Plenary lectures

3D homogeneous geometries and some special surfaces

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The Riemannian manifold (M, g) is called homogeneous if for any $x, y \in M$ there exists an isometry $\Phi : M \to M$ such that $y = \Phi(x)$.

In 1982 W. Thurston conjectured that any maximal, simply connected, three-dimensional geometry which admits a compact quotient is equivalent to one of the eight homogeneous geometries

$$E^3, S^3, H^3, S^2 \times \mathbb{R}, H^2 \times \mathbb{R}, \widetilde{SL(2,\mathbb{R})}, \mathrm{Nil}, \mathrm{Sol}.$$

In 2003 G. Perelman sketched a proof of the Thurston geometrization conjecture using Ricci flow with surgery.

In this lecture we will briefly describe each of the eight homogeneous geometries considering their basic properties. We will devote special attention to understanding of isometry groups of these geometries. We will discuss homogeneous geometries in light of Cayley's famous phrase "projective geometry is all geometry".

The lecture will also outline few important examples of surfaces in twisted product homogeneous geometries $(\widetilde{SL(2,\mathbb{R})}, \mathrm{Nil}, \mathrm{Sol})$ e.g. minimal surfaces, CMC surfaces, parallel surfaces etc.

Key words: homogeneous geometry, isometry group, minimal surface

References

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