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ABSTRACTS

The organizer would wish to express gratitude for succeeding to incorporate all the abstracts specified in the program. Unfortunatelly, we did not succeed to translate all the abstracts so we have decided to publish them in the form in which they were originaly sent in.

Modern Research on Spatial Abilities - An Overview and New Results

R. A. Górska
Faculty of Architecture, Cracow University of Technology
e-mail: rgorska@pk.edu.pl

Research on spatial ability development has been conducted individually and in international co-operation over several subsequent years both in many countries as well as in Poland. Psychologists have always conceived spatial ability as a crucial factor of human intelligence. According to the results received in psychological research, spatial ability is one of the factors which is significant for future careers in technical and engineering professions. Piaget, in his factorial research conducted in the groups of young people, stated that spatial skills of young people rise skyrocketing at their pre-mature age while at the age of being mature the development of these skills slows down. The question arises whether the spatial skills of engineering students undergo enhancement if "trained" with appropriate educational means and methods. In particular, the educators in the field of "visual science", namely the teachers of such subjects as descriptive geometry, engineering graphics, computer graphics and many related subjects keep seeking for the best appropriate methods to prepare students to their future vocational life. Well developed visualization abilities are important conditions to all engineering students, where three-dimensional reality should be created, designed and changed.

Numerous researchers, in particular psychologists but also geometry and mathematics educators, have contributed to the discussion of the concept of spatial abilities. Spatial ability of a human being is directly related to visual perception. There are many theories on visual perception, one of them, which plays an important role in context of spatial visualization is the Gestalt theory. This theory was based on the findings by young psychologists Max Wertheimer, Wolfgang Köhler and Kurt Kofka. They formulated the "Gestalt" laws, among the others: the Law of Grouping, the Figure and the Ground law, the Law of Goodness, the Wholes versus Parts. Another psychological theories on visual perception, which are worth to be mentioned here are the theories developed by psychophysicists, namely the Concept of the Threshold, and the Empirical Approach to perception based on research done by Helmholz and R. Gregory. These theories will be explained during the presentation.

The theories on spatial abilities were proposed by various researches. According to L.L. Thurstone spatial ability is an important component of intellectual ability. He listed seven factors, three of these having to do with visual orientation in space, which he labeled S_1, S_2, and S_3. He identifies these factors by using various tests. J.W. French carried out a number of factorial investigations and called the spatial factors: the **space factor**, **spatial orientation** and spatial **visualization**. Each of these factors has been individually defined. Later, Michael, Guilford, Fruchter and Zimmerman attempted to synthesize the findings of the research on spatial abilities. McGee states that the recent as well as historical factor analytic studies provide strong and consistent support for the existence of at least **two** distinct spatial abilities: **spatial visualization** and **spatial orientation**. **Spatial visualization** is an ability to mentally manipulate, rotate, twist or invert pictorially presented visual stimuli. **Spatial orientation** involves comprehension of the arrangement of

the elements within a stimulus pattern, the aptitude to remain unconfused by the changing orientations of the object or of the body of the observer.

Significant gender differences exist in spatial skills on spatial tasks. As Linn and Petersen state "the sex differences in spatial ability are widely acknowledged, yet considerable dispute surrounds the magnitude, nature, and age of the first occurrence of these differences. Explanations of sex differences in spatial ability depend, to some extent, on when these differences occur. The hypothesized emergence of sex differences in spatial ability in early adolescence had led researchers to suggest explanations linked to pubertal change (Petersen and Waber). If, instead, sex differences are found to emerge prior to adolescence, then a biological explanation based on genetic factors (Witting and Petersen) or prenatal hormonal influences are preferred (Linn and Petersen)".

Various testing instruments have been used in international research on spatial abilities, to give some examples: the **Mental Cutting Test** (MCT: CEEB, 1939), the **Mental Rotations Test** (MRT: Vanderberg and Kuse, 1978), the **Differential Aptitude Test** (DAT, 1990), **TPS** - Spatial Imagination Test (Juščaková, 2003). These tests will be described in details during the presentation.

The results of the research conducted in Poland with the testing instruments listed above will be presented. Simultaneously, together with the testing instruments the special "background" questionnaires were developed to investigate various "background" factors, which were thought that they might have developed spatial skills. These factors included: age, handedness, play with construction toys, previous geometry instruction, vocational training, work experience, and participation in special types of sports (group sports). The responses to the background questions have been statistically analyzed. The research was conducted in the international co-operation at the Cracow University of Technology (CUT) in Poland, at the University of Kaiserslautern (UKL) in Germany and at the Michigan Technological University (MTU) in the USA. The results will be presented during the presentation.

During the last four years a new testing instrument, namely a TPS test has been developed by Zuzana Juščaková from the Technical University of Košice. This testing instrument development has been subsided by the Slovak Ministry of Education and conducted in co-operation with psychologists. A new testing instrument has been a point of interest of many educators in the field of geometry and graphics ("visual science") as it becomes a new and a modern tool to measuring spatial skills. The results of the pilot studies with the use of the TPS test have been presented in the paper. The test still undergoes a procedure of standardization and validation.

A 3-Dimensional Vector Calculus for the Projective Model of Hyperbolic Plane with Applications

E. Molnár

Department of Geometry, Budapest University of Technology and Economics

e-mail: emolnar@mail.bme.hu

A 3-dimensional real vector space V^3 endowed with a scalar product of signature $(+,+,+)$ is well known for describing the spherical geometry S^2. The usual vectorial (cross) product can also be introduced, and we get a complete duality (polarity) among points (unit vectors) and lines (equator circles by the normal unit vectors of spanning planes). Moreover, we can derive the classical formulas of spherical geometry.

We can generalize [1] this concept. First, for the projective sphere by the R^+ - equivalence of vectors (i.e. introducing rays for points), then for the projective plane where opposite rays are identified for a point. The scalar product on V^3 with signature $(-,+,+)$ leads to a hyperbolic metric on the projective sphere and on the projective plane with an absolute conic section and with a hyperbolic metric, respectively.

The hyperbolic analogon of cross product (first denoted by $\wedge$, wedge) can be introduced, the duality (polarity) works just in the same way, only boundary points (at the absolute) and their boundary lines (each touching the absolute in the corresponding boundary point) causes some extra discussions.

The machinery can be applied e.g. for the elementary triangle geometry, related with incycles and outcycles (circle, horocycle, hypercycle), etc. Geometry and algebra visually work together in an aesthetical way [2], [3]. The exact computations cause some difficulties, of course, but computer can help us in a natural manner. This is a planned joint work with Ivanka Babić, Ana Sliepčević and other croatian colleagues.

[1] E. Molnár: Projective metrics and hyperbolic volume. *Annales Univ. Sci. Budapest. Sect. Math.* 32 (1989), 127-157.

[2] A. Bölcskei, E. Molnár: How to design nice tilings? *KoG* 3 (Zagreb, 1998), 21-28.

[3] A. Bölcskei, E. Molnár: Graphische Realisierung der homogenen Dreieckpflasterungen in S^2, E^2 und H^2, Geometrie-Tagung "107 Jahre Drehfluchtprinzip", Vorau, 1-6. Juni 1997, Herausgeber: O. Röschel und H. Vogler, TU Graz, 1999, 11-20.

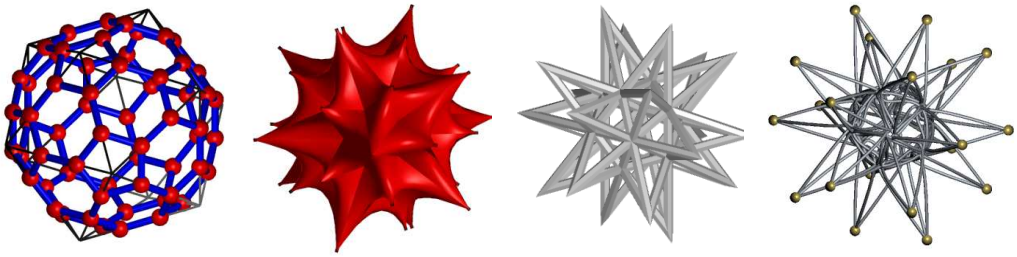
CAD-Packages and the Icosahedral Group: Patterns and Ornaments

O. Röschel
(joint work with S. Mick)
Institute of Geometry, Graz University of Technology
e-mail: roeschel@tugraz.at.

Professional CAD-packages are a powerful tool to study regular polyhedra and their ornaments within the geometric education of students. Now it is possible to focus on the structure and the symmetry of the geometric objects. Former problems concerning the visualisation of these ornaments in complicated hand - drawings are now solved by the CAD-package. This opens this field for the geometric education. The generation of such ornaments offers an excellent possibility to train geometric modeling and the handling of spatial displacements as well.

In our paper we study as an example the Icosahedral group $\boldsymbol{I}$ with its 60 direct displacements in detail. To obtain a structured generation of the ornaments we make use of the concept of blocks: It consists of blocks, which transform a motif M (as input block) into the corresponding ornament $(M,\ \boldsymbol{I})$ (as output block).

Now this structure is used to produce a series of interesting examples. They are gained by interchanging the motif M according to geometric properties of the desired ornament. This includes icosahedral pavements of the sphere, framework ornaments of polyhedra (of the icosahedral group) and some further examples. This fascinating field of ornaments shall stimulate the geometric use of professional CAD-packages.



[1] S. Mick, O. Röschel: Ausgewählte Beispiele für Ornamente und Stabmodelle der Ikosaedergruppe. *IBDG* (2005)(to appear)

Projections and their Reconstruction, a Standard Topic in Descriptive Geometry

H. Stachel

Institute of Discrete Mathematics and Geometry, Vienna University of Technology

e-mail: stachel@dmg.tuwien.ac.at

A central problem in Computer Vision is how to recover a 3D structure from a collection of 2D images. For more than 90 years this has been a standard problem of Descriptive Geometry and Photogrammetry (Remote Sensing), too. It will be demonstrated to which extent results from Descriptive Geometry can contribute to "Epipolar Geometry", which is the new name for the geometry of multiple images. Also the problem of how to distinguish photographs from linear images (e.g., photos of photographs) will be addressed.

New Method for Studying the Isotropic Plane

V. Volenec
Department of Mathematics, University of Zagreb
e-mail: volenec@math.hr

Each allowable triangle in an isotropic plane can be set in a standard position, in which it is possible to prove analytically in a simplified and easier way the geometric properties by means of suitable algebraic theory. For the proving the geometric facts for each allowable triangle it is sufficient to give a proof for a standard triangle. This method will be used for the introducing and studying of some geometric concepts in an isotropic plane.

The concepts of symmedians and the symmedian center of the triangle in an isotropic plane are defined. Some interesting relationships between the introduced concepts and other elements of the triangle in an isotropic plane are also studied. A certain number of these statements results from euclidean geometry, but some of them are new.

Polinomi u konačnim geometrijama

M. Andrić
Građevinsko-arhitektonski fakultet Sveučilišta u Splitu
e-mail: maja.andric@gradst.hr

Izlaganjem se daje uvod u lakunarne polinome (polinomi kojima su jedan ili više uzastopnih koeficijenata poslije vodećeg člana jednaki nuli) nad konačnim poljima. Pokazane su neke primjene ove teorije, kao što je problem određivanja broja smjerova definiranih grafom funkcije nad konačnim poljem, te veličina minimalne blokade u projektivnim ravninama.

[1] A. Blokhuis, S. Ball: *Polynomials in Finite Geometry.* Quaderni di matematica vol.5, Univ. Napoli, 1999.

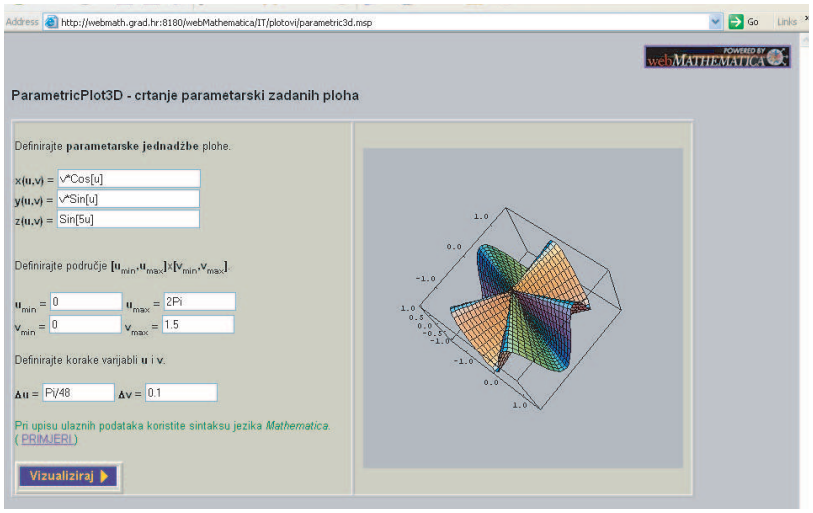
[2] L. Rédei: *Lacunary Polynomials over Finite Fields.* Akadémiai Kiadó, Budapest, 1973.

WebMathematica

V. Benić
Faculty of Civil Engineering University of Zagreb
e-mail: benic@grad.hr

S. Gorjanc
Faculty of Civil Engineering University of Zagreb
e-mail: sgorjanc@grad.hr

WebMathematica is a program which bridges web server and the program *Mathematica* and enables interactive calculations and visualizations on the web pages. The web pages on server have to be written in extended HTML which is defined by the rules of *webMathematica*. Such complex systems with very sophisticated graphics can be applied creatively in teaching geometry. Within the IT project[1] we installed *webMathematica* on the web server at the Faculty of Civil Engineering in Zagreb and designed some interactive web pages powered by this program which can be find at the address:
http://www.grad.hr/itproject_math/Links/webmath/indexeng.html



The print-screen of the web page with ParametricPlot3D. A user writes his inputs in white rectangles and by pressing the command button Visualize starts interactive communication. The result figure is *LiveGraphics3D* on computer.

WebMathematica je program koji povezuje web server s programom *Mathematica* te omogućuje interaktivni račun i vizualizacije na internetu. Stranica na serveru mora biti pisana u proširenom HTML jeziku kojeg određuju pravila *webMathematice*. Taj složeni sistem, s vrlo finom grafikom, može se kreativno upotrijebiti za učenje geometrije. U okviru rada na IT projektu[1] instalirali smo *webMathematicu* na serveru na Građevinskom fakultetu u Zagrebu i izradili nekoliko interaktivnih datoteka koje su dostupne na URL adresi:
http://www.grad.hr/itproject_math/Links/webmath/index.html

[1] http://www.grad.hr/itproject_math

The New SAT

J. Beban–Brkić
Geodetski fakultet Sveučilišta u Zagrebu
e-mail: jbeban@geof.hr

Svakodnevno smo suočeni sa željom naših mladih ljudi za školovanjem u inozemstvu. Najveći interes pokazuje se za studiranjem u Sjedinjenim Američkim Državama. Ako je već tomu tako sretna je okolnost da im se može pomoći u odbiru studija, kako na dodiplomskom tako i na poslijediplomskom nivou, omogućiti polaganje općih i posebnih klasifikacijskih ispita kao i organiziranim pripremama za polaganje istih.

Naime, informacije i usluge za koje je donedavno trebalo putovati izvan Hrvatske, sada se mogu dobiti kod kuće prvenstveno zahvaljujući *Studentskom informacijskom centru*, poznatom pod nazivom *SIC*.

SIC Zagreb je informativni i savjetodavni centar o mogućnostima visokog obrazovanja u Hrvatskoj i inozemstvu. Savjetodavna pomoć se odnosi na pomoć pri određivanju i ostvarivanju obrazovnih ciljeva kao i na stručnu pomoć tijekom procesa aplikacije. SIC Zagreb je jedini *U. S. Educational Information Center* za Hrvatsku ovlašten i potpomognut od *State Deparmenta* za pružanje informacija i savjeta o obrazovnim mogućnostima u SAD-u.

Moja suradnja sa SIC-om Zagreb započela je 2000. godine. Od tada držim pripremne tečajeve za polaganje matematičkog dijela SAT i GRE testova. SAT = *Scholastic Assessment Tests* i GRE = *Graduate Record Examination* su testovi kojima se služe koleđi i fakulteti u SAD-u za evaluaciju pristupnika prilikom upisa. Od 2004. godine u rad je uključena i kolegica mr. sc. Marija Šimić, sa Arhitektonskog fakulteta u Zagrebu.

Međutim, SAT test koji služi za upis na dodiplomski studij je izmijenjen i pod nazivom New SAT primjenjuje se od proljeća ove godine. U svjetlu promjena u obrazovnom procesu koji je kod nas u tijeku, s tendencijom smanjenja obima općeobrazovnih predmeta, iznenadila me činjenica da se u SAD-u javlja test koji je za razliku od prethodnog matematički zahtjevniji. Želja mi je ukazati na razlike i navesti neke primjere.

O četverodimenzionalnoj deskriptivnoj geometriji

Z. Božikov
Građevinsko-arhitektonski fakultet Sveučilišta u Splitu
e-mail: zdravka.bozikov@gradst.hr

N. Blagaić
Građevinsko-arhitektonski fakultet Sveučilišta u Splitu
e-mail: neda.blagaic@gradst.hr

Postoje brojne metode izučavanja koncepta 4-dim prostora. Zajednički im je pristup temeljen na analogiji s prostorima nižeg reda, budući da je izravnu vizualizaciju 4-dim prostora teško postići. Pristup problemu četvrte dimenzije sa stajališta projektivne geometrije ostvaruje se također proširenjem razmatranja analognih problema u 3-dim prostoru, pri čemu teorijska podloga prethodi grafičkom predočenju elemenata 4-dim prostora.

Teorijska podloga izučava relacije koje povezuju sve elemente referentnog sustava 4-dim prostora, s posebnim naglaskom na relaciju okomitosti, dok se za grafičko predočenje koristi prilagođena Mongeova metoda, naslijeđena iz 3-dim deskriptivne geometrije.

Četverodimenzionalni referentni sustav sastoji se od 4 međusobno okomita pravca koji u parovima određuju 6 međusobno okomitih ravnina, a koje pak u trojkama određuju 4 međusobno okomita 3-dim prostora.

Tako je položaj točke, kao osnovnog geometrijskog elementa svakog prostora, određen njenim udaljenostima od svakog od spomenutih 3-dim prostora.

Određenjem grafičkog predočenja točke lagano se ostvaruju i projekcije ostalih geometrijskih elemenata 4-dim referentnog sustava: pravca, ravnine i 3-dim prostora.

[1] C. E. S. Lindgren, S. M. Slaby: *Four-dimensional descriptive geometry.* McGraw-Hill Book Company, 1968.

[2] H. P. Manning: *The Fourth Dimension Simply Explained: A Collection of Essays Selected From Those Submitted in the Scientific American's Prize Competition.* University of Virginia Library Electronic Text Center, 1997.

Constructive Procedure for Transformation of Two Generally Collinear Adjusted Spaces Defined by Irregular and Real Autocollinear Tetrahedron

A. Čučaković
Faculty of Civil Engineering, University of Belgrade
e-mail: cucak@grf.bg.ac.yu

Two generally collinear adjusted spaces are incompatibile if any conventional projecting method is applied. They become compatibile if applied projecting method is based on invariants of two generally collinear adjusted spaces. Main invariant, which is used in this method, assumes that all straight lines of space Σ - parallel to vanishing plane $\mathbf{R}$ of space Σ and to one of the sides of autocollinear tetrahedron - has its corresponding straight line of space $\overline{\Sigma}$ - parallel to vanishing plane $\overline{\mathbf{Q}}$ of space $\overline{\Sigma}$ and to the corresponding side of autocollinear tetrahedron. Each point of two generally collinear adjusted spaces Σ and $\overline{\Sigma}$ is understood as intersection of two straight lines. In space Σ point A is intersection of two straight lines a_1, respectively a_2, is parallel to the line of intersection, of two planes - vanishing plane $\mathbf{R}$ and side $D_2D_3D_4$ of autocollinear tetrahedron, respectively of vanishing plane $\mathbf{R}$ and to the another side $D_1D_3D_4$ of autocollinear tetrahedron. Collinear adjusted point $\overline{A}$ in space $\overline{\Sigma}$, is intersection of two corresponding straight lines $\overline{a}_1$ and $\overline{a}_2$. Straight line $\overline{a}_1$,respectively $\overline{a}_2$, is parallel to the line of intersection, of vanishing plane $\overline{\mathbf{Q}}$ and to the side $D_2D_3D_4$ of autocollinear tetrahedron, respectively of vanishing plane $\overline{\mathbf{Q}}$ and to the another side $D_1D_3D_4$ of autocollinear tetrahedron.

Enhancing *Mathematics for Informatics* (*ICT*) and its Correlation with Student Pass Rates

B. Divjak
Faculty of Organization and Informatics, University of Zagreb
e-mail: blazenka.divjak@foi.hr

Z. Erjavec
Faculty of Organization and Informatics, University of Zagreb
e-mail: zlatko.erjavec@foi.hr

In this paper changes in *Mathematics for Informatics* (*ICT*) at the Faculty of Organisation and Informatics (FOI) in the University of Zagreb are described, and correlated with students pass rates. The main reason for introducing the changes in the curriculum and its delivery, reflecting internal pressures, was the very low student pass rate in Mathematics. The second reason, reflecting external pressure, was to address the requirements of the Bologna declaration in Higher Education (HE) in Croatia. The innovations were introduced in the following areas: content, methodology of teaching and learning, examination methods and support for students. The changes resulted in a significant increase in student pass rates and levels of student satisfaction.

Geometriespaziergang durch Zagreb

B. Hajsig
Fakultät für Architektur und Raumplanung, Universität Zagreb
e-mail: bhajsig@arhitekt.hr

In letzten 5 Jahren habe ich den Studenten im I+II Semester der Architektur und I+II+III+IV Semester Design Studium oft Architekturbeispiele und Designbeispiele gezeigt, die ich von verschiedenen Quellen aus aller Welt gesammelt habe (Bücher, Photos von Studienreisen, Internet etc.). Damit wollte ich die Vorlesungen aus Darstellender Geometrie durch praktische Beispielen bereichern. In einer Studentenumfrage bekam ich auch den Titel "Beste Professorin". Entscheidend war es auch dass ich den Studenten die Theorie & Praxis besonders anschaulich praesentiert habe. Deswegen hatte ich auch die Beispiele in Zagreb gesammelt. Zagreb ist eine alte mitteleuropäische Stadt, die sich als Kultur-, Wirtschafts-,Wissenschaftszentrum entwickelte. Im dritten Jahrtausend schreitet die Haupstadt als eine Millionenstadt mit attraktiver Architektur in die Zukunft, hat aber die ihr eigene Schönheit, Ruhe und entspannte Atmosphäre bewahrt. Mit dem Photoapparat machte ich dann im Sommer einen "Geometriespaziergang" durch die Stadt. Eine Auswahl dieser Photos wird im Vortrag gezeigt.

Affine Regular Icosahedron in GS–Quasigroup

Z. Kolar–Begović
Department of Mathematics, University of Osijek
e-mail: zkolar@mathos.hr

R. Kolar–Šuper
Teacher Training College, University of Osijek
e-mail: rkolar@vusos.hr

A GS–qusigroup is defined as an idempotent quasigroup which satisfies the identities $a(ab \cdot c) \cdot c = b$, $a \cdot (a \cdot bc)c = b$. Some interesting geometric concepts can be defined in a general GS–quasigroup. The concept of the affine regular icosahedron in general GS–quasigroup is introduced by means of twelve ARP relations which are valid for five out of twelve points. The theorem of the unique determination of the affine regular icosahedron will be proved by means of its four vertices which satisfy certain conditions. The connection between an affine regular icosahedron and a dodecahedron in a general GS–quasigroup will be researched. The geometrical representation of the introduced concepts and relations between them will be given in the GS–quasigroup $C(\frac{1}{2}(1+\sqrt{5}))$.

Zlatni rez

D. Kušar
Arhitektonski fakultet Sveučilišta u Ljubljani
e-mail: domen.kusar@arh.uni-lj.si

Pojam zlatnog reza povezuje različite profesije. Prvenstveno pripada matematici i geometriji, a nalazimo ga, kao simbol svega lijepog, i u svim područjima umjetnosti. Pronaći ga se može i u prirodi. Zlatni je rez vjerojatno bio vrlo značajan u antičkom i kasnom tesarstvu - kod dimenzioniranja optimalnih drvenih nosača, gdje možemo tražiti i izvor njegovog imena. Kasnije, proporcija zlatnog reza bila je tehnološki "Know how" i dobro skrivena unutar tesarskog ceha. Tu se vjerojatno skriva i razlog za kasnu pojavu samog imena "Zlatni rez" i sličnost naziva zlatni rez (rezati, sjeći) u različitim europskim jezicima.

Some Mappings Between Surfaces in Cayley-Klein Spaces

Ž. Milin–Šipuš
Department of Mathematics, University of Zagreb
e-mail: milin@math.hr

B. Divjak
Faculty of Organization and Informatics, University of Zagreb
e-mail: blazenka.divjak@foi.hr

We describe some classes of mappings between surfaces in the pseudo-Galilean and simply-isotropic space, such as, isometries between surfaces, conformal and equiareal mappings. We compare these results with the analogues results for the mappings in the Euclidean space. In particular, we also study Minding's isometries of ruled surfaces in these spaces, i.e., isometries that preserve the generators of surfaces.

Iskustva oko izobrazbe studenata učiteljskih studija za nastavu geometrije u osnovnoj školi

Experiences in Education Teacher Studies Student for Teaching Elementary School Geometry

M. Pavleković
Visoka učiteljska škola Sveučilišta u Osijeku
e-mail: pavlekovic@vusos.hr

S. Duka
Visoka učiteljska škola Sveučilišta u Osijeku
e-mail: duka@ffos.hr

U radu se prezentiraju polazišta i navode neka iskustva oko izobrazbe studenata učiteljskih studija za nastavu geometrije u osnovnoj školi. Naglasak je na poticanju pronalazaštva.

In this research the authors present starting points and describe some of the experiences connected to educating students of teacher studies for teaching geometry on elementary school level. The emphasis is placed on encouraging the spirit of discovery in students.

Examples of Multimedial Tools in Geometry Teaching

L. Pletenac
Civil Engineering Faculty, University of Rijeka
e-mail: pletenac@gradri.hr

K. Jurasić[2]
Technical Faculty, University of Rijeka
e-mail: jurasic@rijeka.riteh.hr

Fast development of computer graphics is based mostly on geometry and other mathematical disciplines. Our future engineers need new geometric computer graphics very much. New technological possibilities offer new opportunities, which are not only the new applications of old descriptive geometry (and theories in general) but also need to introduce new theoretical approaches and develop new theories.

The introduction of the new concept is necessary in the process of university education. It is not enough to modernize classical courses of geometry using a computer for presentations. Geometry (or mathematics) can not be only classical and "lonely" any more, but it has to interact with theory and practice of computer graphics. This is the reason for introducing a new interdisciplinary curriculum of computer constructive geometry at the University of Rijeka (Civil Engineering Faculty). In February 2004, the conception of the new curriculum was presented to HDKGIKG, which was oriented to the engineering profession, with CAD as a graphic tool.

Here we present some examples of the multimedia approach in mathematical education of our students. New media in the teaching of geometry are: the examples of animation using the software *Mathematica*, teaching materials available on Internet, lectures in "*MS PowerPoint*" combining CAD, photos and sound, and examination using computers. We use the constructive way to manipulate geometric structures in 3D and their plane projections, using CAD software.

After making some teaching materials available on Internet, we have examined the effect on the students' understanding and learning. About 80% of students have been very satisfied with the new multimedial approach in the geometry teaching. About 60% of students want to learn geometry with graphics and CAD in some combination.

As a conclusion, geometry at universities should be taught along with computer graphics, using CAD: while students are trying to solve the problem, the geometric theorems become the rules of the geometric game.

Examples are prepared as lessons for students, within the project "Actual parts of mathematic and geometry in engineering graphics and practice", which is supported by the Foundation of the University of Rijeka.

[2]Supported by the Foundation of the University of Rijeka

On a Special Class of Parallelogram Quasigroups

M. Polonijo
Department of Mathematics, University of Zagreb
e-mail: polonijo@math.hr

In 1963 F.Ostermann and J.Scmidt introduced the notion of parallelogram space. Many characterizations of such spaces are known. The best one is given by a subtractive groupoid - i.e. parallelogram space is described in terms of subtraction in an abelian group. Parallelogram quasigroup is defined by a very simple implication, and such quasigroup is a natural generalization of the subtractive groupoid. The corresponding binary operation could be expressed by a bijection acting on subtraction in an abelian group.

We will show that the special class of parallelogram quasigroup, for which the previously mentioned bijection is an automorphism, could be given by an identity.

Zrcalne slike u perspektivi

N. Sudeta
Arhitektonski fakultet fakultet Sveučilišta u Zagrebu
e-mail: nikoleta.sudeta@arhitekt.hr
M. Šimić
Arhitektonski fakultet fakultet Sveučilišta u Zagrebu
e-mail: marija.simic@arhitekt.hr

Ako je neka građevina projektirana tako da se nalazi u neposrednoj blizini vodene površine ili ispred vertikalne staklene fasade susjedne zgrade njena se perspektivna slika često upotpunjuje zrcalnim slikama. Analogno vrijedi i za interijer u kojem se nalaze vertikalna ili koso postavljena zrcala ili glatke i sjajne podne ili stropne obloge.

Konstrukcija zrcalne slike na nekoj ravnini temelji se na zakonu geometrijske optike pa je zrcalna slika virtualna slika objekta simetrična s obzirom na zrcalnu ravninu. Pri tom je kut kojeg zatvara upadna zraka svjetla s ravninom zrcala jednak kutu koji čini reflektirana zraka s tom istom ravninom. Svaka zraka sa svojom reflektiranom zrakom leži u ravnini koja je okomita na ravninu zrcala. Zbog toga točka i njena zrcalna slika leže na pravcu okomitom na ravninu zrcala od kojeg su jednako udaljene. Zrcalna slika objekta je određena zrcalnim slikama njegovih točaka, bridova i ploha.

Razmatraju se slučajevi zrcalnih slika u perspektivi ovisno o položaju ravnine zrcala prema ravnini slike i osnovnoj horizontalnoj ravnini. U slučaju kad se ravnina zrcala podudara s horizontalnom ravninom ili je paralelna s njom okomica na kojoj leže perspektivne slike točke i njene zrcalne slike paralelna je s ravninom slike pa se čuvaju omjeri dužina. U primjeru bokocrtne ravnine zrcala pokazuje se konstrukcija zrcalne slike prizme u perspektivi iz njenog ortogonalnog i zrcalnog tlocrta. Koristeći svojstva geometrijske optike lako se dokaže da su nedogledi horizontalnih pravaca na kojima leže bridovi prizme i nedogledi njihovih zrcalnih slika incidentni s horizontom i simetrični s obzirom na glavnu točku O_c. Naravno, moguća je i neposredna konstrukcija perspektivne zrcalne slike bez korištenja zrcalnog tlocrta što se već vidi i u slučaju frontalnog položaja ravnine zrcala. Tada su pravci okomiti na ravninu zrcala ujedno okomiti i na ravninu slike pa je njihov nedogled glavna točka. I ovdje nedogledi horizontalnih bridova i njihovih zrcalnih slika leže na horizontu i simetrični su s obzirom na glavnu točku. Pokazani su i primjeri zrcala koji su donjim horizontalnim bridom pričvršćeni za bokocrtnu, odnosno frontalnu ravninu. Na kraju se promatra vertikalna ravnina zrcala koso položena prema ravnini slike. Konstrukcija nedogleda zrcalnih slika horizontalnih bridova temelji se na mjerenju prave veličine kuta u horizontalnoj ravnini.

[1] W. Abbott: *Theory and Practice of Perspective.* Blackie&Son Limited, London and Glasgow, 1964.

[2] J. Božičević: *Linearna perspektiva.* Zagreb, 1942.

[3] U. Linke: *Architekturperspektive für Studium und Praxis.* Bauverlag, Wiesbaden-Berlin, 1991.

[4] V. Niče: *Perspektiva.* Školska knjiga, Zagreb, 1978.

Karakteristične točke trokuta u projektivno proširenoj hiperboličkoj ravnini

Charakteristische Dreieckpunkte in der projektiv-erweiterten hyperbolischen Ebene

A. Sliepčević
Građevinski fakultet Sveučilišta u Zagrebu
e-mail: anas@grad.hr

I. Babić
Graditeljski odjel Tehničkog Veleučilišta u Zagrebu
e-mail: ibabic@tvz.hr

Na Kleinovom modelu hiperboličke ravnine prikazane su neke osnovne planimetrijske konstrukcije u vezi s dužinama, kutovima i trokutima uz usporedbu sa situacijom u euklidskoj ravnini.

Učinjena je klasifikacija H-trokuta u odnosu na položaj njegovih vrhova i stranica prema apsoluti. U tom smislu su trokuti razvrstani na 27 tipova. Za svaki tip trokuta je utvrđeno koje karakteristične točke posjeduje i koliko.

Karakteristične točke konstruirane su na nekoliko primjera raznih tipova trokuta H-ravnine.

An dem Kleinischen Modell der hyperbolischen Ebene wurden einige gründliche planimetrische Konstruktionen in der Verbindung mit den Strecken, Winkeln und Dreiecken durchgeführt. Dabei wurde der Vergleich mit dem euklidischen Fall gemacht.

Man führte eine Klassifikation der Dreiecke in der H-Ebene nach den Typen der Eckpunkte und nach der Lagen ihrer Seiten im Bezug auf den absoluten Kegelschnitt aus. In diesem Sinn existieren die 27 Dreiecktypen. Für jeden Dreiecktyp stellte man fest wieviel und welche charakteristische Dreieckpunkte besitzt.

Solche Punkte wurden an den einigen Beispielen der verschiedenen Dreiecktypen konstruiert.

[1] I. Babić, B. Kučinić: PM-Modell des hyperbolischen H3-Raums in der Möbius-Ebene. *Rad HAZU* 467 (1994), 67-75.

[2] V.F. Kagan: *Osnovanija geometrii, č.1. Geometrija Lobačevskogo i jejo predistorija.* Moskva - Lenjingrad, 1949.

[3] F. Klein: *Vorlesungen über nicht-euklidische Geometrie.* Berlin, Springer-Verlag, 1928., Nachdruck 1968.

[4] V.I. Kostin: *Osnovanija geometrii.* Moskva-Lenjingrad, 1946.

[5] L. Rajčić: Obrada osnovnih planimetrijskih konstrukcija geometrije Lobačevskog sintetičkim sredstvima. *Glasnik mat. Fiz. Astr.* 5 (1950), 57-120.

Methods of Representation Course at the Graz University of Technology - New Media, Classical Construction, Geometrical Freehand Drawing - New Dimension in Geometrical Education

M. Stavrić
Institute of Architecture and Media, Graz University of Technology
e-mail: mstavric@tugraz.at

A. Wiltsche
Institute for Geometry, Graz University of Technology
e-mail: wiltsche@tugraz.at

H. Schimek
Institute of Architecture and Media, Graz University of Technology
e-mail: schimek@TUGraz.at

This talk discusses the course on Methods of representation that has evolved at the Faculty of Architecture in Graz in conjunction with developments in the modern practice of architectural design. The course focuses on classical geometrical representation methods, geometrical freehand drawing and on solid modelling in CAD. We established web sites (http://www.geometrie.tugraz.at/lehre/dm_ue03 and http://ikg.tugraz.at/dm0/s05/index.phtml), which includes the introduction in teaching, tutorials and VRLM animations to help students understand space geometry. Each of these parts will be worked out and examples of student exercises will complete the presentation.

C^1-continuous Coons-type blending of triangular patches

M. Szilvási-Nagy
Department of Geometry, Budapest University of Technology and Economics
e-mail: szilvasi@math.bme.hu

The presented surface definition is based on the classical interpolation method for triangles, where the constructed function of two variables has given values on the boundary of a given triangle. The original formulation of the solution of this interpolation problem is the following [1].

If the real-valued function $F(x,y)$ is continuous on the triangle T with vertices $(0,0)$, $(1,0)$ and $(0,1)$ in the xy plane, then the function given by

$$\begin{aligned} W(x,y) &= \frac{1}{2}\{ \left[\frac{1-x}{1-y}F(y,y) + \frac{1-x}{1-y}F(1,y)\right] + \left[\frac{x-y}{x}F(x,0) + \frac{y}{y}F(x,x)\right] \\ &+ \left[\frac{1-x}{1-x+y}F(x-y,0) + \frac{y}{1-x+y}F(1,1-x+y)\right] \\ &- [(1-x)F(0,0) + (x-y)F(1,0) + yF(1,1)]\,\} \end{aligned}$$

is continuous over T and interpolates to the values of F on its boundary.

A geometric interpretation of this interpolation problem is the construction of Gordon-Coons triangular surface patches, which is the triangular version of the well-known construction of rectangular Coons patches [2]. A Gordon-Coons surface patch is generated by the above formula from three continuous curve segments forming a spatial curvilinear triangle, which are the boundary curves of the generated patch.

Both surface constructions have been extended to surface constituents. A rectangular Coons-type patch has been constructed from four rectangular surface patches instead of four boundary curves in [3], and the generalized Gordon-Coons patch has been defined by three continuous triangular surface patches in [4].

In this paper a new definition of a triangular Gordon-Coons-type surface patch will be given. It is determined by three differentiable triangular patches, where one boundary curve of each form a curvilinear triangle, and the resulting patch fits these boundary curves and has a C^1-continuous connection to the given constituents along them.

[1] R.E. Barnhill, G. Birkhoff, W.S. Gordon: Smooth Interpolation in Triangles. *Journal of Approximation Theory* 8 (1973), 114-128.

[2] S.A. Coons: Surfaces for computer-aided design of space forms. *Project MAC report*, 1964.

[3] M. Szilvási-Nagy, T.P. Vendel, H. Stachel: C^2 filling of gaps by convex combination of surfaces under boundary constrains. *KoG* 6 (2002), 41-48.

[4] Filling triangular holes by convex combination of surfaces. *Periodica Polytechnica Mech. Engrg.* 47 (2003), 81-89.

Lehrgang der Darstellenden geometrie ohne Geometriewerkzeug

V. Szirovicza
Građevinski fakultet Sveučilišta u Zagrebu
e-mail: szvlasta@grad.hr

Analiziran je stav tehničkih fakulteta u Hrvatskoj prema kolegiju Nacrtna geometrija u smislu nove reforme studija.

Napravljena je usporedba u izvođenju nastave nacrtne geometrije klasičnim načinom i uz pomoć računala (programa Microsoft PowerPoint). Obrazložene su prednosti rada s računalom.

Predstavljen je novi sveučilišni udžbenik u obliku Compact Disca.

Two-Axial Surfaces of Revolution

D. Velichová
Department of Mathematics, Slovak University of Technology in Bratislava
e-mail: daniela.velichova@stuba.sk

The presentation gives short information on a special class of surfaces created by the Euclidean metric transformation composed from two revolutions about two different axes. There is presented a classification of the particular class of surfaces based on the superposition of the axes of revolutions. Different types of surfaces can be recognized and defined on the base of specifying the type of the surface basic figure within separate groups.

Concatenated transformation of general revolution about two axes can be analytically represented as the product of multiplication of two square matrices of type 4x4 of revolutions about given axes. For the sake of easy formulas, the special positions of axes of revolutions are chosen in the coordinate axes or in lines parallel to the coordinate axes.

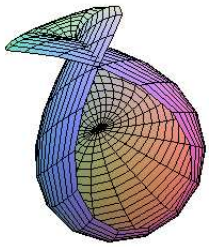
Three general types of superposition of two lines in the space (parallel, intersecting and skew) predetermine the three specific groups of two-axial surfaces of revolution that can be defined as:

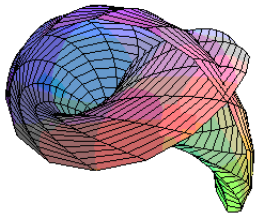
1. Cycloidal-type, 2. Spherical-type, 3. Euler-type.

Determination of the surface basic curve enables to specify several types in all three defined groups. Choosing the line (line segment) as the basic curve, cylindrical, conical and hyperbolical two-axial surfaces can be created within each of the three different groups, while toroidal two-axial surfaces result by circle (circular arcs) chosen as the surface basic curve.

Vector (point) equations of the defined two-axial surfaces of revolution can be easily derived for all specific types and groups. Several illustrations of the special type representatives from the three specific groups of two-axial surfaces are presented by their analytic representations and visualized in the programme system MAPLE.

Two-axial surfaces of revolution can be utilised in mechanical engineering, for determination of the trajectories of composed rotational movements.





[1] H. Goldstein: *The "Euler Angles" and "Euler Angles in Alternate Conventions"*. 2nd ed. Reading, MA: Addison-Wesley, 1980.

[2] D. Velichová: Euler angles. Proceedings of the 3rd International Conference on Applied Mathematics APLIMAT 2004, SjF STU Bratislava, SR, ISBN 80-227-1995-1, 2004, pp. 191 - 198.

[3] S. Zacheriáš, D. Velichová: Projection from 4D to 3D. *Journal for Geometry and Graphics* 4 (2000), No. 1, Helderman Verlag, ISSN 1433-8157, 2000, pp. 55 - 69.

The Role of Graphics in Enhancing Spatial Mobility European Senior Citizens - an International Approach

L. Żakowska
Faculty of Architecture, Cracow University of Technology
e-mail: : lidia.zakowska@neostrada.pl

This work presents qualitative results of the on-going research project SIZE (life quality of senior citizens in relation to mobility conditions), which is founded by the EC within the 5^{th} framework programme, RTD programme "quality of life and living resources", key action "the ageing population and disabilities", contract No QLK6-CT-2002-02399.

The general objectives of SIZE are: (1) to explain and describe the present mobility and transport situation, the problems, needs and wishes of different groups of senior citizens from their own perspective compared with experts' points of view, (2) to motivate action by the authorities and other relevant groups in society who are, or feel, responsible in this area, among others by making discrepancies in problem identification transparent, (3) to identify relevant solutions for existing problems and to provide guidance for setting up and implementing policies aimed at "keeping the elderly mobile".

Nine research partners (representing eight European countries, namely Austria, Czech, Ireland, Germany, Italy, Poland, Spain and Sweden), supported by five senior's organizations, followed the same methodology of interviewing seniors and experts, in order to compare the results. Qualitative studies for both groups, seniors and experts, show many interesting technical solutions for implementation. The most interesting solution, common to all countries and regions, deals with wider introduction of graphical information, instead of the traditional written one.

Legibility of time-tables of public transport and other information in public spaces and buildings are very important to enhance seniors mobility. However, according to our experts, boards on the bus stops and license plates are often illegible and unreadable. There is lack of voice information in public transport due to citizens' protests against loudness. Experts strongly advised making all information about public transport more accessible for elder citizens (flyers, websites, free phone lines, media announcements).

One of the most negative aspect of aging is, that fitness decrease with age. Memory, sight and hearing weaken with age. Coping with new situations also becomes harder. Urban infrastructure is often not adjusted to seniors' needs, thus, seniors report difficulties with handling simple leaving conditions. Generally speaking, elderly are aware of their limited senses and try to adapt their expectations to these limitations.

Due to the technical progress, a lot of useful devices that enhance senior citizens' mobility are employed in every day life. However, technical perspective is always connected with personal one, because seniors often manifest a strong fear to new devices and technologies. It is much easier to older citizens to understand the simple graphical language, than to recognize complicated text in a short time during movement in a dynamic urban environment. The research findings show that graphic is an international, intercultural language, very important in creation of safer and more mobile future for European citizens.