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# Methods for determination of centre of stiffness and torsional radius in multi-storey buildings

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## Abstract

In order to compare the different approach for the calculation of eccentricity of centre of stiffness and centre of mass and torsional radius, four different five storey buildings with different degree of plan irregularity were created. Four procedures for determination of centre of stiffness and torsional radius, which differ among themselves according to the distribution of the lateral loading over height are presented. The value of the torsional radius is inversely proportional to the level of irregularity. By increasing of irregularity degree, the ratio between the eccentricity obtained for the first and for the fourth distribution, decreases. By comparing the results obtained for different lateral load distribution it can be noted that from the results obtained by one method, the structure can be defined as plan regular, while from another as plan irregular. First distribution is most conservative, while the fourth is most flexible. All base isolated models are characterized as regular in plan, which is an indicator of the advantages of the base isolated structures in case of irregularity in plan.

***Keywords:*** *plan irregularity, centre of stiffness, torsional radius*

# Metode za određivanja centra krutosti i torzijskog polumjera višekatnica

## Sažetak

Četiri različite petokatnice, s različitim stupnjem nepravilnosti u tlocrtu, izgrađene su u svrhu uspoređivanja raznih pristupa za proračun ekscentričnosti centra krutosti, središta mase i torzijskog polumjera. U radu su prikazana četiri postupka za određivanje centra krutosti i torzijskog polumjera koji se međusobno razlikuju po raspodjeli bočnog opterećenja po visini. Vrijednost torzijskog polumjera obrnuto je proporcionalna stupnju nepravilnosti. Povećavanjem stupnja nepravilnosti, smanjuje se odnos između ekscentričnosti dobiven za prvu i četvrtu raspodjelu. Uspoređivanjem rezultata dobivenih za razne raspodjele bočnog opterećenja može se ustanoviti da se iz rezultata dobivenih jednom metodom konstrukcija može definirati kao građevina pravilnog tlocrta, dok rezultati druge metoda pokazuju da je tlocrt nepravilan. Prva je raspodjela najkonzervativnija a četvrta najfleksibilnija. Svi bazno izolirani modeli okarakterizirani su kao građevine pravilnog tlocrta, što je indikator prednosti bazno izoliranih konstrukcija u slučaju nepravilnosti u tlocrtu.

***Ključne riječi:*** *tlocrtna nepravilnost, centar krutosti, torzijski polumjer*

## 1 Introduction

Irregularity in plan may have negative implication on the design process and on the behaviour of the structures exposed to earthquake loadin Structures with eccentricity between the centre of the mass and centre of stiffness or with a lack of minimal torsional rigidity can undergo coupled lateral and torsional motions during earthquakes, which can significantly increase the seismic demand. One of the possible solutions for reducing these side effects is the application of base isolation systems. Most of the seismic design codes contain provisions for control of structural irregularities. If the prescript criteria for regularity are not satisfied, certain restrictions related with the selection of method or numerical model for seismic analysis have to be done.

## 2 Criteria for plan irregularity

In principle, conventional designed structures with respect to the lateral stiffness and mass distribution shall be approximately symmetrical in plan with respect to two orthogonal axes. According to EN 1998-1 [1], buildings can be characterized as regular in plan, if six different conditions at all storey levels are satisfied. Some of these conditions are qualitative and can be checked in the preliminary design stage. The conditions that are based on the eccentricity between the centre of mass and the centre of stiffness or torsional radius, Eq. (1) and Eq. (2), are quantities that have to be calculated additionally.

$$e_{ox} \leq 0.3 r_x, \quad e_{oy} \leq 0.3 r_y \tag{1}$$

$$r_x \leq l_s, \qquad r_y \leq l_s, \tag{2}$$

In these equations $e_{ox/oy}$ is eccentricity and $r_{x/y}$ is torsional radius in the considered direction, while $l_s$ is the radius of gyration of the floor mass in plan. In multi-storey buildings only approximate definitions of the centre of stiffness and of the torsional radius are possible, because those parameters are not uniquely defined and depend on the distribution of lateral loading with height.

## 3 Determination of centre of stiffness and torsional radius in multi storey structures

The centre of lateral stiffness is defined as point in plan with the property that any set of horizontal forces applied at floor levels through that point produce only translation of the individual storeys, without any rotation with respect to the vertical axis. Conversely, any set of storey torques produce only rotation at the floors about the vertical axis that passes through the centre of lateral stiffness, without horizontal displacement of that point at any storey. The torsional radius is defined as the square root of the ratio of the torsional stiffness and the lateral stiffness in the considered direction.

$$r_x = \sqrt{\frac{K_\theta}{K_y}}, \qquad r_y = \sqrt{\frac{K_\theta}{K_x}} \tag{3}$$

Torsional radius, as a structural characteristic, represents the potential for torsional vibration of structure exposed to earthquake ground motion.
Because EN 1998-1 does not provide a procedure for determination of centre of stiffness and torsional radius, the national annex should include reference to documents that provide definitions of these structural characteristics in multi-storey buildings. In Figure 1 four procedures for determination of these characteristics, which differ among themselves according to the distribution of the lateral loading over height are presented.

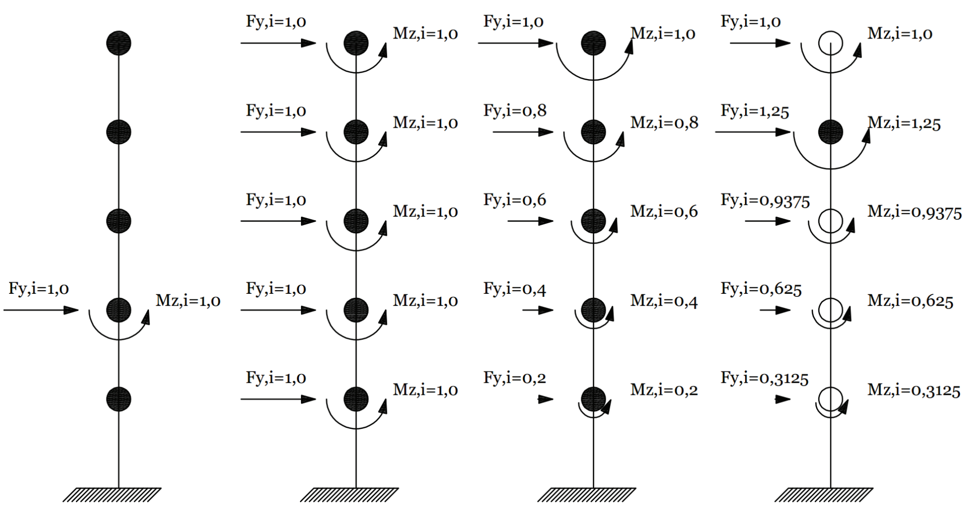


**Figure 1. Schematic representation of methods for determination of centre of stiffness and torsional radius**

According to Fajfar et al. [2], determination of these structural characteristics is performed for each level individually. Three static load cases are necessary to be defined for each storey level, two single forces in x and y direction and one torsional moment about the vertical axis, which are applied in the centre of mass. After determination of the centre of stiffness i.e. eccentricity, similar procedure is repeated for determination of the torsional radius, but in this step the loads are applied in the centre of stiffness. The second and the third method [3] for determination of these structural characteristics differ from the first. In these methods the lateral forces and torsional moments are not applied for each level individually, but to all levels simultaneously. The second method has uniform lateral load distribution over height, while the third has triangular. In the fourth method [4], the centre of stiffness and torsional radius are not determined by individual levels, but for the whole structure. The first step is determining horizontal lateral forces, which are proportional to the product of the floor mass and height i.e. are calculated according to the principle of equivalent static forces. The structure is analysed with applied torsional moments on each level, proportional to the forces. The centre of rotation of each level due to the action of the torsional moments is geometrically determined. The horizontal projection of the centre of rotation to 80 % of the total height of the structure is considered as the centre of stiffness of the whole structure. In the second step, horizontal forces in both directions, numerically equal to the torsional moments of the previous analysis, are applied to the centre of stiffness. With the results of this analysis, the torsional radius can be obtained.

## 4 Numerical example

### 4.1 Description of the analysed structures

In order to compare the different approach for the calculation of eccentricity of centre of stiffness and centre of mass and torsional radius, four different five storey buildings with different degree of plan irregularity were created. All analysed structures are rectangular in plan and consists three frames in x, and five frames in the global y direction, Figure 2. The distance between the frames is 5 m in both direction, while the story height is 3 m. The columns are rectangular with a dimensions 50/50 cm at the first two stories and 45/45 cm on the above 3 stories. All beams are 40/45 cm, as well as the slab's thickness is 15 cm. The first structure is regular. In the remaining three structures the middle column in the first frame in y direction is replaced with the RC wall with dimensions 120/40 cm, 160/40 cm and 200/40 cm respectively.

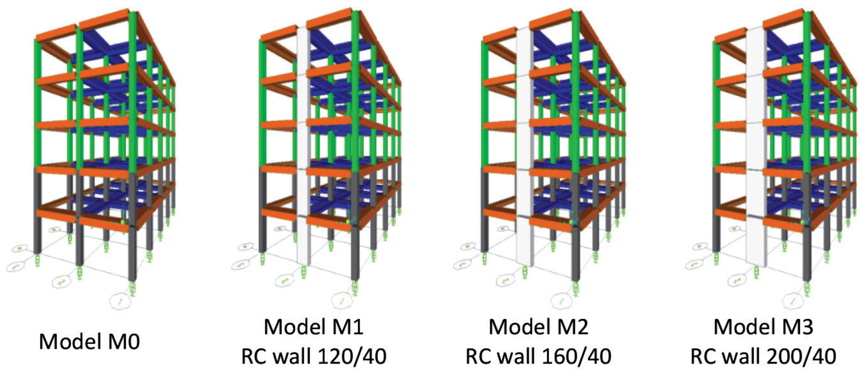


**Figure 2. Mathematical models of analysed structures**

For comparison, besides fixed base, the same models are analysed as base isolated. For the base isolated structures elastomeric bearings made by Italian company FIP Industriale are used. Selected isolator type SI-N 350/125 is with effective secant stiffness $K_{eff}$=620kN/m and maximal displacement of 250 mm. The analyses are performed with the software SAP2000, for the both types of structures.

## 4.2 Comparison of results

In regular structures centre of mass and centre of stiffness are coincident. From the results presented in Figure 3 it can be concluded that by increasing of RC wall width the eccentricity of each level is also increasing. Eccentricity is the largest on the first floor and for structure M3 is in range of 3.23 to 5.51 m for the four distributions of lateral load. The largest differences between the centre of stiffness on the first and the last floor are obtained from the first lateral load distribution. The differences obtained from the second and from the third distribution are smaller. Centre of stiffness obtained from the fourth lateral load distribution is uniquely at all floors. This is due to the fact that centre of stiffness obtained at 80 % of the height of the structure is considered as a centre of stiffness of the whole structure. By increasing of irregularity degree, the ratio between the eccentricity obtained for the first and for the fourth distribution, decreases.

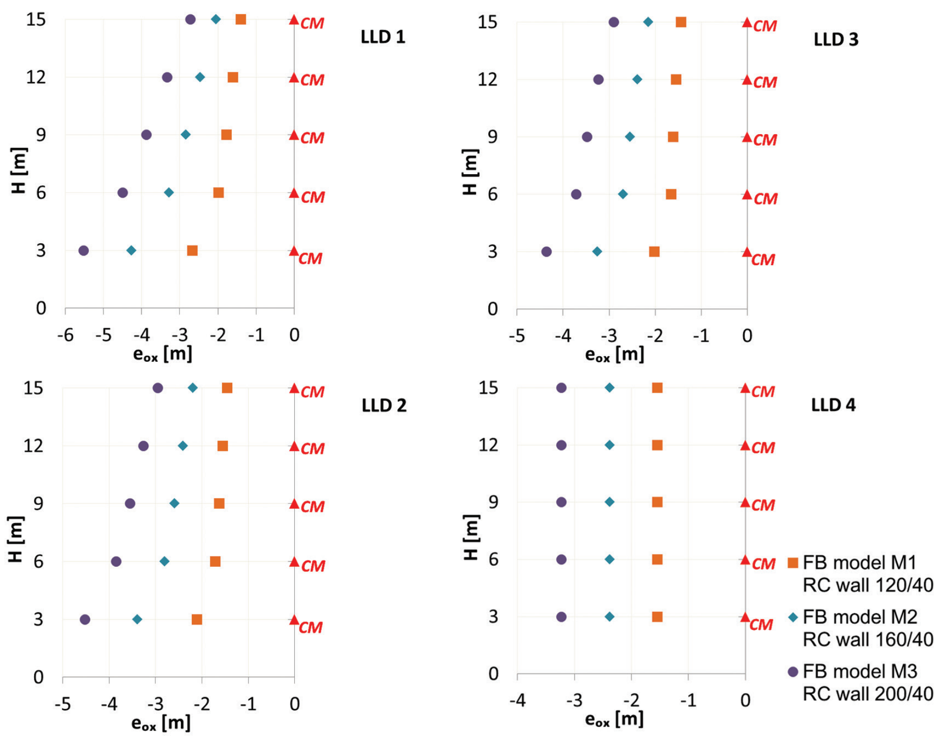


**Figure 3. Eccentricity values of analysed structures for different lateral load distribution**

The value of the torsional radius is inversely proportional to the level of irregularity, Figure 4. By increasing of RC wall width, the value of the torsion radius decreases. For the structure M1 is in range from 8.28 to 8.46 m, while for the structure M3 is in range from 7.40 to 8.39 m. Torsional radius is smallest on the first floor and increases on the upper levels. By comparing the results obtained for different lateral load distribution it can be noted that from the results obtained by one method, the structure can be defined as plan regular, while from another as plan irregular. The first distribution is most conservative, while the fourth is most flexible.

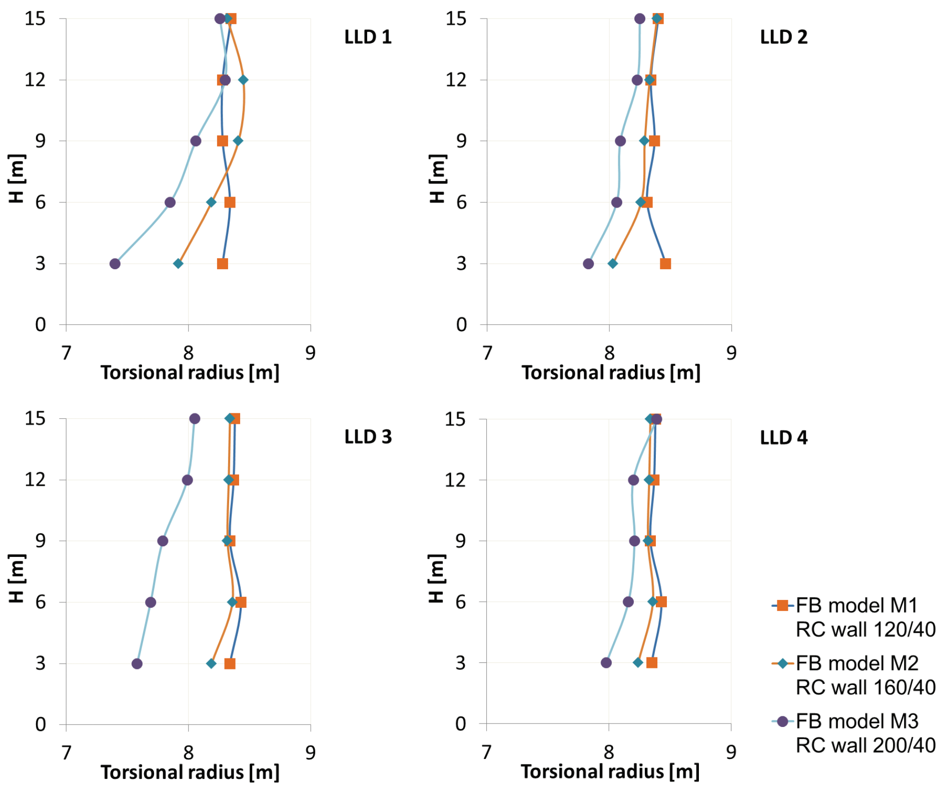


**Figure 4. Values of torsional radius for different lateral load distribution**

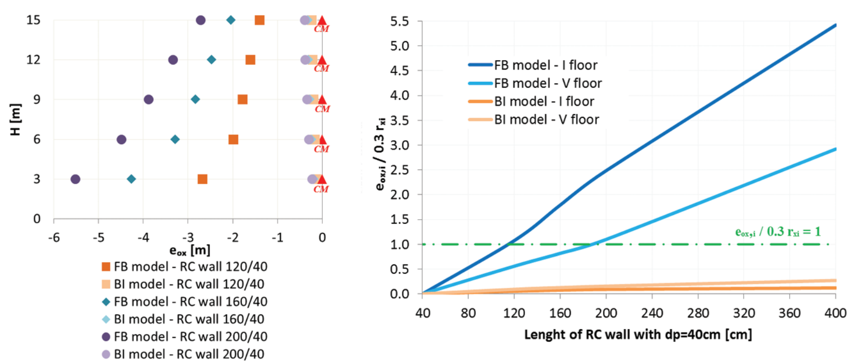


**Figure 5. Obtained eccentricity for fixed and base isolated models and control of plan irregularity**

Figure 5 present the coordinates of centre of stiffness, for fixed and base isolated models, with respect to the centre of mass at all storey levels, for first distribution of lateral load. From the presented results it can be noticed that at the fixed base models the eccentricity has highest value at the first storey and it decreases over height.

Conversely, at the base isolated models the eccentricities are highest at the top level and at the first storey are lower as far as 25 times with respect to fixed base models. From the graph it can be noted that all base isolated models comply the condition $e_{ox,i} < 0.3 \cdot r_{x,i}$ at every story level, therefore they are characterized as regular in plan.

## 5 Conclusion

From the presented results obtained from the performed analysis it can be concluded that the choice of method for determining of centre of stiffness and torsional radius in multi-storey structures has a great influence in the further process of designing seismic resistant structures. By increasing of RC wall width the eccentricity of each level is also increasing. Eccentricities at fixed base models are the largest on the first floor. Conversely, at the base isolated models the eccentricities are the largest at the top level and at the first storey are lower as far as 25 times with respect to fixed base models, which is an indicator of the advantages of the base isolated structures in case of irregularity in plan.

## References

[1] CEN: European Standard EN 1998-1: 2004 Eurocode 8: Design of structures for earthquake resistance, Comite Europeen de Normalisation, Brusells, 2004.

[1] Fajfar, P., Marusic, D., Perus, I.: Torsional Effects in the Pushover – Based Seismic Analysis of Buildings, Journal of Earthquake Engineering, 9 (2005), pp. 831 – 854.

[1] IStructE: Manual for the seismic design of steel and concrete buildings to Eurocode_8, The Institution of Structural Engineers, London, 2010.

[1] OASP: Greek Code for Seismic Resistant Structures – EAK2000, Organization for Earthquake Resistant Planning and Protection, Ministry of Environment Planning and Publik Works, Greece (OASP), 2003.