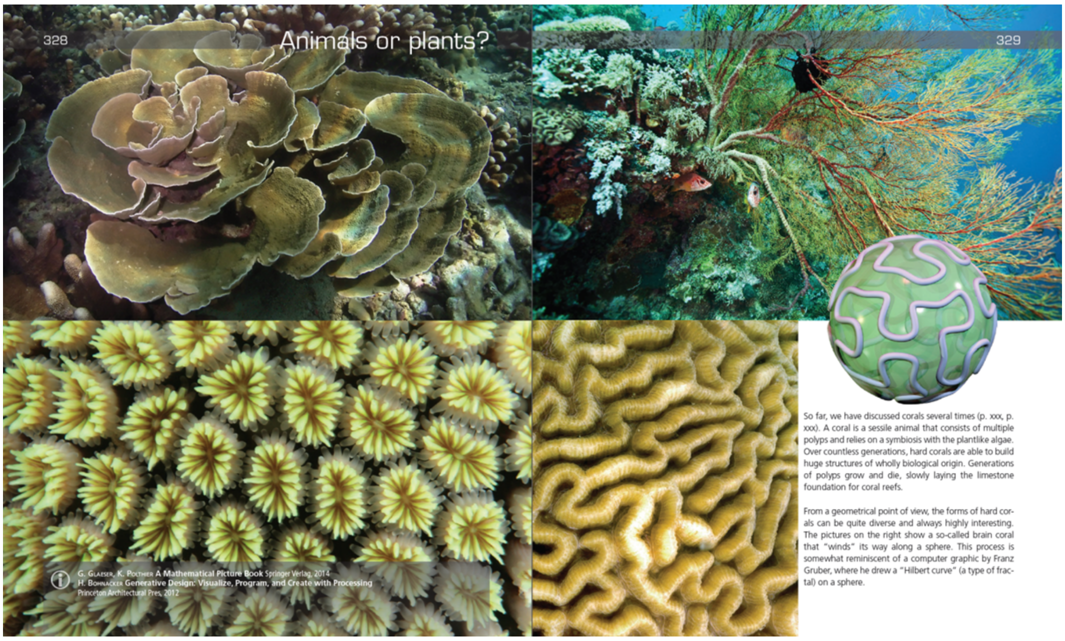

Figure 4: Surface enlargement, growth patterns

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Perspective – optional course for master students at the Faculty of Civil Engineering in Zagreb

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The course *Perspective* was introduced as an optional course for master students at the Faculty of Civil Engineering in Zagreb during implementation of Bologna Process. Since it first started, students have shown some interest in it. In the first few years, 7-9 students enrolled per year. Last academic year we introduced computer 3D modeling which caused huge increase in the number of students. This academic year 34 students enrolled. We will present organisation of classes for that group of students and show some of their achievements.

Most students who enroll in the course *Perspective* (6 ECTS) are in the final semester of master programme. By Faculty's decree, whole class is held during 8 weeks, giving the students enough time to prepare their theses. Hence, work is very intense (8 classes per week) and workgroups consist of 8-9 students. These students were never taught 3D computer modeling. That is the reason we can use our educational material prepared for the course *Descriptive geometry* (held in the first year) in teaching *Perspective*. When generations which were taught 3D modeling in their first year of studying come to master level, our plan is to introduce *Grasshopper* as well as the basics of *Python* scripting for *Rhino* (only on the informational level).

Here is a short overview of the organisation of class:

During the first two weeks, students are introduced to the basics of perspective drawing and are enabled to construct perspective images of simple geometric objects. In the third week, 3D modeling is introduced and they construct perspective images of objects using program *Rhinoceros*. During the next two weeks, students are acquainted with quadric surfaces and ruled surfaces with emphasis on quadric ruled surfaces. Furthermore, examples of these surfaces in civil engineering are shown and they are acquainted with geometric interpretation and visualisation of notions of differential geometry of these surfaces (classification of points on a surface, principal curvatures, normal curvature, principal directions). In the sixth week the topic is conoids of third and fourth order. During the last two weeks, students model situations of earthwork beside roads on terrains.

Our goal was to spend as little time as possible on obsolete approach (manual constructions, the theory on construction of images) and teaching basics of 3D modeling but to dedicate most of the time to applications of geometry, promoting creativity and production of attractive illustrations. For this approach we were inspired by article [1].

Homework assignments are graded. They are made by students at home and uploaded over system *Merlin*. Homework assignments are given only by the theme so the students are free to choose the object they would like to model. These assignments make students search geometrical properties in their surroundings, and they can be used as a source of ideas for future exercises.

In classes students solve exercises available online. Working version of the collection of exercises (in Croatian) is available on line `http://www.grad.hr/sgorjanc/perspektiva-vjezbe.pdf`.

Key words: geometry education, 3D modeling, Rhinoceros, e-learning

MSC 2010: 97G80, 97G40

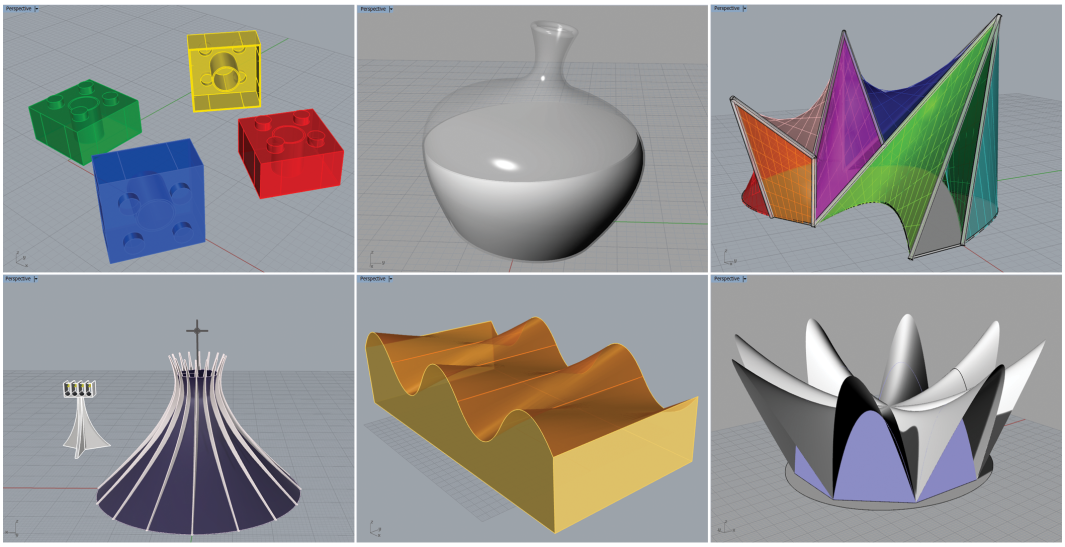

Figure 1: Student homework assignments made by L. Karaula, I. Čusek, B. Kirin, M. Jagatić, A. Zadro and S. Debelec, respectively.

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Curves of the 3rd class obtained by line-inversion in the quasi-hyperbolic plane

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The quasi-hyperbolic plane is one of the nine Cayley-Klein projective metrics where the metric is induced by an absolute figure $\mathcal{F}_{\mathbb{QH}} = \{F, f_1, f_2\}$, consisting of two real lines f_1 and f_2 incidental with the real point F. The line-inversion with respect to a line p and non-degenerated 2nd class curve κ is as an involutive quadratic line mapping where the corresponding lines are concurrent with the line p and conjugate with respect to the 2nd class curve κ. In this presentation the line-inversion with respect to a circle and different positions of a line will be observed. Furthermore, the notion of circularity for curves will be introduced and types of circular 3rd class curves obtained by line-inversion as images of the 2nd class curves will be analysed.

Key words: quasi-hyperbolic plane, line-inversion, circular curve

MSC 2010: 51A05, 51M15, 51N25

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Circular cubics in pseudo-Euclidean plane

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A curve in the pseudo-Euclidean plane is circular if it passes through at least one of the absolute points.

In the presentation we will study the cubics obtained as a locus of the intersection points of a conic and the corresponding line of the projectively linked pencil of conics and pencil of lines. The conditions that the pencils and the projectivity have to fulfill in order to obtain a circular cubic of a certain type of circularity will be determined analytically.

It will be shown that the cubics of all types (depending on their position with respect to the absolute figure) can be constructed by using these results. The results will be first stated for any projective plane and then their pseudo-Euclidean interpretation will given.

Key words: pseudo-Euclidean plane, circular cubics, projectivity, pencil of conics

MSC 2010: 51M15, 51N25

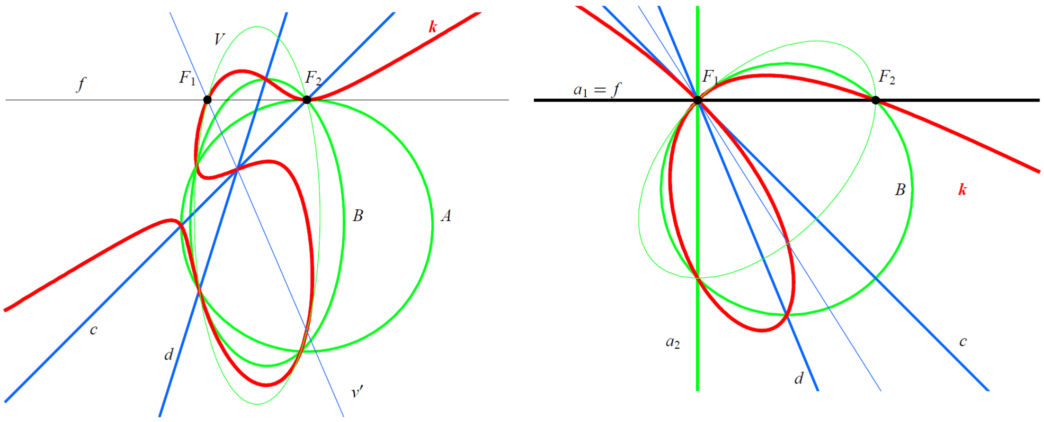

Figure 1: Two circular cubics of type (2,1) in the pseudo-Euclidean plane with the absolute figure $\{f, F_1, F_2\}$.

On visualization of regular polytopes, algorithmic problems

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The theoretical background of our topic is the d-dimensional projective spherical space $\mathcal{PS}^d(\mathbf{V}^{d+1}; \boldsymbol{V}_{d+1}; \mathbf{R}; \sim)$ or projective space $\mathcal{P}^d$, modelled as subspace incidence structure of the real $d+1$-dimensional vector space $\mathbf{V}^{d+1}$ for points or its dual $\boldsymbol{V}_{d+1}$ for hyperplanes, respectively. Here $\sim$ indicates the multiplicative equivalence by positive reals $\mathbf{R}^+$ in case $\mathcal{PS}^d$, or by non-zeros $\mathbf{R}\backslash\{0\}$ for $\mathcal{P}^d$. E.g. non-zero $\mathbf{V}^{d+1}$ vectors $\mathbf{x} \sim c\mathbf{x}$ describe the same point $X(\mathbf{x})$ in $\mathcal{PS}^d$ iff $c \in \mathbf{R}^+$.

In this presentation we report the basic algorithms in visualizing higher dimensional regular polytopes $\mathcal{P}$ of Euclidean d-space $\mathbf{E}^d$, in the computer p-screen Π, projected from a complementary $d-p-1 = s$ centre figure $\boldsymbol{C}$, with visibility and shading (see our references for $d = 4, p = 2, s = 1$ and the homepage [6] for free download, hopefully in a more developed form). E.g. the 4-cube (8-Cell) and its dual the 4-cross-polytope (16-Cell) are illustrated here. Their analogues exist in each dimension. Projection into $p = 3$-space seems to have interesting applications (see L. Vörös' presentation and his former works).

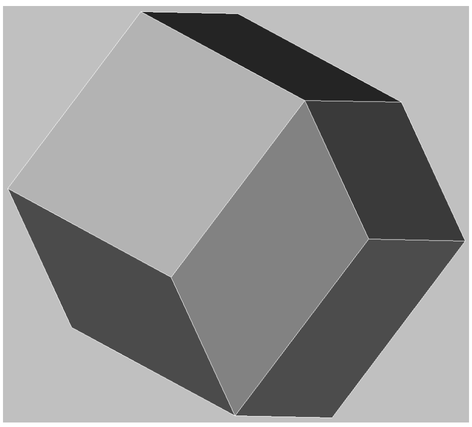

Figure 1: The 4-cube (8-Cell) with C-Sch-symbol (4,3,3)

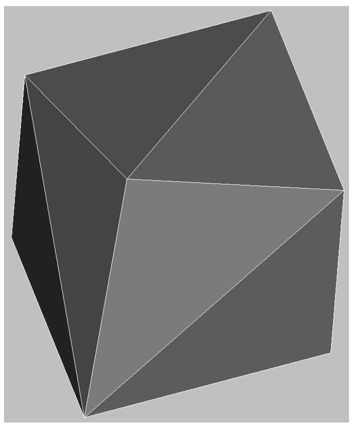

Figure 2: The 4-cross-polytope (16-Cell) with C-Sch-symbol (3,3,4)

Key words: Coxeter-Schläfli theory of regular polytopes, $d \to p$ projection via projective spheres, visibility algorithms

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Architecture and face

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Every day we see our face and easily recognize information, emotions through the face during communication. In architecture we can consider a facade as an analogy. During description of human's face through physical parameters we evaluate its aesthetic and emotional content. During description of a facade the same attitude could be carried on.

Long time ago numerous attempts were held in order to figure out perfect proportions of a human's face, as well as in a field of architecture, for example, by Albrecht Dürer and Leonardo da Vinci. Nowadays some classical canons still remain.

What if perfect proportions are applicable to architectural ratios so that it is able to provide more favorable aesthetic perception? Or perhaps we architects are able to design facade based on proportions of a face of certain person, so he could say: "my house is my face".

While creating space an architect should form new object within a context and understand the rules on which the environment has appeared. Even strictly ordered geometry has completely naturalistic origin. Shevelyov in his book speaks about the fact that verticality, horizontality and right angle have direct origin from physically determined conditions on the Earth, and thus the rules of morphogenesis contribute into natural shapes along those vectors; "straight or vertical line is a reality of gravitation, objects are falling down along this vector; light, electromagnetic and magnetic waves are spread in a direction which is perpendicular to a plane of the wave – which is practically a plane of action of force lines. Horizontality appears due to physical existence of the right angle and vertical line" [1, p. 22].

Along history ratios of the building are compared with proportions of human body in order to translate rules of biological morphogenesis into a building. Vitruvius wrote about it in his treatise "De architectura", canonic system of orders considers a modulus as a base as well as height of a head equals 8 or 9 units along the height of body.

During practical part of this research face and facade details were compared. Position of elements and compositional symmetry was not considered, only functions and meaning of each category were assumed (Figure 1). Then on the base of a chosen facade was defined a face corresponding to it. A scope was to figure out which appearance of the facade could be if it was a human. By means of facial composite software we've built up approximate face according to general compositional features of the facade. As example we choose a main facade of Casa Batlló in Barcelona, architect Antonio Gaudi.

Features of facade distinguished (Figure 2): general composition of facade is balanced; it fits into an elongated vertical rectangle. Following relationships were found: complex rhythmical design; prevalence of curved rhythms and rows which are regularly positioned. Overall curve of the roof completes and integrates a combination of curved elements. Curves are dominating in decorations of facade, profile of the roof, balconies, and shape of oriels. Mezzanine gallery extends above the first tier of arches. Ventilation and fireplace pipes are shaped specifically. Facade is finished with ceramics and glass mosaic, surface of external walls is textured and has vivid coloristic.

An approximate appearance of the face corresponding to facade description was defined. We define key features according to Figure 1: prolonged face shape; medium-sized eyes clear in confines, circular shape. Prolonged extended nose; birthmarks, freckles are possible on the skin. Cheekbones are clear and smooth. Chin is neither expressed, neither bulging; arched eyebrows, active brow ridges - judging by the edge of the roof profile. According to that we can assume that the face holder is a male. High forehead, proportionally big mouth with clear curved contour, possible accent element in the middle closer to the nose. Bulging ears, curved hair. Generally we have soft facial features fitted into prolonged face in its geometrical shape, Figure 3.

Key words: architecture, biological morphogenesis, human face, proportions, perception

Eyes	→	Overview, visualization	←	Windows
Nose	→	Air circulation, ventilation	←	Air ducts, tower
Mouth and lips	→	Intake	←	Entrance lobby, doors
Chin, lower jaw	→	The base, the bottom	←	Socle
Eyebrows	→	Water abstraction	←	Drainage trays
Eyelashes	→	Shadow casting	←	Visors
Eyelids	→	Protective barrier	←	Shutters, lattice
Cheekbone	→	Transition to lateral surfaces and planes	←	Edges of facades
Ears	→	Connection to the outer world	←	Loggias, balconies
Forehead	→	Closure of the structural part	←	Fronton, cornice
Teeth	→	Emotional impact, magnificence	←	Portico, colonnade
Cheeks	→	Filling	←	Walls between support
Hair	→	Shelter, protection from atmospheric phenomena	←	Roof

Figure 1: Comparison of facial and facade elements

Figure 2: Casa Batlló, main facade

Figure 3: Approximate appearance of the face

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On the Maclaurin mapping in the quasi–hyperbolic plane

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The construction given in [3] serves as a basis for defining a birational quadratic mapping in the projective plane $\mathcal{P}_2(\mathbb{R}) \subset \mathcal{P}_2(\mathbb{C})$. Since this construction is credited to Scottish mathematician Maclaurin who made important contributions to geometry and algebra back in 18th century, the associated mapping will be called *the Maclaurin mapping.*

Hence, the synthetic algorithm for constructing curves of different order under the Maclaurin mapping is studied, and necessary conditions for obtaining the curves of certain degrees are analyzed. We combine the nonmetrical concepts of projective geometry with the language and techniques of geometric algebra, [2], [5], [6]. After the results are stated for the projective plane $\mathcal{P}_2(\mathbb{R}) \subset \mathcal{P}_2(\mathbb{C})$, theirs interpretations have been given in one of nine Cayley–Klein projective metric planes, the quasi–hyperbolic (or dual Minkowski) plane, where the metric is induced by two real lines $\overline{f_1}, \overline{f_2}$ intersecting at a point $\mathbf{f}$, [1], [4]. In what follows, special attention is given to the position of the curves to the absolute figure of quasi–hyperbolic plane $\mathcal{F}_{\mathbb{QH}} = \{\mathbf{f}, \overline{f_1}, \overline{f_2}\}$ and based on it some of the properties of the obtained curves are given.

Key words: quasi–hyperbolic plane, projective geometry, geometric algebra

MSC 2010: 51N15, 51A05, 14P99

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Is there a decline in spatial abilities of students?

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Tests of spatial ability have been performed on Faculty of Architecture, University of Ljubljana for more than ten years. MRT is one of the generally recognized tests that was used, irrespective of the fact that emphasizes on only one part of the spatial ability, which is the area of mental rotation. Survey included the first year students of study program Architecture and since 2012 also the students of study program Urban planning. Till now, the test has passed over 2000 students. The conditions of the test have stayed the same throughout the years which enables us to objectively compare different generations. The results are interesting and generally do not deviate from similar studies around the world.

An interesting phenomena occurred throughout the years, that every third generation has worse results. This difference has been statistically proven for 2009 and 2012, while the difference for 2003 and 2006 has not yet been statistically proven. But in generation, there has been decline in spatial ability in the past years, as can be seen in figure.

Key words: MRT (Mental rotating test), spatial ability

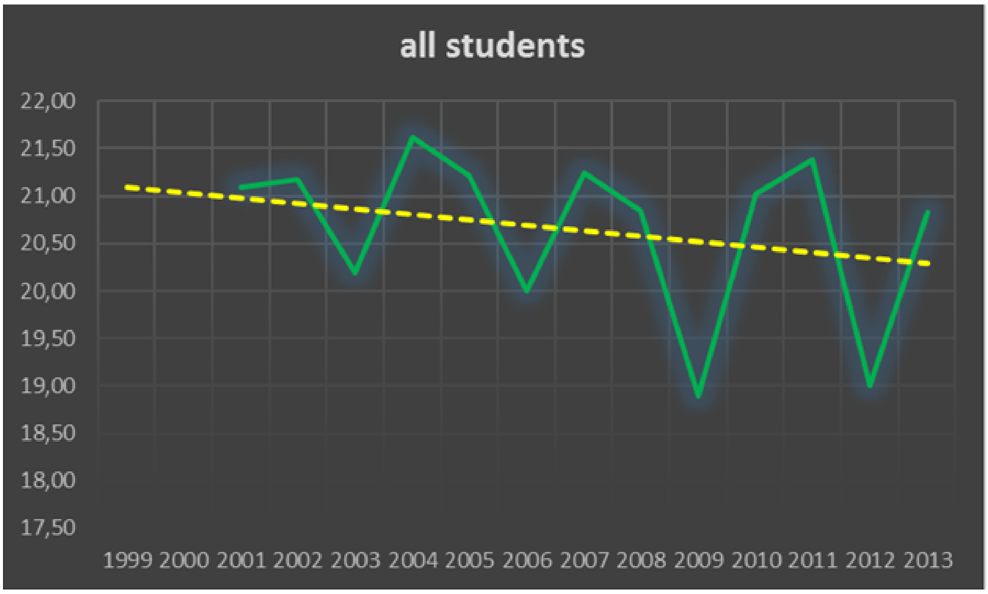

Figure 1: MRT results from 1999 to 2013.

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On DINI-surfaces in Euclidean and Minkowski geometry

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In Euclidean 3-space DINI-surfaces are well known pseudospherical surfaces (surfaces of constant negative Gaussian curvature), usually generated by applying a helical motion to a tractrix. We investigate Euclidean and affine properties of the evolute surfaces of DINI-surfaces. While one of these properties gives a characterization of DINI-surfaces within the class of pseudospherical surfaces, another one is true for every pseudospherical surface Φ: The existence of a second surface which is affinely related to Φ.

The construction of DINI-surfaces and most of the properties keep their meaning in Minkowski 3-space.

Adaptation of image content for users with colour vision deficiencies

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Colour-blindness represents an inability to perceive differences between some of the colours that can be distinguished in the case of normal human colour vision. Since the use of colour to convey visual information in the multimedia content increases, it becomes more important to improve colour information accessibility for colour-blind population. In this article, a new region-based re-colouring method was proposed with purpose to correct the image areas with confusing, problematic neighbour segments which colour-blind people perceive as the same colour and, thus, to make the content understandable for them. There are several algorithms providing such alteration of colours, e.g. the built-in Daltonize algorithm of Google Chrome. The real novelty of our algorithm is the preservation of the original colours of the picture as much as possible, thus making the final picture enjoyable for people with normal vision as well as understandable for people with color vision deficiencies at the same time.

Figure 1: Original image (left), Google Chrome algorithm (middle), our algorithm (right)

Key words: colour-blindness, dichromacy, colour adaptation, region-based algorithm

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A new type of lemniscate

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In the lecture, we present a class of curves derived from a geometrical construction. We take points on two half-lines having a common starting point and including angle α. The first point is on one of the half-lines and the second one is on the other half-line, while the next is again on the first half-line, and so on. The distances of two consecutive points are the unit. These curves are similar to lemniscate and they are defined as the generalizations of the orbits of the considered points when α goes from zero to $\frac{\pi}{2}$. For determining the equations of the curves we use the Chebyshev polynomials. On the surface of a sphere similar nice curves can be defined.

Key words: lemniscate, Chebyshev polynomials

MSC 2010: 51N20, 14H50

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Distances and central projections

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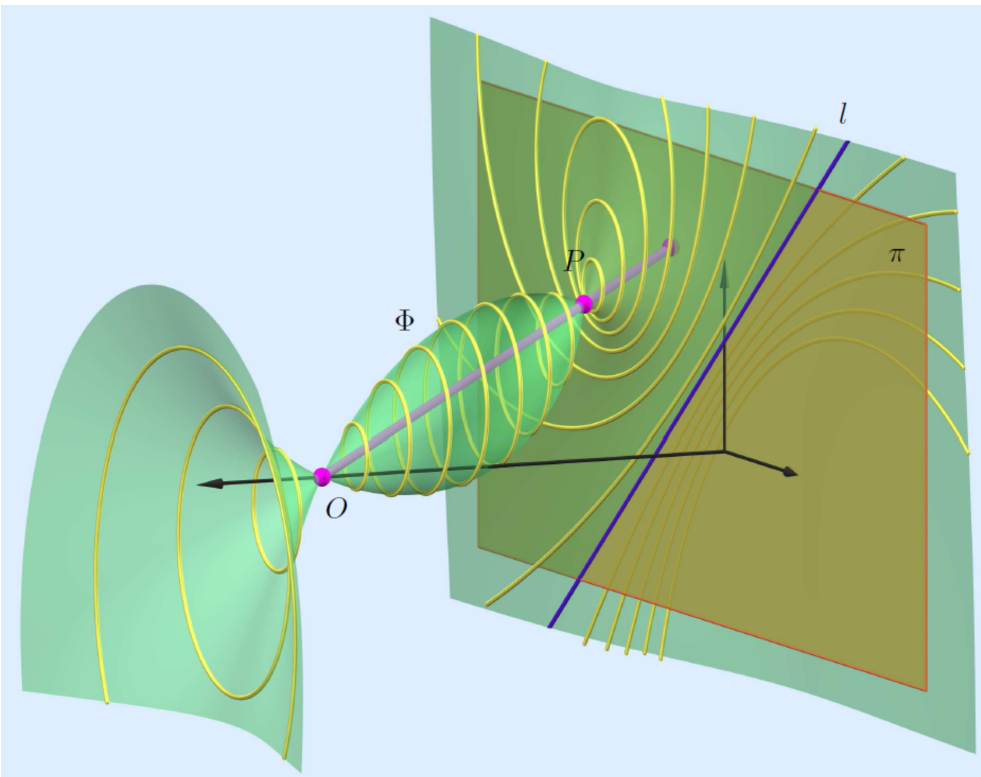
Segments of lines appear undistorted under a central projection, if they are contained in the image plane. However, the line segments in the image plane are not the only segments which agree in length with their images.

Given a point P in three-dimensional Euclidean space $\mathbb{R}^3$, we ask for all points Q such that the distance $\overline{PQ}$ equals the distance $\overline{P'Q'}$ of the image points $P' = \zeta(P)$ and $Q' = \zeta(Q)$ of P and Q under a central projection $\zeta : \mathbb{R}^3 \backslash O \to \pi \cong \mathbb{R}^2$ with eye point O and image plane π. It turns out that the end points Q of those line segments emanating from one point P such that $\overline{PQ} = \overline{P^cQ^c}$ holds, form a quartic surface Φ with two conical nodes. The surface Φ carries a one-parameter family of Euclidean circles and meets the image plane along a line l together with the ideal line of the image plane, the latter with multiplicity three. We shall study this surface and work out some of its properties.

Key words: Central projection, principal line, distortion, length of a segment, quartic surface.



Geometry in science promotion events

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There are many projects to raise public awareness on the importance of natural sciences, technology and innovation. We have regularly been taking part in such events for the last two years. Geometry is one of the most attractive mathematical program components on science festivals even though the role of geometry was decreased in the secondary education. The members of the Institute of Mathematics have not been enthusiastic enough about promoting science, but we gladly undertook this role.
A program named “Playing SPACE” (not Playground!) was developed by us with some variable components. The elements can be adapted to the location of the event, age group of the participants, and any other requests. We are engaged on Researcher's night, SEE Science festival, Girls' day, Special days in schools, “Meet the professors” roadshow, and our program was an accompanying event of a math competition.

In this presentation we show the components of the program, our tutorials and devices, and our experiences and some plans to the future.

Key words: science promoting, informal education, activity

MSC 2010: 97G20, 97G40, 97M10, 65D18, 68U07

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On the rigidity of simplices with prescribed areas of 2-facets

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Due to the Cayley-Menger-formula the volume of any simplex in the Euclidean n-space can be expressed in terms of its edge lengths. R. Connelly posed 2007 the question whether there is also such a formula based on the areas of the 2-facets? In dimension 4 it looks promising since the edges are dual to the 2-faces.

The aim of the presentation is to discuss the problem whether in Euclidean n-space for any full-dimensional simplex S there is an incongruent simplex S' with respective equi-areal 2-facets. For $n = 3$ there is a smooth two-parametric set of S' for each S. For $n = 4$ the existence of any S' is equivalent to a real non-empty intersection between a 10-dimensional linear space and a 10-parametric algebraic variety in a projective 20-dimensional space.

The impact of technology on learning

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This work was created because we are curious to find out in which direction the learning process is heading today and how young adults can use the new devices when they learn mathematics in new technological environment. What could and should the teacher offer to his/her students? How to make lessons more interesting and make students pay more attention.

We know from psychology that students remember the facts better if they are interested and if they have in some way “experienced” that with their own senses. But mathematics is abstract. If we add the fact that every man, in this case student, is a social being, it is logical to ask whether the process of learning to include technological aids to a student’s “experience” using technological aid and being in the company.

Here are the results of research in universities in America on the impact of technological devices on collaboration among students and it compares the learning by using iPads and laptops.

We study to what extent does the use of the iPad for learning mathematics encourage the cooperation of students and is different from the environment in which we grew up, learned and acquired knowledge.

Key words: iPad, laptop, learning math, cooperative behavior in learning

Computer-aided geometry education at Ybl Faculty of Szent István University

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At Ybl Faculty of Architecture of Szent István University we introduced applying computers in the course *Mathematics and Geometry in Architecture* during the academic year of 2013/14.

I would like to report on how we used computers during that course. The instructor used computer software to show some dynamic geometry figures, demonstrating the problems that were considered during classes. The students were asked to present the solution of assigned exercises on computer, creating digital figures of their own. Mainly the dynamic geometry software GeoGebra was used during the course (the current 2D version, and the 3D beta version). I report on the teaching experience and future plans. Some 2D and 3D figures of geometry problems, created by the instructor and the students, will be also presented.

Key words: mathematics education, planar geometry, space geometry, GeoGebra

MSC 2010: 97G40

Fractals in genomic data visualization

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To view and analyze the local and global characteristics of large-scale genome sequences, many different graphical representations can be considered as visualization tools.

At first, the spatial structure of DNA exhibiting self-similar fractal properties is relatively dense, that reminds us of specific space-filling curves (Peano/Hilbert curves). The Hilbert Curve Visualization (HCV) technique has already been used in genomics. The data arranged along the chromosome (in one dimension) is mapped onto a two- (three-) dimensional shape. This visualization technique can complement the genome browsers because it presents the whole chromosome and gives access to further details, as well. Furthermore, the attractor of Iterated Function System can have a fractal-like self-similar appearance for long complex series of symbols. For example, the fractal trajectory of a DNA sequence can be visualized by applying distinct affine transformation for each successive nucleotide encountered in DNA sequence. The main advantage of the IFS method is not only its speed but also the compact representation of genomic information in a visual form.

This lecture is intended to give a brief overview of the genomic data visualization techniques, to present our theoretical results, experiments and plans for the future.

Key words: bioinformatics, information visualization, space-filling curves, fractal representation, iterated function systems

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The density analysis diagrams of the public areas which are affected from user actions with an interactive map

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The use of digital tools that are produced for different purposes and allow the use of interdisciplinary information in the processes of integrated designs becomes the focal point in the representation of the design process. New approaches or ideas which previously not found application because of the limited range of opportunities come to the fore with the help of digital design tools. In the creative process, development of the tools that are used by the designer and exploring the potentials of them are an important factor which provides advantage to the designer. In this study, the changes of digital design tools during the process and the results of them are evaluated in the context of the roles of the designer. Each design has its own systematic fiction. The mathematical background and its effect on the design can be seen when examined, whether a small scale design in the city or a large scale design like urban design. So if this fiction made with easily reproducible dynamic factors, the process of concretization of an image formed in the mind occurs easily in the frame of true information. The diagrams that are used for dynamic design tool are revealed as a productive system model in the process of data transformation. According to Allen's (1993) point of view "Diagrams are not schemas, types, formal paradigms, or other regulating devices, but simply place-holders, formal configurations. They work as abstract machines and do not resemble what they produce."

Key words: dynamic process, digital design tool.

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Parametric microstructure geometry for construction elements

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The emergence of parametric generative design tools and digital fabrication led to significant changes in the field of architecture. These changes are increasing the opportunity to develop innovative forms and structures (new smart geometry of construction elements). Parametric design enables customized forms and individual architectural elements which can only be made using digital fabrication tools. The use of digital fabrication to translate digital models from computer into physical ones enables complexity nonexistent in traditional building.

As part of the wider research project, main focus of this research is to explore new geometries using generative algorithms. The chosen software, for making virtual parametric models/digital models, is a graphical algorithm editor integrated with Rhino's 3-D modeling tools called Grasshopper. Proposed case study is to test and evaluate the use of the parametric generative methodology in design for specific architectural construction elements. Research specific emphasis is on structures geometry (new smart geometry) driven from mathematical algorithms, parametric design, and ability to transform three dimensional density in microstructure which is needed for improved sustainable use of material for load bearing capacity, thermal insulation and acoustic characteristic, or even embedded building installations in structural elements.

After exploring geometry algorithms, analysis and optimization of elements using corresponding parameters is the next stage in design process. Testing of virtual models, their behaviour through evolving design process needs to be confirmed by analysis and testing of digitally fabricated physical scale models in the following steps of the research, and full-size building components in the final stage of the research.

Resulting improved and confirmed algorithms should be in the end implemented in software solutions for architectural design without the need for an architect to engage in the design of intrinsic structure or solutions.

Surely, new methods of digital fabrication and construction open a new way of thinking about construction elements and how their intrinsic geometry could be utilized for improving structural performances.

Key words: computer aided conceptual design, parametric design, smart geometries, digital fabrication

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Minkowski set algebra

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Minkowski set operations defined on point sets in arbitrary geometric space can be used for introduction of an algebraic structure with specific properties valid on point sets as basic elements. Some of the basic notions and concepts such as Minkowski combinations of point sets or Minkowski operators and their properties are derived and applied as modelling tool for generating differentiable manifolds in geometric spaces of arbitrary dimension. Modelling manifolds in $\mathbb{E}^n$ by means of Minkowski operators offers a powerful tool for shape definition in higher dimensional spaces. Such creative design of unusual forms of point sets in $\mathbb{E}^n$ can be used also as a background for modeling new shapes in $\mathbb{E}^3$ obtained through orthographic views of these higher dimensional objects. Interesting examples of surfaces generated as 3D orthographic views of Minkowski triples of three curves positioned in coordinate planes in the space $\mathbb{E}^4$ are presented in Fig. 1. Various forms of Minkowski set combinations can be visualised and dynamically modified in suitable mathematical software packages by changing multiple shaping parameters. Creative manipulations with these flexible objects in dynamic software environment might inspire artists, architects or designers to find among them a possible future sculpture, piece of art, architectonical form or shape interesting from aesthetic point of view and usable in design.

Minkowski linear operator $L_{k_1,\dots,k_m}$ can be generally defined as a mapping from the m-tuple Cartesian product of the potential set of the Euclidean space $\mathbb{E}^n$ to the same potential set, attaching a point set $B \subset \mathbb{E}^n$ to a linear combination of point sets $A_1, A_2, \dots, A_m \subset \mathbb{E}^n$,

$$\begin{gathered}L_{k_1,\dots,k_m} : 2^{\mathbb{E}^n} \times 2^{\mathbb{E}^n} \times \dots \times 2^{\mathbb{E}^n} \to 2^{\mathbb{E}^n}, \quad (A_1, A_2, \dots, A_m) \to B \\ B = L_{k_1,\dots,k_m}(A_1, A_2, \dots, A_m) = k_1 \cdot A_1 \oplus k_2 \cdot A_2 \oplus \dots \oplus k_m \cdot A_m\end{gathered}$$

where $k_1, \dots, k_m, m \subset \mathbb{N}$ are arbitrary real constants and $\oplus$ denotes Minkowski sum of sets. Thus for $m = 1$ a scalar multiple of point set $k \cdot A$ is defined, for $m = 2$ a linear pair of point sets $k_1 \cdot A_1 \oplus k_2 \cdot A_2$ is a point set with specific properties determined by properties of sets A_1, A_2, linear triple $k_1 \cdot A_1 \oplus k_2 \cdot A_2 \oplus k_3 \cdot A_3$ for $m = 3$, etc., while these sets can be further modified changing values of shape parameters $k_i \subset \mathbb{R}$, $i = 1, 2, \dots, m$.

Similarly, Minkowski product operator $P_{k_1,\dots,k_m}$ is defined as the m-tuple Cartesian product of the potential set of the Euclidean space $\mathbb{E}^3$.

$$\begin{gathered}P_{k_1,\dots,k_m} : 2^{\mathbb{E}^3} \times 2^{\mathbb{E}^3} \times \dots \times 2^{\mathbb{E}^3} \to 2^{\mathbb{E}^3}, \quad (A_1, A_2, \dots, A_m) \to C \\ C = P_{k_1,\dots,k_m}(A_1, A_2, \dots, A_m) = k_1 . A_1 \oplus k_2 \cdot A_2 \oplus \dots \oplus k_m \cdot A_m\end{gathered}$$

for arbitrary real constants $k_1, \dots, k_m, m \subset \mathbb{N}$, ordered m-tuple $(A_1, A_2, \dots, A_m)$ of point sets in $\mathbb{E}^3$ and $\oplus$ denoting Minkowski product of sets.

Pair $k_1 \cdot A_1 \oplus k_2 \cdot A_2$ and triple $k_1 \cdot A_1 \oplus k_2 \cdot A_2 \oplus k_3 \cdot A_3$ of point sets etc., for $m = 2, 3, ... \subset \mathbb{N}$, are determined as point sets in $\mathbb{E}^3$ with specific properties. Considering differentiable manifolds as point sets A_i parameterized on sets of real numbers, properties of generated differentiable manifolds are dependent on their parameterizations, mutual superposition and positioning in the space. Interesting geometric properties regarding form of generated differentiable manifolds can be easily derived in the case of point sets in $\mathbb{E}^3$.

Key words: Minkowski sum, Minkowski product, set combinations, Minkowski operators

MSC 2010: 51N25, 53A056

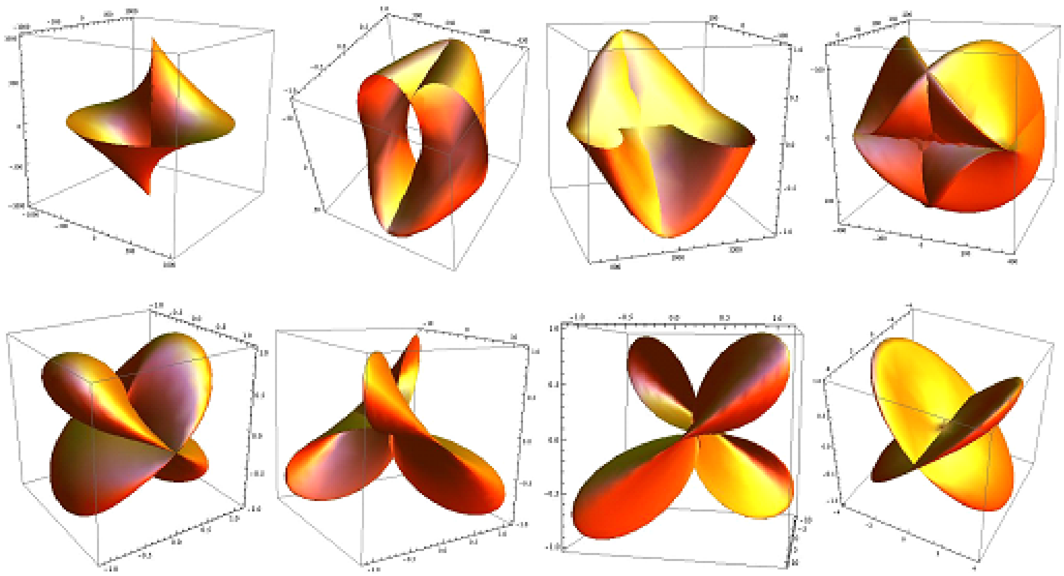

Figure 1: Orthographic views of Minkowski mixed triples of three curves from $\mathbb{E}^4$.

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Newer animations of symmetric patterns based on space-filling zonotopes

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The 3-dimensional framework (3-model) of any k-dimensional cube (k-cube) can be produced based on starting k edges arranged by rotational symmetry, whose Minkowski sum can be called zonotope. Combining $2 < j < k$ edges, we can build 3-models of j-cubes, as parts of a k-cube. The suitable combinations of these zonotope models can result in 3-dimensional space-filling mosaics. The investigated periodical tessellations always hold the 3-model of the k-cube and necessary j-cubes derived from it. Such a space-filling mosaic can have fractal structure as well, since we can replace it with a new one, built from restructured solids. These are composed by addition of 3-models of k- and j-cubes and are similar to the original ones. The intersections of the mosaics with planes allow unlimited possibilities to produce periodical symmetric plane-tiling. Moving intersection planes result in series of tessellations or grid-patterns transforming into each other which can be shown in varied animations.

Planar and spatial symmetry groups are the base of several works in different branches of art. Our symmetric models of the hypercube and the symmetrically arranged periodical tessellations offer several binding points to this field. The newer results can hopefully aid the correspondence among geometry, art and design.

Key words: constructive geometry, hypercube modeling, tessellation, fractal, design

MSC 2010: 52B10, 52B12, 52B15, 65D17

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Non-standard visualizations of Fibonacci numbers and the golden mean

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The well-known Fibonacci numbers, the golden mean value and their generalizations are merely dimensionless numbers. That notwithstanding, they are commonly visualized in the Euclidean plane, i.e., in a two-dimensional setting. The values can also be found in various natural objects of three dimensions. Historically, the golden mean is understood as a special ratio of line segments, an invariant concept of affine geometry. Nonetheless, it is often visualized in Euclidean geometry as well as in other metric geometries.

If an affine plane is additionally endowed with a (general) metric, one gets a so-called Minkowski plane with a well-defined concept of circles. In such a plane it is possible to define "golden spirals" even though there do not exist rotations or general similarities.

Another attempt replaces the frequently used squares for visualizing the Fibonacci sequence by circles, a concept available also in hyperbolic and elliptic geometry. In a hyperbolic plane lines have infinite lengths, compared to an elliptic plane where lines have finite lengths. This necessitates further modifications to visualize "golden objects". Obviously, the proposed procedures can be extended to visualize e.g. metallic means and their corresponding number sequences.

Key words: Fibonacci number sequence, golden mean, non-Euclidean geometry, Minkowski geometry, visualization

Rational trigonometry and relativistic geometries

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We survey basic ideas of rational trigonometry valid for general metrics, in particular for relativistic geometries. Two particular planar relativistic geometries in particular are of interest, both of importance in special relativity, and the intimate connection with Euclidean geometry.

We will present a number of different and interesting applications of thinking about metrical geometry purely algebraically and uniformly.

Posters

Monoid surfaces through the absolute conic in the Euclidean space

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A monoid surface in projective space is a surface of order d which has a singular point of multiplicity $d-1$. We study the properties of a special class of monoid surfaces $\mathcal{S}_{2n}$ in the Euclidean space given by the equation of the form

$$A_2(x,y,z)^n + wH_{2n-1}(x,y,z) = 0,$$

where $A_2(x,y,z) = x^2+y^2+z^2$ and $H_{2n-1}(x,y,z)$ is a product of $2n-1$ linear homogeneous polynomials.

In the paper [1] we show that $\mathcal{S}_{2n}$ is a surface of order $2n$ which touches the plane at infinity through the absolute conic. The surface consists of the separated parts (petals) sharing only one real $(2n-1)$-fold point. We prove that the largest number of petals of $\mathcal{S}_{2n}$ equals $2n^2-3n+2$ and show how this number decreases if some tangent planes at the origin pass through same straight line.

Key words: surfaces, absolute conic, tangent cone, singular point

MSC 2010: 51N20, 51M15

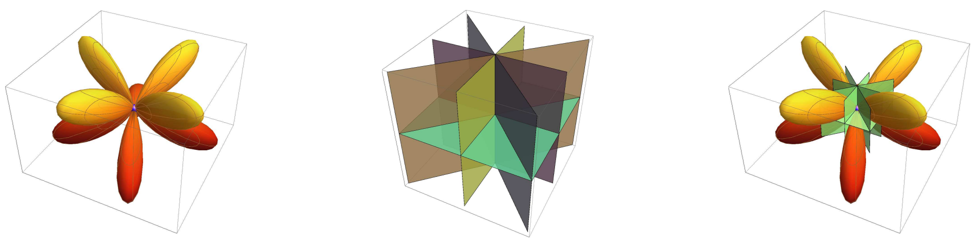

Figure 1: This figure shows one surface $\mathcal{S}_6$ and its tangent cone at the origin that splits into 5 planes, where 4 planes share the common axis z.

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Students' assignments – optional course Perspective

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In this poster we present some examples of students' homework assignments made this academic year for the optional course Perspective at the Faculty of Civil Engineering at the University of Zagreb. The poster is related to authors' presentation titled *Perspective – optional course for master students at the Faculty of Civil Engineering in Zagreb.*

Key words: geometry education, 3D modeling, Rhinoceros, e-learning

MSC 2010: 97G80, 97G40

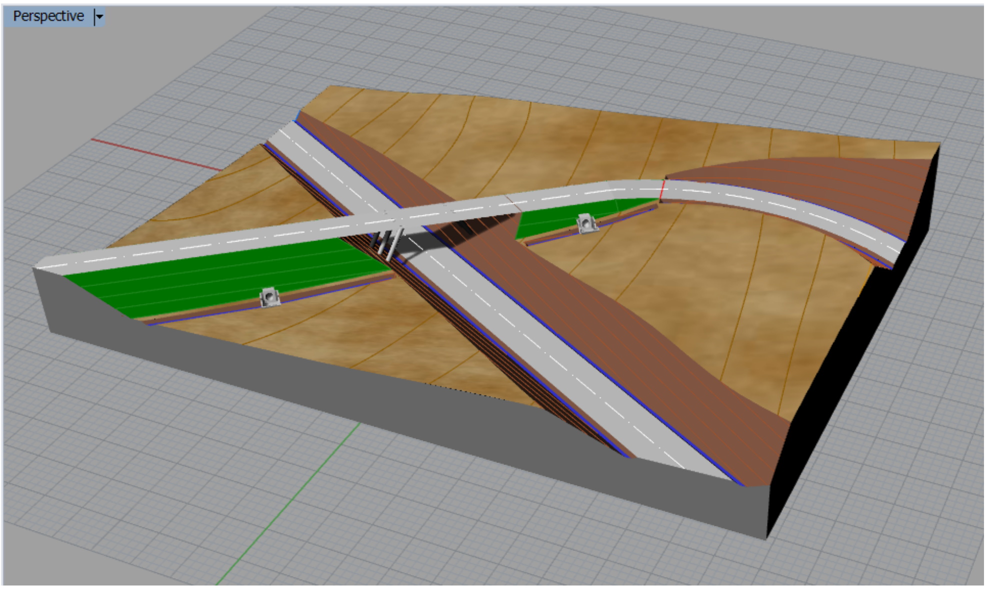

Figure 1: Student assignment related to terrain made by Š. Bezina.

Perceiving architectural space through different folding techniques

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Space and time around us bend in different directions, forming different experiences. For some of those experience architecture is the main actor, and through architecture its creator, the architect.

During the education of an architect a lot of effort is oriented towards his perceiving of space. Through different studies from descriptive geometry, to computer aided design, parametric design and mock up modelling person's awareness of space that surrounds them is expanded. While creating space an architect should be aware of a context and understand the rules on which the environment has appeared in it. Even strictly ordered geometry has comprehensively naturalistic origin, and paper folding techniques have the ability to help expand comprehending geometry / space perception.

Architect should be perceived as sculptor of space. When architect creates space he should be aware of canons and interrelation in space and time, in which he acts, and makes a difference. For an architect it is of great importance to be capable and competent to perceive given space, without actual interference in it. This perception can be deepened through different folding techniques, which along with descriptive geometry and computer aided design give palpable sensation of space, same as mock ups do, only folding techniques give another level of considerations due to its limitation to one piece of material.

Key words: perception, paper folding, descriptive geometry, computer aided design, parametric design, mock up, architecture

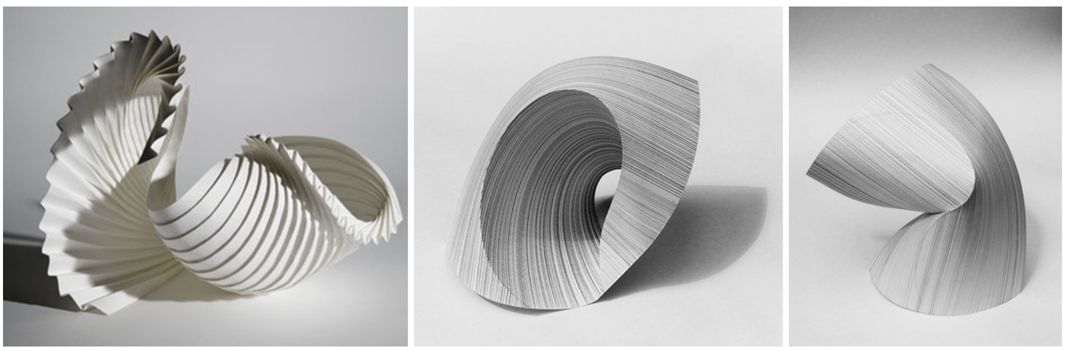

Figure 1: Richard Sweeney 3D structure

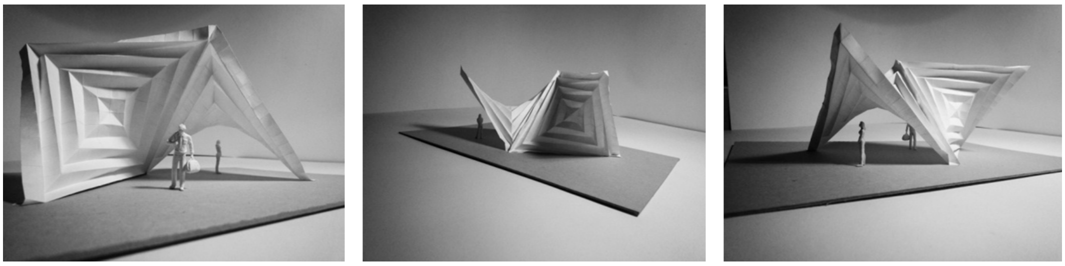

Figure 2: Workshop 21-25 February 2011, TU Graz and AF Novi Sad

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Geometric problems occurring in engineering sciences

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It is five years now that we have published Volume I of the course book "Mathematical Tools in Engineering Applications". This book has been written at the Faculty of Engineering, University of Debrecen for the lectures and seminars of Mathematics I. The main aim of the book is to demonstrate the application of mathematical tools (vectors, matrices, linear functions and complex numbers) on problems that are typical in the field of Physics, Technical Mechanics, Thermodynamics and Electrical Engineering. The exercises in this kind of approach are related to real technical problems. The main goal of the book is to emphasize where and how the different mathematical tools can be applied.

This year we intend to publish the second part of the book in the same form and with the same build up as the first one. To help the students to understand the engineering content and also its relationship with the mathematical content more easily we built GeoGebra animations into the book. Here we are focusing on the geometric relations of presented problems and show examples of these animations.

Key words: geometric problems, engineering applications, teaching mathematics

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GLSL processing in image manipulations

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This poster considers the construction of an image processing software system. This system relies on OpenGL and its GLSL subsystem. Usually image processing systems are using CPU (most widespread usage, but limited only to a few executing threads), perhaps CUDA (which as a proprietary technique is not applicable on many machines) or as a smart choice OpenCL (a lot of threads and a great freedom in coding). All of these standards are great for general computations. However, basically we have pictures, and therefore using shader codes on textures seems a more efficient way to handle specific image processing tasks. Unfortunately, even commercial softwares use OpenGL only for rotating and zooming.

Moreover, there is a feature that almost all of the image editing softwares are missing: if the distortion of the transfer curve of the printing process is known [2], the edited picture may be predistorted before applying the final step [3]. In the case of medical pictures (e.g.: color fundus images), sharpening, edge detection, contrast enhancement, contrast limited adaptive histogram equalization, gray world normalization, histogram equalization or intensity adjustment tasks arise [1]. The speed up factor is the primary concern for the batch processing, while the software is targeted to fulfill the demands for one image in real time.

Key words: OpenGL, GLSL, image processing, real time

MSC 2010: 68U05

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