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Beauty of the Canvas Aspect Ratios 1.357 and 1.441

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Abstract

Recently, a collection of more than 223 thousand paintings have been analyzed and it was established that the average aspect ratio for portraits is 1.357:1, and for landscape-oriented paintings is (close to) 1.441:1. Using the wisdom of the crowd theory, these two numbers should be related to some universal beauty that surpasses individual personal preferences. We show that indeed these values are related to important mathematical proportions (arithmetical mean, Kepler triangle, golden section) and that the difference between aspect ratios of vertically and horizontally oriented paintings is related to the peripheral vision field. These aspect ratios can be used by painters and frame manufacturers to amplify the beauty of artistic compositions taking into consideration the psychology of perception –our ability to innocuously register proportion as beauty. Very few real numbers are so special, that they should be widely known in the artistic world (e.g. golden ratio). It might be that these two numbers could deserve such status.

1 Introduction

The goal of this paper is to try to determine if there is an optimal canvas aspect ratio. This problem is closely related to the long-standing problem of determining if there is aesthetically the most pleasing aspect ratio of the rectangle sides. Fechner [9] introduced three ways to approach this problem almost 150 years ago [13]:

1) “the method of choice (Wahl), in which subjects choose, from among a number of alternatives, the item that they like (or dislike) the most;

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2) the method of production (Herstellung), in which subjects are asked to draw, or otherwise create, an object of a certain kind that has features or proportions they find most agreeable (or disagreeable);

3) the method of use (Verwendung), in which the experimenter examines preexisting objects of the kind being studied, and determines whether they conform to certain hypotheses about the determination of aesthetic pleasure.

In the same paper, Fechner concluded that the most beautiful rectangle is a rectangle with the aspect ratio of its sides equal to the golden ratio - two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities. It is denoted by φ in honor of Greek sculptor Phidias (480-430 BC), painter and architect in whose artworks lots of instances of the golden section have been detected and it is equal to $\varphi = \frac{\sqrt{5}+1}{2} \approx 1.618$. Euclid (300BC) started to study its mathematical properties and since antiquity, this number has attracted scientists and artists –it appears in the abundance of natural phenomena and many artworks incorporate it [5, 16, 17].
Fechner' s observation will steer up quite a controversy. Very early, Scripture [22] and Woodworth' s [28] interpretation of the results of Thorndike [26] strongly supported Fechner' s findings (nice illustrative graphs of both findings can be seen as Figure 2 and Figure 3 in [3]. Throughout the years many more scientists also supported this result [4, 15, 18, 21]. Partial support and partial opposition to this finding can be found in the paper [7] where Fechner' s observation was concurred only for introverts, but disputed for extroverts.
On the other hand, strong opposition to these results can be found in the papers of Godkewitsch [12] and Green [14] which provided strong arguments that the methodology of the previous research had some flaws. Russel [20] finds the average, median, and mode of preferred aspect ratios of experimental subjects all different from the golden section.
As a summarized conclusion of previous research, one may cite Green [13]: "I am led to the judgment that the traditional aesthetic effects of the golden section may well be real, but that if they are, they are fragile as well. Repeated efforts to show them to be illusory have, in many instances, been followed up by efforts that have restored them, even when taking the latest round of criticism into account."
Researchers of the most beautiful aspect ratio for the rectangle almost exclusively used the first two methods that Fechner proposed (Wahl and Herstellung). In this paper, we will analyze what can be learned from the third method (Verwendung). We will compare the results of these findings with

the mathematically indicated ideal aspect ratio of canvas and obtain an almost perfect match.

2 Methods

2.1 Wisdom of the crowd and beyond

Jeroen Visser analyzed a collection of more than 223 thousand paintings: 112476 portraits and 110611 landscape-oriented paintings in his master thesis [27]. He obtained that the average aspect ratio of a portrait is (height to width) 1.357:1 and that the average ratio of a landscape-oriented painting is (width to height) 1.45:1.

At first sight, these two numbers do not have any obvious significance. E.g. the only ratio offered for canvas prints by Saatchi art are $\frac{1}{1} = 1$, $\frac{5}{4} = 1.25$, $\frac{4}{3} \approx 1.333$ and $\frac{3}{2} = 1.5$, none of which is too close to these two numbers.

However, the Wisdom of the crowds theory suggests differently. This theory starts with the famous Francis Galton observation of a cow-weight guessing contest [11] where the average guess of cow's weight was within 0.8% of cow's weight although individual guesses were mostly quite different from correct weight. The basis of this theory is the law of large numbers first discovered by Cardano in the 16th century which implies that if errors of individual guesses are bounded and independent, then the error of the average will be extremely small (for simple proof see [6] and for more details about this theory see [25] and references within).

Suppose that a human's sense of beauty comprises individual preference and the objective concept of beauty. Applying the same methodology as Galton did –i.e. averaging the senses of beauty of multiple individuals, one might be able to distill an objective concept of beauty. If so, then ratios $1.357 : 1$ and $1.45 : 1$ (or ratios very close to these numbers) might have some special status.

Let us note that in fact, the analysis of the average aspect ratio of paintings goes beyond the wisdom of crowds. Namely, the reasonably small price of the ticket in Galton's experiment is quite different from producing a painting in which the artist invests considerable time and effort. Hence, instead of guessing, we could say that painters bet big time on the aspect ratios (among many different painting elements) and experiments where significant betting is included give even better results than simple averaging. There is an old saying "Put your money where your mouth is" (and it is used in the title of the paper Fang, Stinchcombe, and Whinston [8] that analyzes such phenomena).

2.2 Mathematical analysis

When observing a rectangle, there are three lengths that one may observe: each of two sides and a diagonal. Let us denote the smaller side by s, the larger by l, and the diagonal by d. Two most simple regularities that three numbers can show is that the middle one is either the arithmetic mean (average) or the geometric mean of the other two, i.e. that

$$l = \frac{s+d}{2} \text{ or } l = \sqrt{sd}.$$

Considering that the Pythagorean theorem implies that $d^2 = l^2 + s^2$, simple calculation shows that arithmetic mean implies that

$$s : l : d = 3 : 4 : 5.$$

This is the smallest Pythagorean triple (a reader interested in Pythagorean triples is referred to [23]. Calculation using geometric mean implies that

$$s : l : d = 1 : \sqrt{\varphi} : \varphi,$$

where $\varphi = \frac{\sqrt{5}+1}{2} \approx 1.618$ is the golden ratio. The triangle with side ratios $1 : \sqrt{\varphi} : \varphi$ is called the Kepler triangle. Kepler was fascinated by this peculiar connection of golden ratio and Pythagorean theorem stating that: "Geometry has two great treasures: one is the theorem of Pythagoras, the other the division of a line into extreme and mean ratio. The first we may compare to a mass of gold, the second we may call a precious jewel" [10].
Hence, we have two important rectangles –one with the side ratio $\sqrt{\varphi} \approx 1.272$ and the other with the side ratio $\frac{4}{3} \approx 1.333$. Note that when an observer faces a piece of art, he does not face unframed canvas, but framed canvas. Hence, one might wonder what should be the ratio of unframed canvas that would produce a ratio of framed canvas 1.272 and 1.333. Obviously, this depends on the width of the frame. Hence, one might ask if the canvas is given is there some method of calculating the optimal width of the frame?
One of the possible ways is to put such a frame that incorporates nice proportions. We have three areas: unframed canvas area (let us denote it by u), frame area (let us denote it by f), and total framed painting area (which is equal to $u + f$). Hence, it can be required that

$$u : f = (u + f) : u.$$

Then, $(u + f) : u$ is the golden ratio. Such choice of frame width is already advised by many makers of custom-made frames (e.g. see [1, 2]). Let us call such framing golden framing. Let us define the function $g : \langle 1, +\infty\rangle \to$

$\langle 1, +\infty\rangle$ that we call the golden lift. Its input argument is ratio x of the longer side of a framed picture to its shorter side and its result is the ratio $g(x)$ of the longer side of the canvas to the shorter when golden framing is applied. Let us denote the width of the frame by $w(x)$. It holds that

$$[g(x) + 2w(x)] \cdot [1 + 2w(x)] = \varphi\,[g(x) \cdot 1]\,; \tag{2.1}$$

$$[g(x) + 2w(x)] : [1 + 2w(x)] = x. \tag{2.2}$$

From (2.2), we get

$$w(x) = \frac{g(x) - w}{2x - 2}.$$

Inserting in (2.1), we get

$$\left(g(x) + 2\frac{g(x) - x}{2x - 2}\right)\left(1 + 2\frac{g(x) - x}{2x - 2}\right) = \varphi \cdot g(x)$$

which is equivalent to:

$$(g(x))^2 - \left(2 + \varphi x + \frac{\varphi}{x} - 2\varphi\right) g(x) + 1 = 0.$$

Solving it for $g(x)$, we get:

$$g(x) = \frac{2 + \varphi x + \frac{\varphi}{x} - 2\varphi \pm \sqrt{\left(2 + \varphi x + \frac{\varphi}{x} - 2\varphi\right)^2 - 4}}{2}.$$

The solution with the minus sign gives $g(x) < 1$ which is incorrect. Hence,

$$g(x) = \frac{2 + \varphi x + \frac{\varphi}{x} - 2\varphi + \sqrt{\left(2 + \varphi x + \frac{\varphi}{x} - 2\varphi\right)^2 - 4}}{2}.$$

Now we have $g(\sqrt{\varphi}) \approx 1.357$ and $g(\frac{4}{3}) \approx 1.441$.

3 Results and discussion

Note that $g(\sqrt{\varphi})$ coincides in all four digits with the average ratio of the longer to shorter side of the canvas for portraits calculated in [27]. This kind of agreement can hardly be accidental. Hence, two completely different approaches provide the same result. This number $g(\frac{4}{3})$ is 0.6% less than the average of the ratio of longer to shorter side for all landscape-oriented paintings. Hence, there is almost a perfect match. One reason for the small discrepancy is that the aspect ratios of the painting are sometimes divided into three groups (portrait, square, landscape), and sometimes in four groups (portrait, square, landscape, and panoramic). Hence, the wisdom of

the crowd might not imply the result of 1.45, but slightly less than 1.45. This would be in accordance with the calculated value of $g(\frac{4}{3}) \approx 1.441$.
The remaining interesting question is why there is a difference between the average ratio of portraits and landscape-oriented paintings and the answer might be rooted in the shape of the peripheral visual field. Namely, the peripheral visual field is horizontally elongated (see Figure 6 in [19] or detailed review [24] and references within). Hence, for landscape orientation, an obvious choice is the larger of these two possibilities (hence indeed: 1.441), and for portrait smaller of these values (hence indeed: 1.357).

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