Planar kinematics and overconstrained spatial mechanisms

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It is the aim of this presentation to give an overview of last years' development in the field of overconstrained mechanisms. Based on the famous Heureka mechanism and its generalisations (see H. Stachel [8]) the so-called Fulleroid mechanisms were studied in the last few years by G. Kiper et al. ([1], [2], [3]), K. Wohlhart ([9], [10], [11]) and O. Röschel ([4], [5], [6], [7]). All these mechanisms are highly overconstrained and consist of rigid bodies linked by $1R$- or spherical $2R$-joints (a $2R$-joint is called "spherical" if its two rotary axes intersect in a point).

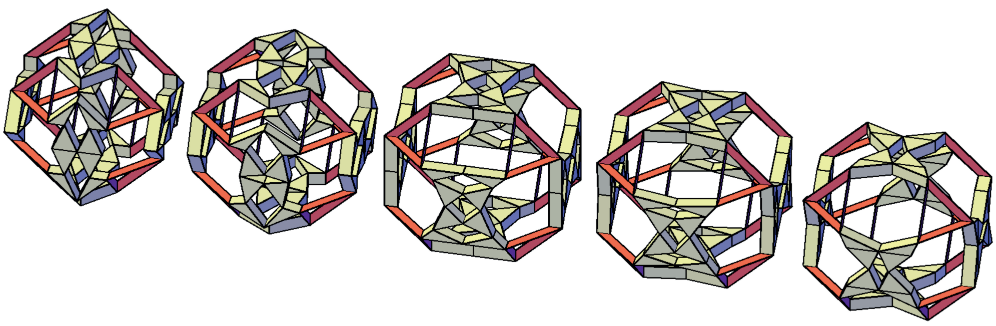
Many of these generalisations are based on observations on special planar mechanisms sharing an interesting property: Some coupler points (connected with the rigid parts of these planar mechansims) describe polygons P homothetic to a prototype polygon all throughout the corresponding self-motion. We present a few examples of such planar mechanisms.

Then we demonstrate how these planar results can be embedded into the faces of a polyhedron. This procedure yields further and new examples and generalisations of Fulleroid linkages. We present a range of new examples of this new type of overconstrained mechanisms. The figure below displays some positions of the self-motion of an example based on this idea (see [7]).

According to the Chebyshev-Grübler-Kutzbach formula we compute the theoretical degree of freedom of these linkages. Our construction uses planar sub-mechanisms - this formula can be specified in order to deliver a more precise value in these specific cases. This yields a modified Chebyshev-Grübler-Kutzbach formula.

Key words: Kinematics, robotics, Fulleroid-mechanism, self-motion, generalisations of Fulleroid-mechanism, modified Chebyshev-Grübler-Kutzbach formula

MSC 2010: 53A17



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